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Application of Discrete Tomographic Methods in Nondestructive Testing and Materials Science

Summary of PhD Thesis
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1 Motivation

Tomography is a tool of image processing for determining (reconstructing) an image (or a function, in general) from a set of measurements over it (called projections). Discrete tomography (DT), a relatively new field of image processing, deals with the special case when the range of the images / functions is a known, finite set. The latter constraint is, actually, rather easy to satisfy in many real-life problems, thus enabling the usage of DT in these cases.

The present thesis discusses two very different applications of DT: The first one is concerned with the reconstruction of images of objects composed of some geometrical primitives like tubes, cylinders and spheres. The specific scenario considered here arose in industrial nondestructive testing of objects using radiographic measurements. The second application, on the other hand, involves the reconstruction of orientation maps and grain maps of deformed polycrystalline material samples from X-ray diffraction patterns. These tasks can be very challenging but they are also crucial for several materials scientific concepts.

2 Reconstruction of Objects Parametrized with Geometrical Primitives

In order to tackle the industrial problem brought up in the motivation, the author devised and developed a new stochastic DT reconstruction method that can reconstruct 2D cross-sectional images of objects. These objects are assumed to possess a specific geometrical structure; namely, they can be described as a composition of discs and annuli. Moreover, the object is allowed to be composed of 4 kinds of homogeneous materials that will appear as different pixel intensity levels. The algorithm expects a small number of projections as input taken with parallel beam geometry. To improve robustness against measurement errors, the reconstruction problem is formulated as an optimization task so that solutions are represented in terms of the parameters of constituting geometrical primitives.

Next, the author extended the above approach to enable the reconstruction of 3D objects containing tubes, cylinders and spheres. Instead of being reduced to 2D sub-problems, the algorithm provides a native 3D reconstruction.

Driven by the need to have a large selection of test images, the author also developed an algorithm that can automatically generate random configurations of object parameters in a way that any configuration is chosen with equal probability.

In order to ensure a faster convergence of the optimization process, the author devised and implemented a method for automatically and deterministi-

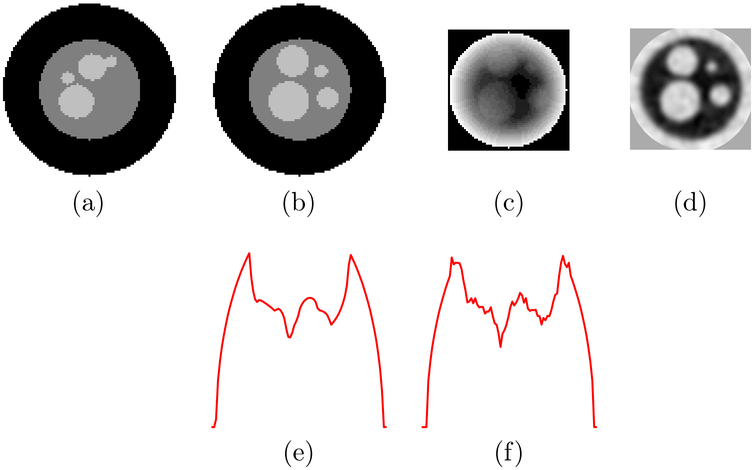

Figure 2.1: Reconstruction of Test Case I from 4 projections. **(a)** Initial configuration. The two smaller discs were added randomly. **(b)** Reconstructed configuration. **(c)–(d)** Filtered back-projection (FBP) reconstructions. **(e)** Simulated projection of (b) at $\vartheta = 0°$. **(f)** Input projection at $\vartheta = 0°$.

cally constructing initial configurations based on the input projections. Since the precise pixel intensities are sometimes unavailable for physical measurements, the algorithm also provides estimates for these values.

The effectiveness of the reconstruction techniques, as well as their sensitivity to various factors, were first investigated using simulation studies. The following parameters were considered: the geometrical complexity of the configuration, the number of input projections, the amount of noise present in the projections, and the quality of the initial configuration. In order to get closer to the circumstances found in physical measurements, the author implemented an additive noise model that can be used to distort simulated projections.

Besides using an appropriate initial configuration to start the optimization procedure, further speed-ups were achieved by carrying out several optimizations in the algorithm logic.

Finally, the author also had the opportunity to test the algorithms with several physical measurements. These included 2D (see Figure 2.1) as well as 3D (see Figure 2.2) objects examined with X-ray, neutron and gamma radiation.

