In our dissertation we deal with single and double trigonometric series with nonnegative coefficients which are absolutely convergent. Since absolute convergence implies uniform convergence, the sums of these trigonometric series exist at each point and they are continuous functions. It is well known that if a trigonometric series is uniformly convergent, then it is the Fourier series of its sum. We investigate the order of magnitude of the coefficients in order that the sum of the trigonometric series in question belong to one of the Lipschitz classes $\mathrm{Lip}\,\alpha$, $\mathrm{lip}\,\alpha$ and Zygmund classes Λ_*, λ_* in the case of single series; and to one of the classes $\mathrm{Lip}(\alpha,\beta)$, $\mathrm{lip}(\alpha,\beta)$, $\Lambda_*(1,1)$, $\lambda_*(1,1)$ in the case of double series. We give necessary and sufficient conditions in terms of the coefficients.

1. Known results: single trigonometric series

Given a sequence $\{a_i : i = 1, 2, \ldots\}$ of nonnegative numbers such that

$$\sum_{i=1}^{\infty} a_i < \infty, \tag{1.1}$$

then the sum of the cosine series

$$\sum_{i=1}^{\infty} a_i \cos ix =: f_c(x) \tag{1.2}$$

and the sum of the sine series

$$\sum_{i=1}^{\infty} a_i \sin ix =: f_s(x) \tag{1.3}$$

are continuous functions, due to uniform convergence. Thus, the series in (1.2) and (1.3) are the Fourier series of their sums $f_c(x)$ and $f_s(x)$, respectively.

Relying on the relevant papers by R.P. Boas [2] and J. Németh [11] we briefly summarize the basic definitions and theorems relating to cosine and sine series.

First we present the results on the Lipschitz classes $\mathrm{Lip}\,\alpha$ and $\mathrm{lip}\,\alpha$.

We say that a 2π-periodic function ϕ belongs to the *Lipschitz class* $\mathrm{Lip}\,\alpha$ for some $\alpha > 0$ if there exists a constant $C = C(\phi)$ such that for all x and h we have

$$|\phi(x + h) - \phi(x)| \le C|h|^{\alpha}.$$

Furthermore, we say that a 2π-periodic function ϕ belongs to the *little Lipschitz class* $\operatorname{lip}\alpha$ for some $\alpha > 0$ if

$$\lim_{h\to 0} |h|^{-\alpha}[\phi(x+h) - \phi(x)] = 0$$

uniformly in x.
It is clear that $\operatorname{lip}\alpha \subset \operatorname{Lip}\alpha$ for all $\alpha > 0$.

We note that in the case $\alpha = 0$ we may define $\operatorname{Lip} 0$ as the class of bounded functions and the class $\operatorname{lip} 0$ as the class of uniformly continuous functions. It is well known that if $\phi \in \operatorname{Lip}\alpha$ for some $\alpha > 1$ or if $\phi \in \operatorname{lip}\alpha$ for some $\alpha \geq 1$, then ϕ is a constant function. Therefore, in the sequal we consider the classes $\operatorname{Lip}\alpha$ for $0 < \alpha \leq 1$ and $\operatorname{lip}\alpha$ for $0 < \alpha < 1$.

R.P. Boas proved in 1967 the following theorems.

Theorem 1.1 (Boas [2]). *Let* $\{a_i : i = 1, 2, \ldots\}$ *be a sequence of nonnegative numbers such that condition* (1.1) *is satisfied and denote by* f *either* f_c *or* f_s *defined in* (1.2) *and* (1.3). *Then* $f \in \operatorname{Lip}\alpha$ *for some* $0 < \alpha < 1$ *if and only if*

$$\sum_{i=m}^{\infty} a_i = O(m^{-\alpha}), \qquad m = 1, 2, \ldots \tag{1.4}$$

or equivalently

$$\sum_{i=1}^{m} i\, a_i = O(m^{1-\alpha}), \qquad m = 1, 2, \ldots\ . \tag{1.5}$$

Theorem 1.1 remains valid if we replace $\operatorname{Lip}\alpha$ by $\operatorname{lip}\alpha$ and "O" by "o".

