UNIVERSITY OF SZEGED
Faculty of Science and Informatics
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PhD School in Computer Science

Designing Topology-Preserving Thinning Algorithms and Quantitative Comparison of Skeleton Approximations

Theses of PhD Dissertation

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Introduction

Skeleton-like shape features (i.e., the topological kernel and medial curve in 2D, furthermore the medial surface in 3D, see Figs. 1 and 2) play an important role in various fields of image processing, computer vision, and pattern recognition.

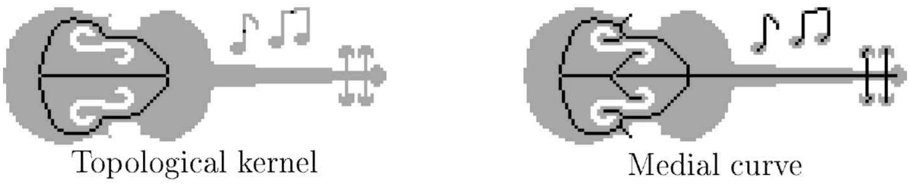


Figure 1: Examples of 2D skeleton-like shape features. The middle gray shapes represent the elongated original objects, while the black curves superimposed on them are the topological kernel and the medial curve.

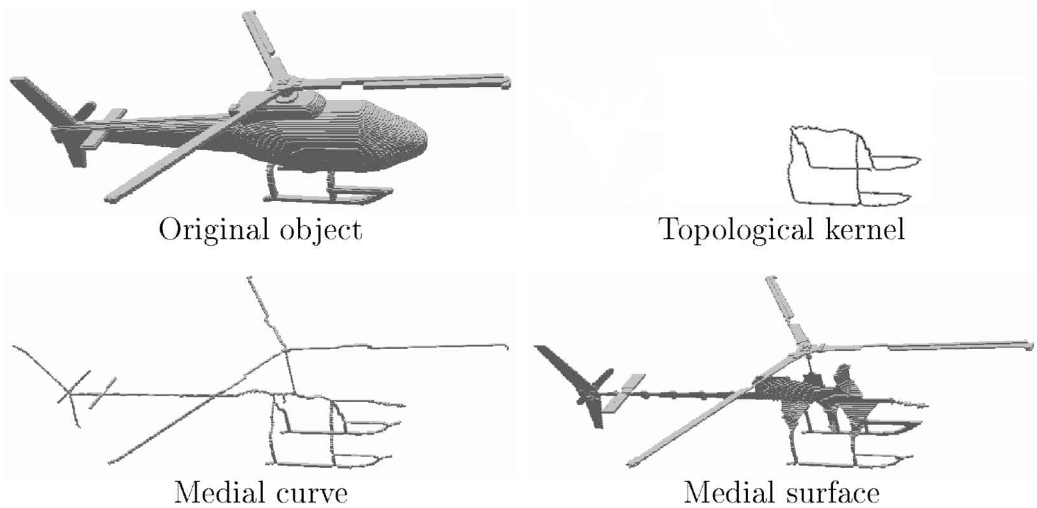


Figure 2: Examples for the three 3D skeleton-like shape features.

Thinning (i.e., an iterative object reduction [2, 23]) is a frequently used method to extract skeleton-like shape features. The advantage of thinning compared to the other two general skeletonization techniques (i.e., the distance-based and the Voronoi-based methods) can be summarized as follows:

- thinning is the most efficient method of skeletonization,
- all the three skeleton-like shape features can be extracted,
- it can be efficiently parallelized, and
- topology preservation can be guaranteed.

One of the most important requirements of skeletonization of binary objects is the topology preservation (i.e., the results of skeletonization must be topologically equivalent to the original objects [4]). The thinning algorithms are composed of reduction operations, which delete a set of object points (i.e., change them to white ones) corresponding to the deletion conditions. Sequential thinning methods delete only one point at a time, while parallel thinning algoritms may delete a set of object points simultaneously. If a thinning algorithm is composed of topologically correct reduction operations, then the topology preservation of the entire algorithm is guaranteed.

To validate the topological correctness of thinning algorithms is a difficult task, especially in 3D. Although there exist already some sufficient conditions for topology preservation of parallel reductions [3,7,17,21], their verification leads to long proofs.

Although skeletonization algorithms, including thinning ones too, have been published for half a century, their quantitative comparison is still an open question. The drawback of the proposed comparison methods is that they consider similarity measures that do not take the elongated object into account.

My dissertation presents the results in the following three thesis points:

- Some new sufficient conditions for topology-preserving reduction operations are proposed, which investigate fewer conditions than the earlier ones. By this way, on one hand they need shorter proof parts, and on the other hand they consider deletion of individual points instead of point-configurations.