The results shown above have been published in [2, 3, 5–9].

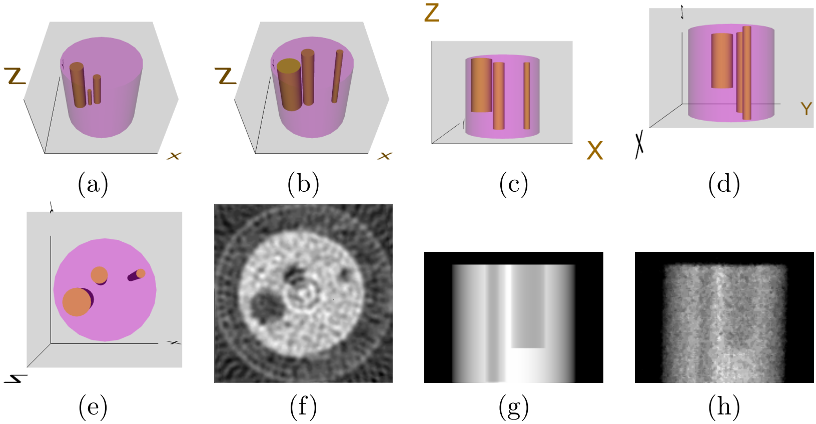

Figure 2.2: Reconstruction of Test Case II from 4 projections. **(a)** Initial configuration. The smallest cylinder was added randomly. **(b)–(e)** Different views of the reconstructed configuration. **(f)** Filtered back-projection (FBP) reconstruction of a cross-section. **(g)** Simulated projection of (b)–(e) at $\vartheta = 180°$. **(h)** Input projection at $\vartheta = 180°$.

3 Reconstruction of Deformed Polycrystalline Samples

The scope of the second industrial application is to get an insight into the microstructure of polycrystalline specimens. As a first attempt targeting the general case, the author devised and implemented a new stochastic DT reconstruction technique that can reconstruct the orientation map in a 2D cross-section of a deformed polycrystal from X-ray diffraction measurements. The sample is presumed to be in monophase, i. e. being composed of one material and a single crystal structure. For better robustness against measurement inaccuracies, the reconstruction problem is formulated as an optimization task that seeks a solution in the space of all possible orientation map images. After considering several alternatives, the author eventually decided to represent orientations with unit quaternions and model orientation maps as a Markov random field.

Since the aforementioned general case seemed to be hard to handle efficiently, the author extended the reconstruction method to be able to simultaneously produce both an orientation map and a grain map, both of which are modeled as Markov random fields. This approach is applicable to moderately deformed specimens where one can derive a grain map from the orientation map in a sensible way.

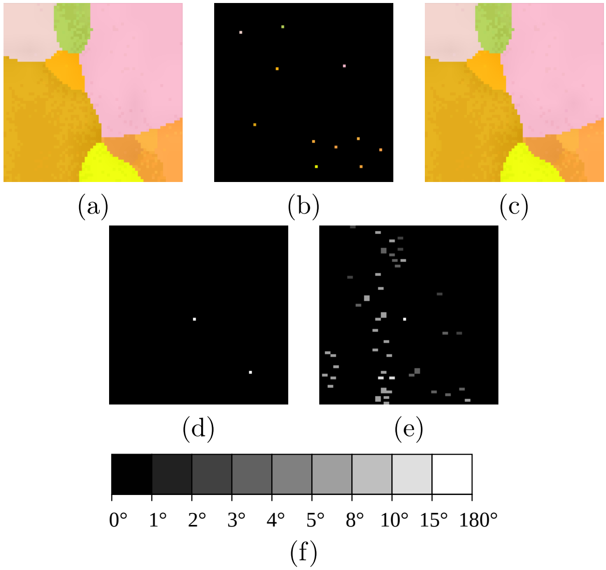

Figure 3.1: The reconstruction of Test Case I using noiseless projections. **(a)** Reference orientation map. **(b)** Initial orientation map. **(c)** Reconstructed orientation map. **(d)** Difference of the reference and the reconstructed grain maps. Black pixels denote identical grain labels, white pixels represent mismatching ones. **(e)** Difference of the reference and the reconstructed orientation maps. The intensity of the pixels is determined by the distance (disorientation angle) of corresponding orientation pairs, as shown in (f).

For both techniques rely on the notion of orientation similarity, the author defined and realized a process for expressing and efficiently computing this quantity in the presence of crystal symmetries.