If $\alpha = 1$, then the situation is more complicated, since in this case the cosine and sine series behave differently. In particular, in case $\alpha = 1$ Theorem 1.1 and the equivalence of conditions (1.4) and (1.5) are no longer true. It is not difficult to check that in case $\alpha = 1$ condition (1.5) implies (1.4), but not conversely. However, the stronger condition (1.5) is necessary and sufficient for a sine series to belong to the class $\operatorname{Lip} 1$, as the following theorem shows.

Theorem 1.2 (Boas [2]). *Let* $\{a_i : i = 1, 2, \ldots\}$ *be the same as in Theorem* 1.1, *and let* f_s *be defined in* (1.3). *Then* $f_s \in \operatorname{Lip} 1$ *if and only if*

$$\sum_{i=1}^{m} i\, a_i = O(1), \qquad m = 1, 2, \ldots\ .$$

We note, that the sufficiency part of Theorem 1.2 was proved by Boas in a different way. Our method provides a new proof of the sufficiency part of Theorem 1.2.

Boas [2] presents examples of cosine series to demonstrate that in case $\alpha = 1$ the stronger condition is not necessary, while the weaker condition (1.4) is not sufficient for $f_c(x)$ to belong to the class $\mathrm{Lip}\,1$. However, condition (1.4) together with another condition are necessary and sufficient for $f_c(x)$ to belong to $\mathrm{Lip}\,1$, as the following theorem shows.

Theorem 1.3 (Boas [2]). *Let* $\{a_i : i = 1, 2, \ldots\}$ *be the same as in Theorem* 1.1. *If* $\alpha = 1$, *then* $f_c \in \mathrm{Lip}\,1$ *if and only if*

$$\sum_{i=m}^{\infty} a_i = O(m^{-1}), \qquad m = 1, 2, \ldots$$

and

$$\sum_{i=1}^{m} i\, a_i \,\sin ix = O(1), \qquad m = 1, 2, \ldots$$

uniformly in x.

Before stating the next theorems we recall the definitions of the Zygmund function classes $\Lambda_*(\alpha)$ and $\lambda_*(\alpha)$.

The *Zygmund class* $\Lambda_*(\alpha)$ $(\alpha > 0)$ consists of all continuous, 2π-periodic functions ϕ for which there exists a constant $K = K(\phi)$ such that for all x and h we have

$$|\phi(x+h) + \phi(x-h) - 2\phi(x)| \le K\, h.$$

Furthermore, the *little Zygmund class* $\lambda_*(\alpha)$ $(\alpha > 0)$ consists of all continuous, 2π-periodic functions ϕ for which

$$\lim_{h \to 0} h^{-1}[\phi(x+h) + \phi(x-h) - 2\phi(x)] = 0$$

uniformly in x.

It is plain that $\lambda_*(\alpha)$ is a proper subclass of $\Lambda_*(\alpha)$. It is well known that

$$\Lambda_*(\alpha) = \mathrm{Lip}\,\alpha \quad \text{and} \quad \lambda_*(\alpha) = \mathrm{lip}\,\alpha, \quad \text{if} \quad 0 < \alpha < 1.$$

On the other hand, we have

$$\Lambda_* := \Lambda_*(1) \supset \mathrm{Lip}\,1 \quad \text{and} \quad \lambda_* := \lambda_*(1) \supset \mathrm{lip}\,1.$$

Boas proved that for cosine series the weaker condition (1.4) in the case $\alpha = 1$ is necessary and sufficient to belong the class Λ_*, which is narrower than all the classes $\mathrm{Lip}\,\alpha$ with $0 < \alpha < 1$ and broader than the class Lip 1. J. Németh (see [11, Theorem 3] in the special case when $\omega_1(h) := h$) proved the same necessary and sufficient condition in the case of sine series, too.

Theorem 1.4 (Boas [2], Németh [11]). *Let* $\{a_i : i = 1, 2, \ldots\}$ *and* f *be the same as in Theorem* 1.1. *Then* $f \in \Lambda_*$ *if and only if*

$$\sum_{i=m}^{\infty} a_i = O(m^{-1}), \qquad m = 1, 2, \ldots .$$

Theorem 1.4. remains valid if we replace Λ_* by λ_* and "O" by "o".

