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ANALYSIS OF MATHEMATICS WORD PROBLEM-SOLVING ABILITY AMONG 9-13-YEAR OLD MAJORITY AND LEARNING DISABLED STUDENTS

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THEORETICAL BACKGROUND

Significant efforts are made in Hungary as well as internationally toward developing mathematics education to be able cope with the challenges of the changing world (*Csapó*, 2005; *OECD*, 1999, 2004, 2007, 2009). From the several objectives of mathematics education, the *National core curriculum* (*OKM*, 2007) places major emphasis on the everyday applicability of the acquired mathematics knowledge, on independent thinking, on the development of problem apprehension, on the acquisition of problem-solving strategies, and on the development of productive skills instead of delivering large amounts of factual knowledge. This emphasis results in increased attention to problems embedded in realistic contexts besides equations and algorithmic, symbolic calculation exercises, which problems both satisfy the "reality-modelling" and the "problem-solving" requirements (*Felvégi*, 2005; *Molnár*, 2006).

The solution of word problems is one of the fields of mathematics where the concrete use of abstract concepts can be demonstrated and chance of direct application and real problems testing may take place. The research on the solution of word problems is considered to be one of the most intensive orientations of mathematics education today (*Csíkos*, 2002, 2003; *Csíkos* and *Dobi*, 2001; *De Corte*, 1997; *Józsa* and *Székely*, 2004; *Kelemen*, 2006, 2007; *Kelemen* and *Csíkos*, 2008; *Kelemen*, *Csíkos* and *Steklács*, 2005; *Sternberg* and *Ben-Zeev*, 1998; *Vidákovich* and *Csapó*, 1998; *Wyndhamn* and *Säljö*, 1997).

Any task in mathematics is considered to be a word problem that is textually represented and essentially requires the application of some field of mathematics in the process in its solution. Hence, word problems can be constructed in various field of mathematics. The deep structure of the task that are rooted in number theory, higher algebra or function analysis, but combinatorics, probability calculation or geometry might also be mentioned as topics of elementary and secondary level word problems. However, the following investigations and analyses are based on a narrower sense of the concept. It involves only those word problems that are built on a deep structure of basic arithmetic operations. These might also be referred to as arithmetic word problems (*Csíkos*, 2003).

Mathematics word problem-solving ability is defined here as the complex ability needed for successfully solving various mathematics word problems. In *József Nagy's* psychic compontent-system of personality (2000) this ability can be represented as a complex ability of cognitive existential competence. *Vidákovich* and *Csíkos* (2009) use mathematical knowledge as a generic term and perceive it as a set of all the components that affect mathematics learning and the application of mathematics both in- and outside school. Although they fail to describe the situation and relation of components or define subgroups within this generic term, it is assumed that they attribute a special role to the subset that incorporates all the component needed for solving word problems. This assumption in based on the authors' choice of presenting the investigations of word problems aiming to enhance mathematical problem-solving as a pillar of research on the cognitive components of mathematics knowledge in their comprehensive description of the research in this field.

Since *Verschaffel, De Corte* and *Lasure*'s publication in 1994 “realistic" flag before certain mathematical word problems in the relevant literature means that the solution of

certain tasks require the students to transfer their experiences from everyday situations. Thus, the "realistic" nature of a task refers to its content, i.e. to the knowledge elements and their application required by the successful solution of the problem.

The findings of the research on realistic mathematics word problems, deriving from a number of international and Hungarian (*Csíkos*, 2003; *Kelemen*, 2004) investigations, has shown that only a maximum of 20-50 percent of the students provide realistic responses to realistic problems, depending on the task. These international findings are widespread and consistent enough to justify the thesis that students tend to ignore realistic considerations and everyday experiences in the process of solving mathematics word problems.

Although there is no dispute about the complexity of word problem-solving ability, the separation of the involved knowledge elements and skills is a complicated issue. One of the major questions of educational ability tests is if and how complex abilities might be traced back to basic skills. *Csíkos* and *Dobi* (2001) claim that there is no available system that would intend to explicitly describe mathematical skills and abilities integrating their components into one coherent system.

The present thesis intends to capture those attributes of mathematics word problems that affect the success of solution. Therefore, in the investigation of word problem-solving ability, three levels are distinguished concerning the structure of the tasks: (a) the deep structure of the task which covers the mathematical, generally arithmetic, operation involved in the problem, (b) the content level which is about the scenario of the task, and (c) the context of solving the problem which describes the environmental factors characteristic of the problem-solver and the circumstances in the process of solving the task.

