Critical behaviour of the two-dimensional models in the presence of inhomogeneous perturbations

PhD thesis

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Preliminaries and method of investigation

In all fields of physics homogeneous systems have the simplest properties and thus play a particular role. On the other hand, all real systems are inhomogeneous in one way or another. The most common example is the almost always inevitable presence of impurities or other lattice defects in crystals. The theoretical description of this kind of feature of real systems established the concept of disorder, and started the investigation of disordered models.

Among disordered models special attention has been paid to those models which exhibit a phase transition. In this case the basic question is, what consequences the introduction of disorder has on the properties of homogeneous systems. According to experiences the disorder can change the nature of phase transition in different degree. It is possible that the disorder may cause a shift in transition temperature. Or it may change the order of the phase transition, namely the first order transition can turn into a continuous one. In all cases one wants to know which universality class determines the behaviour of the disordered model.

The heuristic relevance-irrelevance criterion for the stability of the pure system fixed point against the disorder was worked out by Harris in the case of site diluted Ising model. The idea can be generalized to other kind of random models. The Harris criterion predicts the randomness to be relevant if the critical exponent of the specific heat in the pure system is negative. In this case the critical behaviour of the system is described by a new disordered fixed point. Intensive numerical and analytical work has been started to clarify the universality class of various disordered models.

Perhaps one of the simplest examples for inhomogeneity is the system with free surface. To obtain it from a homogeneous system one has to cut an infinite number of bonds. The case of planar surfaces was first treated for two-dimensional Ising model and subsequently studied in great detail. It was found that, connected with the surface, there is a set of critical exponents with values different from those in the bulk.

Random fields are one of the possible sources of disorder. Placing a plane of random fields in the bulk often constitutes a relevant perturbation as it is the case for two-dimensional Ising model. Thus a new fixed point appears, which controls the local critical behaviour. This fixed point is expected to be a surface one since random fields tend to destroy the local order and the bulk defect then acts as an effective cut. On the other hand random fields can be present at the surface. In this case the relevance of perturbation depends on the decay exponent of the local order parameter correlations in the pure system.

The presence of disorder in the couplings may cause differences between the critical properties of the random model and the pure one. In most studies on disordered systems the parameters are expected to be independent and identically distributed random variables among which there is no correlation. However, there are systems in which the random variables exhibit some kind of correlations. This effect can change the random universality class of the phase transition.

There is another class of inhomogeneity in which two or more systems meet at an interface, each having different type of bulk and surface critical properties. If the critical temperatures of the two subsystems are significantly different, the nature of transitions at the interface is expected to be the same as for a surface. The local critical behaviour at the interface can be more complex if the critical temperatures of the subsystems are the same or if their difference is much smaller than the deviation from their mean value. In this case an interplay or competition between the two different bulk and surface critical behaviours can result in a completely new type of interface critical phenomena.

In this work we have examined the previously mentioned inhomogeneities in two dimensions. We have used the Harris criterion to show the relevance of perturbations. The analytical results were justified by confronting with extensive Monte Carlo simulations.

In the case of random surface field we have used the simple spin flip Metropolis algorithm for non zero reduced temperature. The largest linear size of system we have investigated was 1000, and over 1000 runs with different realization of the random surface field have been performed. In the critical point we have used the Swendsen-Wang algorithm to avoid the critical slowing down.

We have analyzed the impact of disorder in the couplings with Wolff algorithm, which is an effective cluster algorithm. For different strength of disorder we have made a finite-size scaling, where the larges system size was 116×116. During the investigation of the conformal properties of the model we have used the Swendsen-Wang algorithm, because of the fixed boundary conditions. For the measuring we have used over 60 thousand CPU times.

In the case of the investigation of the critical behaviour at the interface between two systems we have used the mean field theory. Solutions of the mean-field equations are obtained by adjusting the order-parameter profiles of the two semi-infinite subsystems through the introduction of appropriate extrapolation lengths on the two sides. Numerically testing these results we have used a special form of the Swendsen-Wang algorithm, where the energy corresponding to interface was built into the acceptance ratio of the flipping of a cluster.

Results

A/1. First we have introduced a random field at the surface of the two-dimensional Ising model. The random field had zero mean and its variance was used to characterize the strength of disorder. This is a marginally irrelevant perturbation. Using the replica trick and the differential renormalization group one can determine the leading logarithmic correction of the critical exponent of the surface magnetization in the weak disorder limit. Measuring the temperature dependent effective exponent we have studied the theoretical result with extensive Monte Carlo simulations. For various strength of disorder we have showed the leading logarithmic correction in different ways [1].

A/2. We have also examined the previous model at the critical point. In this case the effective surface exponent, which determines the initial increase of the magnetization profile, depends on the finite size of the system. Using the relation between the correlation length and the reduced temperature we have calculated the temperature dependent effective surface magnetization exponent from the measurement of the size dependent effective surface exponent. These results fit very well to the theoretical curve close to the critical point [1].

B/1. Besides the above we have studied the critical behaviour of the square lattice Ising model with biaxially correlated disorder. Due to our parameterization the system remained self-dual. So at the exactly known critical point we have studied the finite-size scaling behaviour of different magnetic and thermal quantities. We have determined their critical exponents. Our results were consistent and using the scaling laws we have calculated a lot of critical exponents of the model [2].

B/2. We have investigated numerically the conformal properties of the two-dimensional Ising model with correlated disorder. At the critical point using the Schwarz-Christoffel transformation we have determined the scaling dimension of the energy density. The value was in accord with the previous result we got from the finite-size scaling. The magnetization density profiles have shown strong correction to scaling. Nevertheless, the asymptotic behaviour was in agreement with the value obtained by finite-size scaling [2].

C/1. Finally we have analyzed the critical behaviour at an interface which separated two semi-infinite subsystems The two subsystems have had the same critical temperature, but they had belonged to different universality classes. By varying the interface couplings, we have monitored the order at the interface and studied the behaviour of the order parameter profile as the critical temperature has been approached. We have provided a detailed analytical solution of the problem in the framework of mean-field theory. Basically three types of interface critical behaviour have been observed [3].

C/2. We have generalized the mean-field results using phenomenological scaling consideration. The critical exponent of the interface magnetization was expressed in terms of bulk and surface exponents of the two semi-infinite subsystems. We have also discussed the smooth or discontinuous nature of the order parameter profile. These results have been tested through large scale Monte Carlo simulations, in which the critical behaviour at the interface between two-dimensional Ising, Potts and Baxter-Wu models was studied and satisfactory agreement has been found [3].

Publications

[1] M. Pleimling, F. Á. Bagaméry, L. Turban, F. Iglói, *Logarithmic corrections in two-dimensional Ising model in a random surface field*, J. Phys. A: Math. Gen. **34**, 8801-8809 (2004).

[2] F. Á. Bagaméry, L. Turban, F. Iglói, *Two-dimensional Ising model with self-dual biaxially correlated disorder*, Phys. Rev. B **72**, 094202 (2005).

[3] F. Á. Bagaméry, L. Turban, F. Iglói, *Critical behavior at the interface between two systems belonging to different universality classes*, Phys. Rev. B **73**, 144419 (2006).