Thus, the sine and cosine series behave in the same way in the case of Λ_* in spite of the fact that they behave differently in the case of the narrower class Lip 1.

We note that it is enough to require the fulfilment of conditions like (1.4)–(1.5) only for large enough m, say $m > m_0$, where m_0 is a positive integer.

2. New results: double trigonometric series

Now, let $\{a_{ij} : i, j = 1, 2, \ldots\}$ be a double sequence of nonnegative numbers such that

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} < \infty, \tag{2.1}$$

then the double cosine series

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} \cos ix \cos jy =: f_{cc}(x, y), \tag{2.2}$$

the double sine series

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} \sin ix \sin jy =: f_{ss}(x, y) \tag{2.3}$$

and the double cosine-sine series

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij}\, \cos ix\, \sin jy =: f_{cs}(x,y) \tag{2.4}$$

converge uniformly, and in particular, their sums f_{cc}, f_{ss} and f_{cs} are continuous functions. If we interchange the roles of cosine and sine in (2.4), then we obtain the symmetric counterpart of (2.4), that is, the sine-cosine series

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij}\, \sin ix\, \cos jy =: f_{sc}(x,y).$$

Each theorem on series (2.4) can be naturally reformulated for the latter series. Therefore, in the sequal we do not deal with this fourth series.

As in the one-variable case, we characterize those double trigonometric series whose sum belong to some double (multiplicative) Lipschitz or Zygmund class. We extend a few theorems of the previous chapter from single to double trigonometric series.

We begin with the definitions of the two-dimensional multiplicative Lipschitz classes $\mathrm{Lip}(\alpha,\beta)$ and $\mathrm{lip}(\alpha,\beta)$, where $\alpha,\beta > 0$. The definitions are due to Ferenc Móricz [8].

A continuous function $\phi(x,y)$, 2π-periodic in each variable, is said to belong to the *two-dimensional Lipschitz class* $\mathrm{Lip}(\alpha,\beta)$ for some $\alpha,\beta > 0$ if there exists a constant $C = C(\varphi)$ such that for all x, y, h and k, we have

$$|\phi(x+h,y+k) - \phi(x+h,y) - \phi(x,y+k) + \phi(x,y)| \le C|h|^{\alpha}|k|^{\beta}.$$

Furthermore, a function $\phi(x,y)$ is said to belong to the *two-dimensional little Lipschitz class* $\mathrm{lip}(\alpha,\beta)$ for some $\alpha,\beta > 0$ if $\phi(x,y)$ belongs to $\mathrm{Lip}(\alpha,\beta)$ and if

$$\lim_{h,k\to 0} |h|^{-\alpha}|k|^{-\beta}[\phi(x+h,y+k) - \phi(x+h,y) - \phi(x,y+k) + \phi(x,y)] = 0$$

uniformly in (x,y) as h and k tend to 0 independently one another.
Similarly to the one-dimensional case, we consider only the case when $0 < \alpha,\beta \le 1$.

The following theorems are motivated by the corresponding one-dimensional theorems.

The first result is the extension of Theorem 1.1 from single to double series. Since Theorem 1.1 is valid for both cosine and sine series, we expected that its extension would be valid not only for double cosine and sine series, but for the mixed cosine-sine series as well. Indeed, this is the case as the following theorem shows.

Theorem 2.1 (Fülöp [6], [7]). *Let* $\{a_{ij} : i, j = 1, 2, \ldots\}$ *be a double sequence of nonnegative numbers, such that condition* (2.1) *is satisfied, and let* f *be either* f_{cc}, f_{ss} *and* f_{cs} *are defined by* (2.2)–(2.4). *Then* $f \in \mathrm{Lip}(\alpha, \beta)$ *for some* $0 < \alpha, \beta < 1$ *if and only if*

$$\sum_{i=m}^{\infty}\sum_{j=n}^{\infty} a_{ij} = O(m^{-\alpha}n^{-\beta}), \qquad m, n = 1, 2, \ldots, \tag{2.5}$$

or equivalently

$$\sum_{i=1}^{m}\sum_{j=1}^{n} ij\, a_{ij} = O(m^{1-\alpha}n^{1-\beta}), \qquad m = 1, 2, \ldots\ . \tag{2.6}$$