Mathematical deep structure is meant here to cover the mathematical elements which, translated into mathematical language, represent an exercise of algebraic and arithmetic symbols. In today's mathematics instruction word problems are grouped according to their deep structure and used as means of repetitive training after the introduction of a certain operation (addition, subtraction, multiplication, division). Despite the fact that the students come across word problems grouped according to their deep structures at school, they have difficulties when having to do this grouping themselves (*Kercood*, *Zentall* and *Lee*, 2004). By the time of entering grade 5, students do not find it difficult to solve arithmetic operations, provided they are represented with figures and mathematical symbols, however when embedded in word problems the same tasks are solved successfully at a much lower rate. This fact is confirmed by several research in which traditional tasks that could be solved by one or two arithmetic operation were presented together with realistic word problems (*Verschaffel*, *De Corte* and *Lasure*, 1994; *Csíkos*, 2002, 2003; *Russer* and *Stebler*, 1997; *Kelemen*, 2004).

The appearance of the content of a task is an attribute of the wording of the task. The teaching of thinking – or any other cognitive functioning – in many cases is executed at an abstract content plane, assuming that thereby the acquired processes would not be attached to any specific content, resulting in easily transferable knowledge. However, there is little evidence that these programs would really be successful in the long term (*Csapó*, 1999a). In contrast with this, it is widely known that from isomorphic tasks those that have more familiar content are easier to solve (*Eysenck* and *Keane*, 1997).

The context of word problems mean the circumstances of their presentation, i.e. the verbal and non-verbal communication concerning the situation. According to *Butterworth* (1993), context has no generally accepted definitions, but it is mostly understood as the physical, social and cultural characteristics of the appearance of a task.

A number of international (Japanese, Dutch, Irish and German) experiments are known *(Verschaffel, Greer and De Corte*, 2000) which attempted to increase the rate of

students' realistic responses in paper-and-pencil tests by changing the context of the tasks. These context changes that were limited to the paper-and-pencil testing process remained ineffective, they did not result in a significant increase in realistic student responses (*Reusser* and *Stebler*, 1997; *Verschaffel*, *De Corte* and *Lasure*, 1999; *Verschaffel* and *De Corte*, 2000).

The first and the largest yet Hungarian investigation was conducted by *József Nagy* (1973). His research accomplished the systematization of word problems that can be solved by one or two basic operations and explored the structure of different ways of solving them. As a result of the research, a parameterized task bank was created and the development of word problem-solving ability was explored (*Nagy*, 1973).

The next important investigations concerning the cognitive knowledge base of the mathematics word problem-solving ability were conducted just before the turn of the millennium by the Institute of Education at the University of Szeged. The sample used in the studies was nationally representative regarding the size of the place of residence and school-type (*Vidákovich* and *Csapó*, 1998). These investigations can be considered as the adapted reproductions of the 1972 studies adjusted to the social and educational changes of the past 15 years.

The following stage of Hungarian research in the field is related to *Csaba Csíkos* (2002). Shortly after the turn of the millennium, connecting to the international research direction (*Greer* 1993*; Verschaffel, De Corte* and *Lasure,* 1994; *Reusser* and *Stebler*, 1997) he investigated students' performance on realistic mathematics word problems and their realistic responses in these tasks. The findings, confirmed by a number of international and Hungarian studies (*Csíkos*, 2003; *Kelemen*, 2004), has shown that only a maximum of 20-50 percent of the students provide realistic responses to realistic problems, depending on the task.

Successful solution of word problems require a high level of development of several skills and abilities. The optimal level of acquisition is essential in case of basic mathematical skills, such as counting skill (*Nagy,* 2007), but problem representation also has a significant role in the process (*De Corte*, 2001, *Mayer* and *Hegarty*, 1998). The decoding of textual information obviously requires further the development of word reading skill, and the understanding and interpretation of the text itself necessitates an appropriate level of text comprehension (*Józsa*, 2006).

