# Quantum Teleportation

# on Generic Hilbert Spaces

# and in Optics

summary of the
PhD thesis
of

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## I. **The subject of the dissertation and the aims of the research**

The end of the 20th century can be regarded as the second golden age of quantum mechanics, the core of all fields of modern physics. After its birth in the 1920's, it induced a revolution of physics, and became a frontier in the human thinking about the nature of reality. The theory was extremely successful: it yielded an appropriate model for the structure and interactions of matter, generation and absorption of light, and many other applications. There was a price to be paid for these achievements. Due to the more sophisticated mathematics employed, it has lost its understandability for everyday (non-physicist) people. On the other hand, even those who "speak" the language of mathematics still do not possess the opportunity of visualization or even imagination of phenomena in argument. And though quantum mechanics provides us with recipes to connect the abstract theory with the actual measurement results, it leaves a lot of counterintuitive ideas, interpretational or even more general philosophical problems, open questions behind. Most of these originate from two main concepts: *quantum measurement* and *entanglement*.

Were these problems, which are so much in the center of today's research interest, not strikingly visible from the very beginning? They were there definitely, but probably in spite of their extreme conceptual importance, they must have been regarded as marginal questions from a practical, experimental point of view. Some peculiar features of quantum entanglement for instance were very well understood in 1935 by Einstein, Podolsky and Rosen, but probably not many physicists thought of these as something one can see in the laboratory, or even apply for certain purposes.

The feasibility of entangled quantum states had another important consequence: the concept of quantum information was born. A new field of information and computation theory appeared. A theory of quantum communication and computation is being developed. This is a generalization of classical information theory in which the unit of information is a *qubit*, a two level quantum system, which is a quantum generalization of the classical bit. In contrast with the latter case, which can take values of either $0$ or $1$, a qubit can be in any complex superposition of the basis states $|0\rangle$ and $|1\rangle$ spanning its state space. The appearance of the concept of *information* in physics is also promising, especially in understanding fundamental problems of quantum mechanics. The non-existence of “quantum hardware”, a painful fact for quantum information scientists, provides experimental physics and technology with a great challenge.

In my thesis I have investigated a basic constituent of quantum communication and computation, namely *quantum teleportation*. This is one of the most interesting applications of the so-called Einstein-Podolsky-Rosen (EPR) correlations occurring in bipartite quantum systems. Quantum teleportation itself means a disembodied transport of a quantum state with the aid of a classical communication channel and EPR correlations, suggested by Bennett in 1993. It relies on two key concepts: quantum entanglement and quantum measurement.

In classical physics, one may measure and record all data of a physical system. In possession of these data, it is possible at least theoretically to prepare a physical system, which is an exact copy of the original one. In contrast with this, quantum mechanics forbids „cloning” of physical systems. On the other hand, destroying a physical system can make it possible to regain it again at some other point of spacetime, but classical information is insufficient for this purpose. EPR correlations are needed as an entangled resource of this kind of quantum communication.

A part of my considerations are not close related to any particular realization of quantum teleportation, hence I shall call these considerations concerning “generic Hilbet spaces”. The other part of them are related more closely to some of the optical realizations.

One of my aims was to describe quantum teleportation in several different formalisms. Each of these descriptions reveals different interesting details of the phenomenon. These results may be helpful in our getting used to behavior of quantum systems. I have also investigated a classical limit of quantum teleportation to benefit the understanding of the quantum case. I have described finite dimensional quantum teleportation in terms of discrete Wigner-functions defined by Wootters, and I gave a direct description of the Braunstein-Kimble teleportation scheme for continuous quantum variables by the application of certain coherent-state superpositions. I also gave a compact operator description of certain teleportation schemes using antilinear representation of bipartite states, which is the starting point in defining relative state representations in quantum channel theory.

Modern optical techniques have a lion’s part in the experimental investigations of quantum information communication, especially nonlinear optical processes, detectors, and interferometric arrangements containing these devices. Most experimental realizations of quantum teleportation are good examples of these. The other part of my results is related to certain optical arrangements.

I have studied optical schemes, where modes of the electromagnetic field with a very low photon number interfere. Specifically, I have generalized a quantum state preparation method, which is mainly based on an optical implementation of quantum teleportation. I have found, that the well-known application of $SU(2)$ symmetry in description of beam splitters helps in understanding the operation of this interferometric scheme. Motivated by that, I described passive lossless linear optical six-ports with the aid of $SU(3)$ symmetry.

## II. New scientific results

In the following I summarize the main points I have considered. The list of related publications follows this summary page.

1. I have shown, that the Bennett scheme for quantum teleportation of a qubit is the quantum mechanical generalization of the “one-time-pad” or Vernam’s cipher. The latter is the theoretically most reliable way of secure classical communication, but as it requires a huge number of correlated true random bits, it is rarely used in practice.