Theorem 2.1 remains valid if we replace $\mathrm{Lip}(\alpha, \beta)$ by $\mathrm{lip}(\alpha, \beta)$ and "O" by "o":

Theorem 2.2 (Fülöp). *Let* $\{a_{ij} : i, j = 1, 2, \ldots\}$ *and* f *be the same as in Theorem* 2.1. *Then* $f \in \mathrm{lip}(\alpha, \beta)$ *for some* $0 < \alpha, \beta < 1$ *if and only if condition* (2.5) *or equivalently* (2.6) *as well as the following condition is satisfied:*

$$\sum_{i=m}^{\infty}\sum_{j=n}^{\infty} a_{ij} = o(m^{-\alpha}n^{-\beta}), \qquad m, n \to \infty$$

or equivalently

$$\sum_{i=1}^{m}\sum_{j=1}^{n} ij\, a_{ij} = o(m^{1-\alpha}n^{1-\beta}), \qquad m, n \to \infty,$$

where m *and* n *tend to* ∞ *independently one another.*

In the case when $\max(\alpha, \beta) = 1$ – like in the case of the one-dimensional series – the cosine and sine series behave differently. Furthermore, Theorem 2.1 and the equivalence of conditions (2.5) and (2.6) are no longer true. Namely, condition (2.6) is stronger than (2.5) if $\max(\alpha,\beta)=1$.

Next, we extend Theorem 1.2 to double sine series. Similarly to the one-dimensional case, the stronger condition (2.6) is the necessary and sufficient one for the sum $f_{ss}(x,y)$ to belong to $\mathrm{Lip}(1,1)$.

Theorem 2.3 (Fülöp [6]). *Let* $\{a_{ij} : i,j=1,2,\ldots\}$ *be a double sequence of non-negative numbers such that condition* (2.1) *is satisfied and let* f_{ss} *be defined by* (2.3). *Then* $f_{ss}\in\mathrm{Lip}(1,1)$ *if and only if*

$$\sum_{i=1}^{m}\sum_{j=1}^{n} ij\,a_{ij}=O(1), \qquad m,n=1,2,\ldots\,. \tag{2.7}$$

We observe that for double sine series condition (2.7) formally coincides with (2.6) when $\alpha=\beta=1$. It turns out that the class $\mathrm{Lip}(\alpha,1)$ for $0<\alpha<1$ can also be characterized by condition (2.6).

Theorem 2.4 (Fülöp [6]). *Under the conditions of Theorem* 2.3 $f_{ss}\in\mathrm{Lip}(\alpha,1)$ *for some* $0<\alpha<1$ *if and only if*

$$\sum_{i=1}^{m}\sum_{j=1}^{n} ij\,a_{ij}=O(m^{1-\alpha}), \qquad m,n=1,2,\ldots\,.$$

The symmetric counterpart of Theorem 2.4 gives a criterion for f_{ss} to belong to $\mathrm{Lip}(1,\beta)$ for $0<\beta<1$.

Without claiming completeness, we raise the following problems which may be the targets of further research.

(i) It is an open problem of how to characterize those double cosine series whose sum belong to one of the classes $\mathrm{Lip}(\alpha,\beta)$ when $\max(\alpha,\beta)=1$, that is, to $\mathrm{Lip}(1,1)$ or $\mathrm{Lip}(\alpha,1)$ for some $0<\alpha<1$. We recall that the characterization of single cosine series whose sum belong to $\mathrm{Lip}\,1$ and $\mathrm{Lip}\,\alpha$ for some $0<\alpha<1$ was different. Therefore, we even did not try to formulate a conjecture in the case of double cosine series.