Students' motives, self-concept and attitudes are also to be mentioned when talking about the success of mathematical word problem solving (*Dobi*, 2002). In many cases, students claim themselves incompetent in solving the tasks only on the basis of looking at it and hence refuse to start deliberating on the problem. This aversive motive, which emerges in several students, hinders the development of word problem-solving ability. One of, or perhaps the most important element of successful mathematics education is the elimination of these aversive motives and negative attitudes (*Józsa*, 2007).

The most comprehensive set of individual problems regarding school learning and the cognitive development of the majority is referred to as learning barriers by *Gaál* (2000). She distinguishes three separate groups within this broad concept.

Those children and adolescents who learn with heavy difficulties permanently due to development disorders in their learning skills are called learning disabled, on the basis of *Mesterházi's* (1997) definition. Learning disability is a permanent learning barrier that is present in several skill domains. This concept was introduced into scholarly discourse based on the experiences deriving from monitoring learning problems (*Mesterházi*, 2008), which explains the nature of the learning disability umbrella term. A subset of children with learning disabilities is the group of students considered mild mental disabled by expert and rehabilitation committees. Although they are not exclusively disadvantaged in

their intellectual abilities, mild mental disabled children are characterized by slower developmemt of cognitive functions and an IQ score between 50-69 (*Mesterházi* and *Gerebenné*, 1998). Nowadays, experts attention has turned from the promotion of biological criteria to learning abilities (*Papp*, 2004). As a consequence, the concept of the learning disability is expanded to integrate those students who do not satisfy the criteria of mild mental disability, but permanently struggle with difficulties in learning. This group includes a very large proportion of children from disadvantaged or low-SES families (*Gaál*, 2000). The following sections of the present thesis provide an overview of the special education areas in mathematics instruction. Although there are increasing efforts toward the synchronization of Hungarian and international terminology in the literature (*Fejes* and *Szenczi*, 2009; *Gordosné*, 2004; *Kelemen*, *Szenczi* and *Fejes*, 2009; *Mesterházi*, 1998), the matching of the contents of concepts is still ambiguous.

The crossroads of mathematics and special education is the meeting point of several research areas and terminological approaches. The group of poor performers in mathematics, the issue of dyscalculia, the international term mathematics learning disabilities (MLD) and the cognitive development of children with learning disabilities are all related to this field. Dyscalculia is the one most often dealt with in Hungarian literature of these problems. MLD is an internationally popular term, which does not yet have an exact counterpart in Hungarian literature. The large-sample Hungarian research on the mathematics achievement and mathematical development of students with learning disability has not been published.

RESEARCH OBJECTIVES AND METHODS

The present thesis explores the characteristics of mathematics word problem-solving ability, applying quantitative as well as qualitative research methods. It aims at enriching the circle of educational basic research by the discovery and analysis of the features and development of mathematics word problem-solving ability. In Hungary no preliminary research programme has ever intended to explore the structure and functioning of mathematics problem-solving ability together with the entire spectrum of its external and internal influencing factors.

The discussion about the solution of mathematics word problems is centred around three aspects. (1) The effect of the three levels of task structure – deep structure, content and context – on the process of solving the problems will be analysed. (2) The relationship of mathematics word problem-solving to other mathematical skills, word reading, text comprehension, intelligence as well as to affective factors, motives and social background variables will be tested. (3) The special features of the mathematics word problem-solving ability of students with learning disabilities will be explored and compared to majority students.

The presented work of research is connected to those efforts that attempt to bridge special and majority education – in accordance with international trends – applying identical means and methods in empirical assessment, if possible. Furthermore, the research aims at facilitating the successful application of mathematics word problems in public education. One condition to this success is to know the characteristics of the development of students' word problem-solving ability, the age specialities of this development and its relationships with other cognitive and affective domains.

Hypotheses

The empirical research questions of the present thesis are primarily connected to basic research by nature. The hypotheses concerning the results of the investigations are centred around six issues.

1. *The investigation of mathematics word problem-solving ability*

a) The level of mathematics word problem-solving ability development can be well assessed with paper-and-pensil test in grades 3-7 among the majority as well as among the learning disabled population.
b) It is possible to construct a measurement instrument that is suited for comparatively investigating majority and learning disabled students' mathematics word problem-solving ability in grades 3-7.

2. *The development of mathematics word problem-solving ability*

a) The mathematics word problem-solving ability of majority students develop intensively in grades 3-7.
b) The mathematics word problem-solving ability of learning disabled students develop intensively in grades 3-7.
c) The mathematics word problem-solving ability of learning disabled students fall behind that of majority students.

