QUANTUM TESTS FOR NON-INERTIAL AND GENERAL RELATIVISTIC EFFECTS

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I. Introduction

For the most part, the thesis is concerned with the possibility of gravitational and non-inertial effects on spin-½ particles.

It is beyond any doubt that any experiment performed on the Earth is done under the effect of gravity. Gravitation, one of the four basic interactions governing the structure and behaviour of the world, has by far the smallest effect. The gravitational coupling is so weak that the gravitational attraction between two protons is 10^{39} times less than the electric repulsion; an alternative comparison would show that the order of magnitude of the gravitational term in the Hamiltonian is approximately 10^9 times less than the rest mass energy term when a particle in the Earth's field is considered. For these reasons it is a standard practice to ignore the effect of gravity in case of laboratory experiments or equivalently, to apply the physical theory in flat, rather than in curved space. It sounds even more plausible that this procedure is above all justifiable in the quantum regime: whoever thought that gravitational effects would manifest themselves at the quantum level?

Contrary to the latter it is now more than twenty-five years since Colella, Overhauser and Werner (COW) succeeded in performing an experiment which detected gravitational effects in neutron interferometry, where the behaviour of neutrons were simultaneously governed by gravity and quantum mechanics. The formula describing the phase shift was the first to contain both Planck's constant and gravitational acceleration. By giving direct evidence of gravitational effects the COW experiment established a demand for describing general relativistic effects on quantum systems, leading to a conceptual problem when one tried to combine general relativity and quantum mechanics.

The COW phase shift was explained using Newtonian mechanics, and this was a satisfactory approximation, within the order of the experimental error involved. Since 1975, however, new experiments have been suggested, which are expected to increase the accuracy by a factor of 10^{10}, which will take us to the regime where relativistic corrections become relevant.

The effect of Earth on quantum systems has been examined in a series of experiments: it was demonstrated that neutrons are subject to gravitational acceleration. The COW neutron interference experiment proved, that gravity and quantum mechanics play an essential role, simultaneously. Non-inertial effects (caused by rotation and acceleration of the setup) have also been studied experimentally. These experiments, although involving atoms and neutrons, were not sensitive to spin effects, thus it was not necessary to use the Dirac equation in analysing them. In the studies of Xia and Wu, however, it was found that the spin polarisation of spin-½ particles in the Earth's field is also affected, therefore in the analysis of experiments involving elementary particles in the Earth's field the use of the Dirac equation is necessary.

II. Method of Investigation

The aim of the thesis is to find a method of analysing the behaviour of spin-½ particles in an Earth-bound laboratory, focusing on gravitational and non-inertial effects. The method used is to find the Dirac Hamiltonian in various circumstances, modelling the effect of Earth on elementary particles. These Hamiltonians are then being compared to give the applicability of the models.

In the thesis it is explained how to write the Dirac equation in general Riemannian spaces using Weyl's tetrad formalism. This method is described in detail, as are the problems of using different coordinate sets and moving reference frames, working out the correct measure for spatial integration and the transformation for the proper non-relativistic limit. Various methods of finding the connection coefficients are compared and summarised.

III. Scientific Results

1. To avoid further misinterpretation of results, already found in the literature, a procedure is given for finding the **form of the momentum operator in coordinate representation in curved spaces.** [1]

2. **The effect of a stationary gravitational source on a spin-½ particle** is examined via calculating the Dirac Hamiltonian in Schwarzschild space-time. Interpretation of the terms in the Hamiltonian and their order of magnitude are given for a thermal neutron in the Earth's gravitational field.

[1]

3. **A test of the medium strong equivalence principle** is gained by comparing the Hamiltonians describing the effects of a gravitational field and an accelerated frame on spin-½ particles. It is found that the difference between the Hamiltonians consists of quantum terms only, including a spin term.

[1]

4. A proper analysis of terrestrial experiments takes into account the rotation of Earth in two aspects. First, the space-time outside a rotating gravitational source is described by the Kerr metric. Second, a laboratory on Earth is rotating relative to the fixed stars, causing non-inertial effects. The **Dirac Hamiltonian** is derived **for a particle in a rotating frame in Kerr space-time.** [2]

5. **The Earth's effect on spin-½ particles were studied in three cases:** rotating frames in Kerr and Schwarzschild space-times and an accelerated

frame in Minkowski space-time. Comparison showed that (for a thermal neutron) the difference between rotating frames in Kerr and Schwarzschild space-times becomes apparent at energies of 10^{-19} eV. The difference between an accelerated frame in Minkowski space-time and a frame in Schwarzschild field is of the order of 10^{-11} eV. [1, 2]

6. Experimental data has previously been compared with a model using Newton's theory of gravity. From a fundamental point of view, however, this is somewhat unsatisfactory. Also, new experiments, already proposed, are expected to increase the accuracy such that a higher order description is required. The theoretical expression for the **phase shift** is therefore **derived on a general relativistic basis.** [3]

IV. Final remarks

The research presented in the thesis is an attempt to apply the laws of quantum mechanics and general relativity simultaneously for describing spin-½ particles; but as all models in Physics it is by no means finished.

An obvious extension of this study would be to carry out the above calculations up to higher orders that would enable one to describe situations where the mass or the angular velocity of the gravitating source is more substantial than in case of the Earth. This could be applied to situations such as rotating black holes or the Big Bang.

As mass curves space-time in GR, it is suspected that spin (the other conserved quantity connected to space-time) might also have a dynamical manifestation; this would be a generalisation of GR and the idea of torsion. Theories of torsion have a long history, but the attempts to verify it experimentally on the cosmological scale have not yet been successful. The extension of the model including torsion might suggest a possible test for it in the quantum domain.

V. Related publications

Condensed version of new results has been published in

[1] Katalin Varjú, Lewis H. Ryder, „*The effect of Schwarzschild field on spin 1/2 particles compared to the effect of a uniformly accelerating frame*", Physics Letters A, **250**, no. 4-6 (1998) pp. 263-269

[2] Katalin Varjú, Lewis H. Ryder, „*Comparing the effects of curved space and noninertial frames on spin 1/2 particles*", Physical Review **D 62**, no.2 (2000) 024016

[3] Katalin Varjú, Lewis H. Ryder, „*General relativistic treatment of the Colella-Overhauser-Werner experiment on neutron interference in a gravitational field*", American Journal of Physics, **68**, no. 5 (2000) pp. 404-409