   The understanding of the relation between the “one-time-pad” and teleportation helps understanding the nature of the quantum process. The classical-to-quantum transition is illustrated by an analysis of a gedanken experiment. [II]

2. I have considered probabilistic quantum teleportation processes utilizing pure states as entangled resource and projective measurement, on finite dimensional Hilbert-spaces. There exists a description of states of bipartite quantum systems in terms of antilinear operators. Maximally entangled states are described by antiunitary operators in this formalism. Antiunitary operators are generally related to time reversal symmetry in physics. This is another interesting application of these operators, having several advantages, such as independence of the Hilbert-space basis. This representation is deeply related to the relative state representation of quantum states and channels. Utilizing the antilinear representation, I have expressed quantum teleportation as a quantum operation or quantum channel in a compact, convenient form. [VI]

3. A prevalent alternative description of quantum phenomena is the application of quasiprobability distributions. One of the most frequently used quasiprobability distribution is the Wigner function. Quantum teleportation of continuous variables (i.e. oscillator states) was first described in this formalism, while Bennett's original quantum teleportation scheme was introduced in terms of quantum states directly, on a finite dimensional Hilbert-space.

   There are several ways of defining Wigner functions for finite dimensional Hilbert spaces. Choosing one of these formalisms, I have described Bennett's quantum teleportation in terms of discrete Wigner-functions. This new description clearly demonstrates the process of teleportation in the discrete phase space, revealing the connection with teleportation of continuous quantum variables. This was one of the first applications of discrete Wigner functions in quantum information context. [IV]

4. Low-dimensional coherent-state representations have proven to be useful in describing nonclassical states of light. Their multimode generalization and application to quantum communication arrangements is therefore tempting.

   I have shown, that the Braunstein-Kimble teleportation scheme for teleportation of a single-mode electromagnetic field can be described in terms of coherent-state superpositions in a direct way. The entangled states utilized in the process are described by means of conjugate coherent state pairs, which also appear in the theory of quantum cloning. In the superposition integral it suffices to integrate over two real dimensions instead of four. The method applied for this is an alternative to the number-state or phase-space representation method in the treatment of multi-mode fields. [III,V,VII.]

5. By application of optical multiports (arrangements of beam-splitters and phase shifters) and photodetection, it is possible to prepare certain quantum states of light.

   I have presented such an interferometric scheme, the “generalized quantum scissors”, which is capable of generating superpositions of few-photon Fock-states. The device teleports the first few Fock-components of an electromagnetic field mode. Its design relies on the application of beam splitters with optimized parameters. [I]

6. It is known, that both $\mathfrak{su}(2)$ and $\mathfrak{su}(3)$ Lie-algebras have bosonic realizations. Based on this, on may describe beam-splitters in terms of $SU(2)$ symmetry.

   I have analyzed the teleportation aspect of the “generalized quantum scissors” by exploiting $SU(2)$ symmetry of beam splitters. That turned out to be useful in understanding the operation of the device. I also gave a description of passive, lossless, linear optical six-ports (tritters) in terms of $SU(3)$ symmetry, in analogy with $SU(2)$ theory of beam splitters. [I,VIII]

These results have already received some citations. Up to my knowledge, my pioneering Wigner-function description of teleportation led to an introduction of an alternative finite dimensional Wigner function concept, which is more convenient in quantum information purposes. The application of coherent-state superpositions was further developed: a complete basis was found to describe arbitrary two-mode states, and a description of quantum teleportation in a noisy environment was given. The teleportation of a superposition of vacuum and one-photon state has been realized recently, and the possibility of experimental realization of quantum scissors was also investigated by several authors since.

## III. List of related publications

I. M. Koniorczyk, Z. Kurucz, A. Gbris and J. Janszky: *General optical state truncation and its teleportation*, Phys. Rev. A **62**, 013802 (2000).

II. M. Koniorczyk, T. Kiss, and J. Janszky: *Teleportation: from probability distributions to quantum states*, J. Phys. A (Math. Gen.) **34** pp. 6949-6955 (2001).
`arXive:quant-ph/0011083`

III. J. Janszky, M. Koniorczyk and A. Gbris: *One-complex-plane representation approach to quantum teleportation*, Phys. Rev. A **64** 034302 (2001).

IV. M. Koniorczyk, V. Bužek, and J. Janszky: *Wigner-function description of quantum teleportation in arbitrary dimensions and continuous limit*, Phys. Rev. A **64** 034301 (2001).
`arXive:quant-ph/0106109`

V. J. Janszky, A. Gbris, M. Koniorczyk, A. Vukics and P. Adam: *Coherent-state approach to entanglement and teleportation*, Progress of Physics, **49** pp. 993-1000 (2001).

VI. Z. Kurucz, M. Koniorczyk, and J. Janszky: *Teleportation with partially entangled states*, Progress of Physics **49** pp. 1019-1025 (2001).

VII J. Janszky, A. Gábris, M. Koniorczyk, A. Vukics and J. Asboth: *One-complex-plane representation: a coherent-state description of entanglement and teleportation*, J. Opt. B.: Quantum Semiclasss. Opt. **4**, pp. S213-S217 (2002).

VIII. M. Koniorczyk and J. Janszky: *Photon number conservation and photon interference*, invited paper in “First International Workshop on Classical and Quantum Interference”, Jan Peřina, Miroslav Hrabovský and Jaromir Křepelka Editors, Proceedings of SPIE **4888**, pp 1-8. (2002).
`arXive:quant-ph/0110170`