3. *The influence the structure of mathematics word problems to the solution of these problems*

a) The deep structure of mathematics word problems affect the solution of these problems.
b) The content of mathematics word problems affect the solution of these problems.
c) Realistic content of a mathematics word problem reduces the success of solution.
d) The context of solving mathematics word problems affect the solution of these problems.

4. T*he Criterion system model*

a) It is possible to construct and empirically justify a model that describes the essential cognitive factors of the proper functioning of mathematics word problem-solving ability.
b) Word reading, text comprehension and counting skill are essential prerequisites of mathematics word problem-solving ability.
c) It is possible to find a certain reference-level in word reading, test comprehension and counting skill below which mathematics word problem-solving ability can not function properly.
d) Criterion system model is valid for the population of learning disabled students as well.

5. *Affective influencing factors of mathematics word problem-solving, motives and beliefs*

a) Students' attitude towards mathematics significantly declines between grades 3 and 7.
b) Students' attitude towards mathematics correlates with their mathematics word problem-solving ability and counting skill..
c) Mathematics self-concept significantly declines betweem grades 3 and 7.
d) Mathematics self-concept correlates with mathematics word problem-solving ability and counting skill in grades 3-7.
e) Students possess certain beliefs concerning mathematics word problem-solving that hider realistic considerations in grades 5 and 7.
f) Grade 5 students are able to reliably compare the level of mathematics word problems' difficulty in case of isomorphic deep structures.
g) Grade 5 students find those tasks least interesting from isomorphic word problems which contain only figures and symbols in their description.

6. *The role of background variables on mathematics word problem-solving ability*

a) The level of parents' education affects the development of mathematics word problem-solving ability.
b) Disadvantages status affects the development of mathematics word problem-solving ability.
c) There are no significant gender differences in the development of mathematics word problem-solving ability.
d) The level of the mother's education is the factor exerting the biggest influence on mathematics word problem-solving ability from family background variables.

Research

The central study of the present thesis is a multi-variable, large-scale study. On the one hand, it provides the possibility of analysing the relationships of word problem-solving ability to other cognitive domains (word reading, text comprehension, counting skill, IQ), on the other hand, due to the composition of its sample, it is possible to compare the achievement of students with learning disabilities to that of majority students and to analyse the process of solving the tasks that is characteristic to the subsamples. The study was part of a large-scale research organized by the *Bárczi Gusztáv Faculty of Special Needs Education of Eötvös Lóránd University* under the supervision of *Krisztián Józsa*.

The general aim of the research project was to explore the social and cognitive skill development as well as the learning motivation of students with learning disabilities, and also to compare of the development processes of majority students and those with learning disabilities. The abundance of measurement instruments provide the possibility of exploring the system of relations between skills, IQ and learning motivation as well as of analysing the effects of background variables (family background, disadvantages status, minority status). In the project where the test of mathematics word problem-solving was administered to the students, more than 800 variables were measured. This provides the possibility of analysing the relationships of word problem-solving ability to other cognitive and non-cognitive domains, on the other hand, it was possible to assess the differences of majority and learning disabled students' development.

The cross-sectional data collection of the large-scale study covered the cohorts of grade 3, 5, and 7. Altogether 1670 students participated in the study, 730 of whom struggled with learning difficulties. 940 were majority students.

Some analyses in this thesis consider only majority students, ignoring the disabled subsample. The sample of 940 majority students fulfil the requirements of generalizability in terms of the results. First, the size of the sample is large enough to provide large enough subsamples when divided according to school grades, fulfilling the criteria of analyzability in educational research (*Csíkos*, 2009). Second, the majority sample is nationally representative in respect of the mothers' level of education. The learning disabled subsample is presumably also representative in this respect.

There are no Hungarian large-scale results available concerning the mathematical knowledge of students with learning disability. The following analyses shed light on the effect learning disability has on mathematics word problem-solving ability and counting skill. The study design had the advantage of applying the same instruments in the data collection both in case of majority students and with the learning disabled group, thus making the two set of data comparable in the field of mathematics word problem-solving ability and counting skill.

The central large-scale study is supported by three supplementary studies in the empirical part of the present thesis, all of which supplementary studies focus on the solution of mathematics word problems, yet approaching to the issue from diverse perspectives.