(ii) Likewise, the characterization of the mixed cosine-sine series is also open in the case when $\max(\alpha, \beta) = 1$. Motivated by Theorems 2.1 and 2.3, we guess that

$$\sum_{i=1}^{m} \sum_{j=1}^{n} ij\, a_{ij} = O(m^{1-\alpha}), \qquad m, n = 1, 2, \ldots$$

is the necessary and sufficient condition for the sum $f_{cs}(x, y)$ of series (2.4) to belong to $\mathrm{Lip}(\alpha, 1)$ for some $0 < \alpha < 1$. We have managed to prove the sufficiency of this condition, but we were unable to prove the necessity part so far.

Next, we characterize the two-dimensional multiplicative Zygmund classes $\Lambda_*(1, 1)$ and $\lambda_*(1, 1)$. The definitions are due to Ferenc Móricz [8].

The *Zygmund class* $\Lambda_*(1, 1)$ consists of all continuous functions $\phi(x, y)$, 2π-periodic in each variable, for which there exists a constant $K = K(\phi)$ such that for all x, y, h, k we have

$$|\Delta(\phi; x, y; h, k)| \leq K\, h\, k,$$

where

$$\Delta(\phi; x, y; h, k) := \phi(x + h, y + k) + \phi(x - h, y + k)$$
$$+ \phi(x + h, y - k) + \phi(x - h, y - k) - 2\phi(x, y + k)$$
$$- 2\phi(x, y - k) - 2\phi(x + h, y) - 2\phi(x - h, y) + 4\phi(x, y).$$

The *little Zygmund class* $\lambda_*(1, 1)$ is a subclass of $\Lambda_*(1, 1)$ consisting of all those functions $\phi(x, y)$ for which

$$\lim_{h,k \to 0} h^{-1} k^{-1} \Delta(\phi; x, y; h, k) = 0,$$

uniformly in x and y, where h and k tend to 0 independently one another.

We remark that in a similar way to the one-dimensional case we can define the two-dimensional function classes $\Lambda_*(\alpha, \beta)$ and $\lambda_*(\alpha, \beta)$ $(\alpha, \beta > 0)$. Since the relations among these classes and the Lipschitz classes $\mathrm{Lip}(\alpha, \beta)$ and $\mathrm{lip}(\alpha, \beta)$ have not been known yet in the literature, we do not deal with these classes in our dissertation.

In the case of the one-dimensional Zygmund classes, the cosine and sine series behave in the same way. Therefore we expected that this will be the situation in the case of the double cosine, double sine and mixed cosine-sine series, too. Indeed, this same behavior is shown by the following theorem.

Theorem 2.5 (Fülöp [4], [5]). *Let* $\{a_{ij} : i,j = 1,2,\ldots\}$ *be a double sequence of nonnegative numbers, such that condition* (2.1) *is satisfied, and let* f *be either* f_{cc}, f_{ss} *and* f_{cs}, *where* f_{cc}, f_{ss} *and* f_{cs} *are defined by* (2.2)–(2.4). *Then* $f \in \Lambda_*(1,1)$ *if and only if*

(2.8) $$\sum_{i=m}^{\infty}\sum_{j=n}^{\infty} a_{ij} = O(m^{-1}n^{-1}), \qquad m,n = 1,2,\ldots .$$

We see that condition (2.8) formally coincides with condition (2.5) in Theorem 2.1 in the special case when $\alpha = \beta = 1$. We have noted earlier that between the two equivalent conditions that characterize the belonging to the Lipschitz class $\mathrm{Lip}(\alpha,\beta)$ for some $0 < \alpha,\beta < 1$, in the extended case $\alpha = \beta = 1$, condition (2.8) is weaker than condition (2.6) in the extended case $\alpha = \beta = 1$.

The counterpart of Theorem 2.5 for the class $\lambda_*(1,1)$ is also valid.

Theorem 2.6 (Fülöp [4], [5]). *Under the conditions of Theorem* 2.5, $f \in \lambda_*(1,1)$ *if and only if condition* (2.8) *is satisfied and*

$$\sum_{i=m}^{\infty}\sum_{j=n}^{\infty} a_{ij} = o(m^{-1}n^{-1}), \qquad m,n \to \infty,$$

where m *and* n *tend to* ∞ *independently one another.*

We note that analogously to the single series it is not difficult to check that it is enough to require the fulfilment of conditions like (2.5)–(2.7) for large enough m and n, say $m > m_0$ and $n > m_0$, where m_0 is a positive integer.