In order to make the reconstruction procedure fast enough for practical usage, the author applied numerous optimizations in the algorithm logic.

Both reconstruction methods were subject to a series of simulation studies using undeformed (see Figure 3.2) as well as moderately deformed (see Figure 3.1) orientation maps that were all acquired during physical experiments. These investigations helped the author determine the influence of some factors on the reconstruction quality: magnitude of orientation spread within grains, degree of morphological complexity of grain maps, and the amount of noise present in the projections. So as to get a more realistic test setup imitating physical measurements, the author implemented a multiplicative noise model that can be used to distort simulated projections.

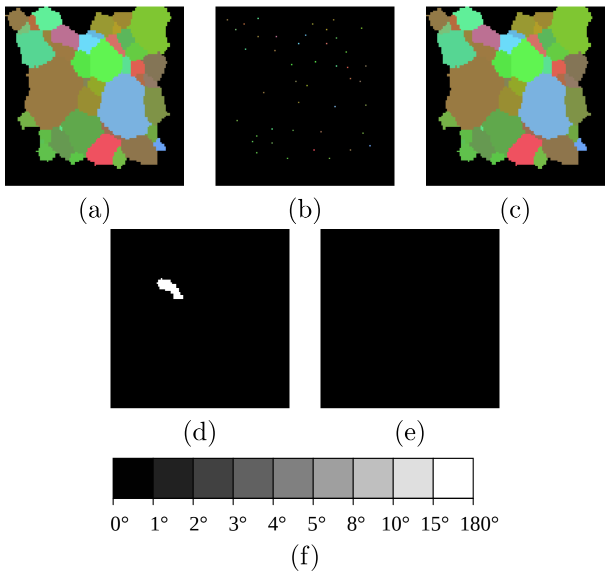

Figure 3.2: The reconstruction of Test Case V using noiseless projections. Map arrangement and gray scales as for Figure 3.1. Black pixels in orientation maps represent void regions. (Note: see dissertation text for an explanation of the error in (d).)

The results presented herein have been published in [1, 10, 11].

4 Conclusions

Two industrial problems have been considered. While quite dissimilar in nature, they nonetheless had a couple of common points: both assumed that the range of the image to be retrieved is a finite, known set, and also that only a limited number of measurements are typically available. These issues together pose serious obstacles to classic tomographic approaches.

With the help of some *a priori* information about the image sought, DT techniques can still be successful in such situations. In the cases presented in the thesis, these priors included an expected geometrical structure, as well as a statistical distribution of grain morphologies and some other local image features. As has been demonstrated, the new methods developed by the author could deliver good quality reconstructions by incorporating prior knowledge into the reconstruction process.

Key Points of the Dissertation

The findings of the research presented in the dissertation can be divided into two thesis groups. Table 4.1 shows which thesis point is described in which publication by the author.

Reconstruction of Objects Parametrized with Geometrical Primitives

The results were published in the conference proceedings [8], papers [2, 5–7, 9], and the book chapter [3].

I/1. The author devised and developed a new stochastic DT reconstruction method that is able to reconstruct 2D discrete images of objects which can be described as a composition of simple geometrical primitives (namely, discs and annuli), and are composed of 4 kinds of homogeneous materials (represented as different pixel intensities). The algorithm expects a small number of projections as input taken with parallel beam geometry. The reconstruction problem is formulated as an optimization task operating in the configuration space, that is looking for solutions in terms of the parameters of constituting geometrical primitives. [8, 9] (Section 4.2)

I/2. The author extended the approach to enable the reconstruction of 3D objects containing tubes, cylinders and spheres, still composed of 4 kinds of homogeneous materials. Unlike some other techniques, this method provides a native 3D reconstruction, i. e. the 3D result is not obtained by simply stacking the reconstructions of individual 2D cross-sections. [3, 5–7] (Section 4.3)

I/3. In order to be able to test the efficiency of the reconstruction approach, the author developed an algorithm that can automatically generate random configurations of object parameters. This automated process strives to be able to generate all valid potential configurations with equal probability. [5, 8, 9] (Section 4.4)

I/4. The author devised and implemented a method for automatically and deterministically constructing initial configurations based on the input projections, which can then become the starting point of the reconstruction methods. The approach chosen is based on geometrical principles combined with some heuristics. As part of this procedure, the author enhanced the algorithm so that it can provide estimates for the pixel intensity levels needed for the reconstruction of the object, which may not be known accurately enough in case of physical measurements. [5] (Section 4.5)