The research called „Realistic mathematics" investigated the successfulness of 126 students from grade 7 in solving realistic mathematics word problems with modified contents. In addition to the comparison of the results of traditional and realistic word problems, the study provides the possibility of analysing students' beliefs attached to the solution of word problems.

The study entitled „Thinking strategies – student interviews" used different methods from the other studies described, since all the other research collected data quantitatively with paper-and-pensil tests and questionnaires, but in this study interviews were conducted, measuring students qualitatively as well. During the interviews students had to solve complex, realistic problems while thinking aloud. 20 students from grade 4 participated in the study. The interviews lasted for 15 minutes each and were videotaped for the sake of exact analizability. The study contributed to the investigations of the content and contextual level of realistic word problems.

The third supplementary study was entitled „Student beliefs". It focused on students' beliefs concerning mathematics word problems regarding their level of difficulty and interest. Grade 5 students were asked to rate the level of difficulty and interest of various, textually differently composed tasks with isomorphic deep structures. Altogether 3650 questionnaires provided the data for analysis.

The multi-aspect investigation of mathematics word problem-solving ability is connected to the educational research on students' cognitive development, skill and ability characteristics. The essential knowledge elements of solving mathematics word problems are presented integrated to one single model that is based on the relationships of mathematics word problem-solving ability and its cognitive influencing factors. The model is called *Criterion system model*, which indicates that the essential knowledge elements of solving mathematics word problems can be represented as criteria which might be described as a system. *Criterion system model* was tested and justified empirically.

The assessment of students with learning disabilities together with majority students contributes to the results of comparative research and to the relationship between majority and special education as well (*Józsa* and *Fazekasné*, 2006a, 2006b, 2009).

Furthermore, the research aims at facilitating the successful application of mathematics word problems in public education. The role of mathematics word problems in serving the aims of contemporary public education is increasing, hence comprehensive investigations in this domain together with the multi-aspect analysis of the issue are essential for research-based educational policy. A precondition of this is to explore the characteristics of students' mathematics word problem-solving ability. The findings of the present studies might shed light on the dissonance between educational practice and educational research, indicating the possibilities of productive changes.

SUMMARY OF THE RESULTS

The thesis provides an overview of the theoretical framework of mathematics word problem-solving ability and analyses the nature of this ability empirically with 9-13-year-old students from multiple aspects. One central and three supplementary studies created the possibility of comprehensive analysis. The central study, giving the essence of the thesis was conducted with a sample of 940 majority students and 730 students with learning disabilities. Alongside the development and age specialities of mathematics word problem-solving ability, its relationship with other cognitive skills, intelligence, affective factors, motives, beliefs and background variables was also examined. Furthermore, a model was constructed based on the empirical results, which integrates the essentially important knowledge elements of mathematics word problem-solving ability. Additionally, the thesis covers the exploration and analysis of learning disabled students word problem-solving ability as well as its comparison to the majority students' achievements. The following section presents the research results along the above described hypotheses.

(1a) The large-scale study aimed at investigating the level of development of mathematics word problem-solving ability in grades 3, 5 and 7 using paper-and-pensil tests. The reliability indices of the different grades indicated that the test functioned properly in all three subsamples. The research project involved students with learning disabilities as well. The reliabilities of this assessment were also satisfactory in all three grades.

(1b) The same instruments were used in measuring the learning disabled and the majority subsamples. The high reliability indices of the two subsamples and the distributions of test achievements suggest that such an instrument had been constructed that provides a good estimation of the level of mathematics word problem-solving ability development both of majority and of learning disabled students in the involved age groups. Furthermore, the test makes it possible to compare the characteristics of the two subsamples.

(2a) The development of mathematics word problem-solving ability was assessed with a cross-sectional design among 9-13-year-olds. The data collection was conducted in grades 3, 5 and 7. According to the results of the majority subsample, this ability develops intensively in the investigated age period. The test results indicate significant differences between the individual age groups.

(2b) The development of word problem-solving in this period is obvious in case of learning disabled students as well. Significant increase was found in the achievement between the individual age groups. It can be concluded that mathematics word problem-solving ability is subject to intensive improvement between grade 3 and grade 7.