3. Auxiliary results

The following auxiliary results play key roles in the proofs of one- and two-variables Theorems.

Lemma 3.1 (Boas [2], Móricz [9]). *Let* $\{a_i : i = 1,2,\ldots\}$ *be a sequence of nonnegative numbers such that condition* (1.1) *is satisfied.*

(i) *If* $\gamma > \mu \geq 0$ *and*

$$\sum_{i=1}^{m} i^\gamma a_i = O(m^\mu), \qquad m = 1, 2, \dots \tag{3.1}$$

then

$$\sum_{i=m}^{\infty} a_i = O(m^{\mu-\gamma}), \qquad m = 1, 2, \dots \tag{3.2}$$

(ii) *If* $\gamma \geq \mu > 0$, *then the converse implication is also valid.*

Comparing statements (i) and (ii) shows that in case $\gamma > \mu > 0$, conditions (3.1) and (3.2) are equivalent. In fact, this equivalence was stated by Boas [2, Lemma 1] without proof. The inclusion of the endpoint cases (that is, when $\gamma = \mu$ or $\mu = 0$) is due to Ferenc Móricz [9, Lemma 1].

We note that in the proofs of the one-dimensional theorems Boas made use of another lemma (see [2, Lemma 2]). We have managed to simplify the proofs of these theorems so that Lemma 3.1 above was enough to complete the proofs.

The following version of Lemma 3.1 is also true.

Lemma 3.2. *Under the conditions of Lemma* 3.1*, conditions* (3.1) *and* (3.2) *are equivalent in the case* $\gamma > \mu > 0$ *when the "O" is replaced by "o" in* (3.1) *and* (3.2)*.*

We give the extensions of Lemmas 3.1 and 3.2 to double series.

Lemma 3.3 (Fülöp [6]). *Let* $\{a_{ij} : i, j = 1, 2, \dots\}$ *be a double sequence of nonnegative numbers such that condition* (2.1) *is satisfied.*

(i) *If* $\gamma > \mu \geq 0$, $\delta > \nu \geq 0$ *and*

$$\sum_{i=1}^{m} \sum_{j=1}^{n} i^\gamma j^\delta a_{ij} = O(m^\mu n^\nu), \qquad m, n = 1, 2, \dots \tag{3.3}$$

then

$$\sum_{i=m}^{\infty} \sum_{j=n}^{\infty} a_{ij} = O(m^{\mu-\gamma} n^{\nu-\delta}), \qquad m, n = 1, 2, \dots \tag{3.4}$$

(ii) *If* $\gamma \geq \mu > 0$ *and* $\delta \geq \nu > 0$, *then the converse implication is also valid.*

Comparing statements (i) and (ii) shows that if $\gamma > \mu > 0$ and $\delta > \nu > 0$, then conditions (3.3) and (3.7) are equivalent.

Lemma 3.4 (Fülöp [7]). *Under the conditions of Lemma* 3.3, *if* (3.4) *is satisfied in the case* $\gamma > \mu > 0$ *and* $\delta > \nu > 0$ *then*

$$\sum_{i=1}^{m}\sum_{j=1}^{n} i^{\gamma}\, j^{\delta}\, a_{ij} = o(m^{\mu} n^{\nu}), \qquad m, n \to \infty$$

and

$$\sum_{i=m}^{\infty}\sum_{j=n}^{\infty} a_{ij} = o(m^{\mu-\gamma} n^{\nu-\delta}), \qquad m, n \to \infty$$

are equivalent, where m *and* n *tend to* ∞ *independently one another.*

The interested reader may find further information on the one-dimensional Lipschitz and Zygmund classes in the monographs by N.K. Bary [1], R. DeVore and G.G. Lorentz [3] and A. Zygmund [12] (each of these is available in English) as well as by I.P. Natanson [10] (which is available in Hungarian).

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