I/5. The author investigated the efficacy of the reconstruction techniques using numerous simulation experiments, which included mostly randomly generated configurations as well as a few manually constructed ones in both 2D and 3D. The goal of this study was to quantify how sensitive the algorithms were regarding the following factors: the geometrical complexity of the configuration, the number of input projections, and the amount of noise present in the projections. The influence of using an automatically determined initial configuration vs a random one was also determined. In order to simulate the imprecise nature of physical measurements, the author implemented an additive noise model that can be used to distort simulated projections. In all the cases, the precision of reconstruction results was measured with several figure-of-merit functions; one of them was devised and implemented by the author. For the sake of improving reconstruction performance, the author carried out several optimizations in the algorithm logic. As a crucial step, the evaluation of the objective function—required for the optimization process—was sped up by executing computations in an incremental way, so that the current value of the objective function gets updated according to the alteration made in the proposed configuration. [3, 5–9] (Sections 5.1.2 and 5.3)

I/6. The author tested the algorithms with several physical measurements. For the 2D scenario, two test objects were reconstructed whose projections had been acquired with X-rays and neutron radiation, respectively. The technique was also evaluated in case of reconstructing 3D objects; in particular, the same test object was reconstructed from X-ray, neutron and gamma radiation based projections. The accuracy of reconstructions was verified by comparing 2D cross-sections with the result of a classical reconstruction technique (FBP). [2, 3, 5–9] (Section 6.2)

Reconstruction of Deformed Polycrystalline Samples

The results were published in papers [10, 11] and the book chapter [1].

II/1. The author devised and implemented a new stochastic DT reconstruction technique that can reconstruct the orientation map in a 2D cross-section of a deformed polycrystal from X-ray diffraction measurements. Unlike some other approaches, this method utilizes the unaltered projections (diffraction patterns) and produces a discrete solution. The algorithm is applicable to specimens consisting of one known material and a single crystal structure. The reconstruction problem is formulated as an optimization task operating in the space of all possible orientation map images, where each pixel represents the local orientation of the crystalline lattice at that location expressed as a unit quaternion. Orientation maps are modeled as a Markov random field based on a combination of a homogeneity term and a collection of clique configurations conveying local image features (namely, grain boundaries). [10] (Section 7.2)

II/2. The author extended the reconstruction method to be able to simultaneously produce both an orientation map and a grain map. Besides the input projections (X-ray diffraction patterns), the algorithm also requires some *a priori* data in the form of statistics about typical grain morphologies, approximate locations and basic orientations. The extended approach is applicable to moderately deformed specimens where a grain map can be meaningfully derived from the orientation map. In order to take advantage of local image features, both maps are modeled as Markov random fields. [1, 11] (Section 7.3)

II/3. The author defined and realized a process for expressing the similarity of orientations in the presence of crystal symmetries. Besides the basic definition, the author also gave a formulation for the efficient computing of this measure. [1, 10, 11] (Section 7.4)

II/4. For the sake of improving reconstruction performance, the author carried out several optimizations in the algorithm logic. One such enhancement was the use of look-up tables for computation-intensive expressions. Additional speed-ups were gained by relying on quantized

Table 4.1: The connection between the thesis points and the author's publications.

	Publication									
Thesis point	[1]	[2]	[3]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
I/1.							•	•		
I/2.			•	•	•	•				
I/3.				•			•	•		
I/4.				•						
I/5.			•	•	•	•	•	•		
I/6.		•	•	•	•	•	•	•		
II/1.									•	
II/2.	•									•
II/3.	•								•	•
II/4.	•								•	•
II/5.	•									•

unit quaternions to represent orientations, and by updating the objective function value incrementally according to the alteration made in the proposed map or pair of maps. [1, 10, 11] (Section 8.1.2)

II/5. The author run numerous simulations to quantitatively characterize the quality of the reconstructions, based on an undeformed and 4 moderately deformed orientation maps that were all acquired during physical experiments. The aim of these investigations was to determine the sensitivity of the algorithms regarding the following factors: magnitude of orientation spread within grains, degree of morphological complexity of grain maps, and the amount of noise present in the projections. In order to mimic the inaccuracies found in physical measurements, the author implemented a multiplicative noise model that can be used to distort simulated projections. In all the cases, the precision of reconstruction results was measured with a pair of figure-of-merit functions, one defined over the grain map and the other over the orientation map. [1, 11] (Section 8.3)

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cited on: 6, 8, 9

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