(2c) Students with learning disabilities performed significantly poorer in all three age groups than majority students. The achievement of grade 7 learning disabled students still fell below that of grade 3 majority students. In case of majority students, the most intensive development was detected between grades 3 and 5, while it was found to be more moderate between grades 5 and 7. On the contrary, students with learning disabilities develop more intensively in the latter period. The measured increase in their achievement between grades 5 and 7 was three times higher than that between grades 3 and 5.

Based on the results it can be assumed that the two subsamples show different characteristics in terms of development, there is a time incongruency between their intensive periods. The level of learning disabled students' mathematics word problem-solving ability displays a minimum 4-year delay in average compared to the majority students.

(3a) Three levels were distinguished in the structure of mathematics word problems: (a) the deep structure of the task, (b) the content of the task and (c) the context of solving the task. The deep structure of the problem is related to the abstract mathematics that provides the substance of the task. Problems with different deep structures display different student achievements. Certain deep structures revealed larger differences between the age groups, whereas others showed insignificant differences. Presumably deep structures influence the success of solution at such a degree that deep structure-characteristic streams of development could have been found.

(3b) In the studies presented in this thesis, mathematics word problems were analysed from the perspective of their realistic nature, which is one of the aspect of the content of tasks. There are more possible dimensions for characterizing the content level of the tasks. In case of word problems, such content characteristic might be the situation presented in the wording of the problem or the content of school subjects as well as the differenced of inside and outside school contents.

(3c) The effect that realistic modifications of the task content has on the success of solving the task was measured on a sample of grade 7 students. They were administered a test with tasks of traditional and of realistic contents having isomorphic deep structures, in identical context. Correct solution of the problems required considerations of real life regularities in each case. The level of successful solutions were found to be lower in every single task when it was of realistic content compared to the traditional content. Hence, it can be declared that in the special case when the modification of task content points toward realism, it influences the chances of successful solution significantly negatively.

(3d) From the several possible modifications of context level, the effects and regularities of in- and outside school context were measured. Although it is impossible to create outside school context inside the school building, still students beliefs about school mathematics and outside school problems of mathematical nature were assessed in grade 7, using the method of individual interview and think aloud protocols. Students were prompted to declare the characteristics of their problem-solving processes by having to think aloud both in case of school tasks and outside school problems. Data derived from the interviews suggest that students are aware of the unsaid rules of problem-solving in school context. On the other hand, they are also aware of the possibilities of problem-solving in everyday life context as well, however, the two contexts are sharply discriminated in them, and they are frustrated when asked to cross the border between the two well-separated system of rules, i.e. if they are to use the features of everyday problem-solving in school context.

(4a) As an integration of cognitive influencing factors of mathematics word problem-solving ability the Criterion system model (*Figure 1*) was constructed which models the functioning of the essential cognitive prerequisites of mathematics word problem-solving ability. Those skills and abilities that are needed for word problem-solving ability are called filters in the model. Four filters were identified altogether: word reading, text comprehension, problem representation and counting skill. The characteristics of the model were justified by the empirical investigation of three of the four filters, word reading, text comprehension and counting skill.

(4b) It has been justified that word reading, text comprehension and counting skill influence the success of solving mathematics word problems in accordance with the characteristics of the filters defined in the model.

(4c) The major feature of a filter is its criterion character. This means on the one hand that a critical level of development can be described in case of skill and ability development which has to be achieved in order for the mathematics word problem-solving ability to function successfully. The analysis of the effect word reading, text comprehension and counting skill have on mathematics word problem-solving, it was found that they all satisfy the conditions described in the definition. A reference level was outlined from the data in all three domains, above which students were much more likely to perform well on the word problem-solving test than below it.

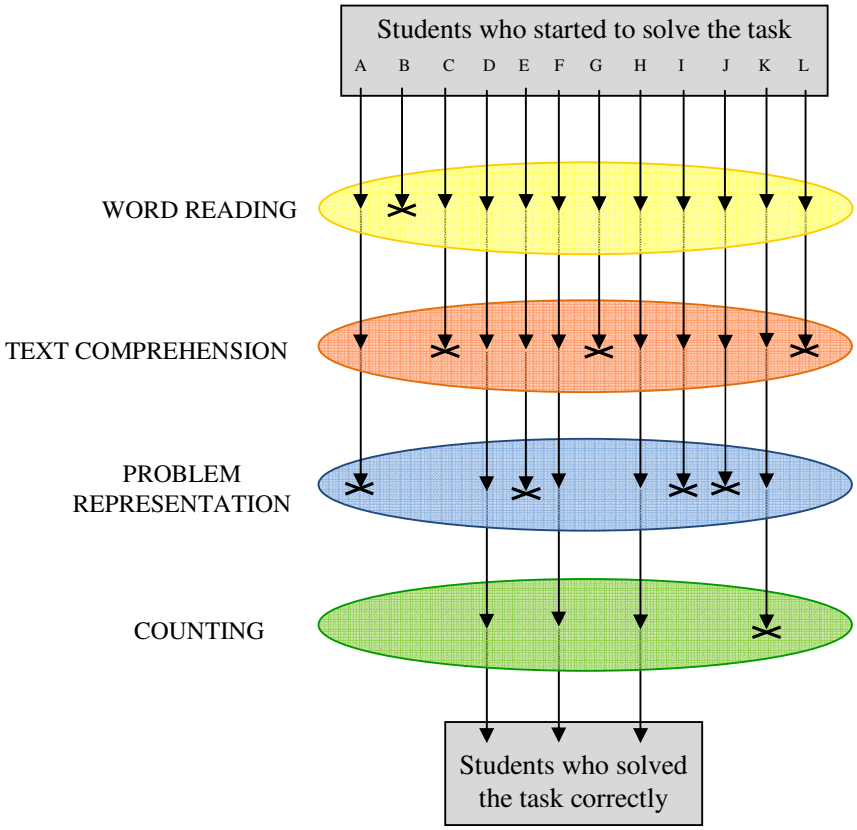

Figure 1
Criterion system model

The criterion nature means on the other hand that above the reference level filters do not have a significant influence on achievement, which means that among the students achieving the reference level of the filter, one can find good as well as medium or bad performance on the mathematics word problem test. Empirical evidence suggests that word reading, text comprehension and counting skill satisfy this criterion, thus their position as filters in the Criterion system model is empirically justified. As a consequence of the criterion nature of filters it can be declared that the conditions for successfully solving mathematics word problems are ensured only with the simultaneous achievement of all filter's reference level. It has been justified that the word problem-solving performance of those students who managed to achieve above the reference level in all three filter criteria

is significantly higher than that of those who failed to reach the reference level at least in one filter criterion. From the perspective of mathematics word problem-solving it is irrelevant how many of the criteria are not fulfilled.

The role of these filters were tested with various types of tasks in different age groups. It was found that word reading and text comprehension have a constantly important predicting factor while the role of counting skill decreases with age. Analysis according to test types also confirmed that the rate of importance is different in case of different filter criteria. It means that the reference levels of filter criteria are related to the characteristics of the sample and the word problems used in the assessment.

(4d) Criterion system model was further tested with the sample of learning disabled students. The validity of the model was confirmed by this sample as well, though filters had a less determinative role than in case of the majority sample. It is assumed that cognitive functions of these children and their relations are rather to be assessed involving more than three variables.

(5a) From the affective influencing factors of mathematics word problem-solving ability students attitudes towards mathematics are presented. A significant decline is detected in this field with age. This decrease in students' attitudes are relatively constant from grade 3 to grade 5 and further to grade 7.

(5b) The attitude towards mathematics was found to have a significant, but weak correlation with word problem-solving ability. The correlation is stronger in the higher grades, however, no significant differences were found in the level of mathematics word problem-solving ability in the sample grouped according to students' attitudes – ignoring the two extremes – even in grade 7. Students' attitude towards mathematics correlates significantly stronger with counting skill than with the achievement on the mathematics word problem-solving test in all three grades.

(5c) Mathematics self-concept was also found to decrease with age. The negative trend was manifested in significantly lower scores in higher grades. The decrease between grades 5 and 7 is higher than between grades 3 and 5.

(5d) The correlations between mathematics self-concept and mathematics word problem-solving ability are different in all three age groups, but display an increasing tendency. The correlations are significant in all cases, however correlation is rather weak in case of grade 3, and increases to be medium strong by grade 7. The correlation coefficients of counting skill are not significantly different in different grades, the relationship is rather weak.

(5e) Students' beliefs on mathematics were tested indirectly, with the help of a realistic word problem test in grade 7. Inhibitory beliefs were associated with the characters of the tasks. In this way, the rate of realistic responses to the given task provided information about the functioning and strength of the associated belief as well. The results justified the presence of all identified student beliefs in the solution process.

(5f) The study aiming at exploring mathematics word problems' level of difficulty measured the beliefs of students about problems with isomorphic deep structures in grade 5. Each task was based on the 8+4 basic mathematical operations. Students found arithmetic tasks described with figures and symbols easiest, followed by the worded but uncontextualized arithmetic expressions. They believed that word problems embedded in realistic situations were more difficult, and found realistic problems with a number of unneeded information the most challenging. Their beliefs displayed a similar sequence according to the number of characters in the task, which indicates that they might consider the length of task descriptions when judging the level of difficulty.

(5g) The study about task interest attributed to word problems was also conducted in grade 5. Students had to decide how interesting they find four differently composed word

problems that had isomorphic deep structures. Each task was based on the $\frac{1}{4}+\frac{1}{2}+\frac{1}{3}$ operations. Students voted least interesting the problem described with only figures and symbols of operations. This was followed by the problem embedded in everyday situation and the one that personified numbers in an imaginary world. They found a task with geometric content the most interesting, however, only this one was illustrated with a hand- and computer-drawn picture and did not include any figures in the task description.

(6a) The large-scale study was conducted as part of a bigger research project, which created the possibility of analysing the influence of several background variable on mathematics word problem-solving. Hence, the roles of parents' level of education, roma ethnicity, disadvantaged status, gender differences, mathematics grades and intelligence were considered. The first results presented are related to the influence of the mother's level of education on students' mathematical word problem-solving ability.

Mathematics word problem-solving ability is highly correlated with the mother's level of education in all three grades, yet the degree of correlation is different, getting stronger with the increase of age. Significant differences were found between the word problem-solving achievement of grade 7 students from the two lowest, two medium and the highest category according to their mothers' level of education.

(6b) The mathematics word problem-solving ability of disadvantaged students was significantly poorer than that of their non-disadvantaged mates. The difference between the two subsamples was found constant across grades. The achievement of disadvantaged students reflect two years delay compared to the level of non-disadvantaged students' word problem-solving ability.

The collected data provided the possibility of analysing the results from a cultural perspective as well. Information about the roma ethnicity of students was provided by their teachers on a teachers' questionnaire. In each grade roma students performed significantly poorer than majority students and, unlike in case of the disadvantaged subsample, the differences increased with age. By grade 7, roma students are falling behind majority students by more than twice the degree of the grade 3 difference. The curves representing the achievement of the two subsamples reflect the image of opening scissors. The common effects of disadvantaged status and roma ethnicity on mathematics word problem-solving ability were analysed by multifactor ANOVA. It showed that the two variables are not independent from each other and that disadvantaged status becomes more influential in terms of mathematics word problem-solving by grade 5.

(6c) No significant gender differences were found either in the results of the mathematics word problem-solving test or of the counting skill test in either grades.

(6d) The joint analysis of the variables of family and social background tried to answer the question which factor has a deeper effect on the development of mathematics word problem-solving ability. In grade 3, the most important influencing factor turned out to be the level of the mother's education. In higher grades, the effect of the level of the mother's qualification was less prominent, while the major influence was exerted by the disadvantaged status. 20 percent of students' achievement in each grade is determined by the level of their mothers' qualification, their roma ethnicity and disadvantaged status together.

Possibilities of further research include the description and systematization of the potential modifications of the levels in the triarchic (deep structure, content and context) structure of mathematics word problems. Therefore such investigations of task characteristics would be desirable where problems are modified in one level besides the strictly fixed other two.

The justification of the Criterion system model would also call for additional empirical evidence. Designing and conducting studies that focus on problem representation is considered to be an important task and great challenge of further research. The generalization of the model would require the exploration of other cognitive domains that play a role in solving mathematics word problems and the integration of these factors into the model.

The biggest work to do in research of mathematics word problem-solving is in the field of learning disabilities. The description of learning disabled students' cognitive skills and abilities on the basis of empirically collected data has become widespread in Hungary in the recent years. Diagnostic methods of assessing skills development are fundamental for successful integration. Yet, prior to them exploratory large-scale basic research are needed in every field, e.g. in the field of mathematical knowledge, to identify the standards of the entire student population providing a basis for comparison between majority and learning disabled students.

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