- The new sufficient conditions combined with geometric constraints yield a family of topologically correct parallel thinning algorithms. Moreover, a general scheme for parallel thinning algorithms on conventional sequential computers is also presented.

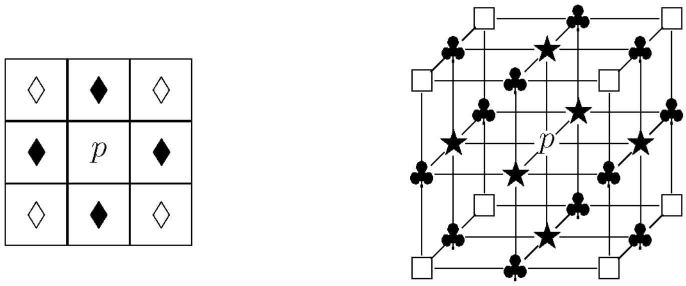
- A novel method is proposed for quantitative evaluation and comparison of skeletons. The two key components of our method are the image database including the reference images with their reference skeletons, and the introduction of a new measure for skeletons. Our method is validated with the help of a new family of skeletons called sequence-skeletons. Applying this method we evaluated and compared 92 types of 2D thinning algorithms (where 80 of them are developed by us).

My dissertation summarizes the current results of my research work. Some open questions and further works are discussed too.

1 Definitions and basic notions

An *n-dimensional* $(k, \overline{k})$ *digital binary image* can be described by the quadruple $(\mathbb{Z}^n, k, \overline{k}, B)$ [4], where

- $\mathbb{Z}^n$ is a set of points with integer coordinates,



(a) (b)

Figure 3: Frequently used adjacencies in $\mathbb{Z}^2$ (a) and $\mathbb{Z}^3$ (b). The set of points $N_4(p)$ contains p and the four points marked $\blacklozenge$. The set of points $N_8(p)$ contains $N_4(p)$ and the four points marked $\Diamond$. The set of points $N_6(p)$ contains p and the six points marked $\bigstar$. The set of points $N_{18}(p)$ contains $N_6(p)$ and the twelve points marked $\clubsuit$, while the set of points $N_{26}(p)$ contains $N_{18}(p)$ and the points marked $\square$. Note that the set $N_j^*(p) = N_j(p) \setminus \{p\}$ $(j = 4, 8, 6, 18, 26)$

- $B \subseteq \mathbb{Z}^n$ is the set of black points, while $\mathbb{Z}^n \setminus B$ is the set of white points,
- k is the adjacency of black points, and
- $\overline{k}$ is the adjacency of the white points.

My thesis deals with 2D $(8, 4)$ and 3D $(26, 6)$ images, whose adjacencies are presented in Fig. 3.

The adjacencies k and $\overline{k}$ are reflexive and symmetric, hence their reflexive and transitive closure, i.e., the k- and $\overline{k}$-connectivity are equivalence relations. The equivalence classes of the set B forms the *k-components* or *the objects*, consequently they partition the set B and the set $\mathbb{Z}^n \setminus B$ into black k- and white $\overline{k}$-components, respectively. A binary image is finite if B is a finite set. The only one infinite white $\overline{k}$-component is the *background*, and the finite white $\overline{k}$-components are called *cavities*. In 3D, the *hole* or *tunnel* is a topological concept that does not have any alternatives in 2D. A black pont is called *border point* if there exits a white point $\overline{k}$-adjacent to it.

The point p is the *smallest element* of the point set Q if $p \prec q$ for each $q \in Q \setminus \{p\}$, where $\prec$ denotes the lexicographical order of point coordinates.

Reduction operations delete some black points, while the white points will not be altered. A reduction operation is topology-preserving if:

1. it does not split any object;
2. it does not delete any object completely;
3. it does not merge cavities with each other or with the background;
4. it does not create any new cavity;

5. it does not merge any hole with another ones (in 3D);

6. it does not eliminate any hole (in 3D);

7. it does not create any hole (in 3D).

A black point is *simple* if its deletion is a topology preserving reduction [3, 8]. The simplicity for $(8, 4)$ and $(26, 6)$ pictures is a local property, which can be decided by investigation the 8- or the 26-adjacency of the point in question.

The iterative object reduction, *thinning*, is the most efficient method of skeletonization [2, 22, 23], which is able to extract the topological kernel, the centerline, and the medial surface in 3D.

A thinning algorithm is topology-preserving if it is composed of topology preserving reduction operations. In order to verify the topogical correctness of thinning algorithms, some sufficient conditions for reduction operations have been given. The existing sufficient conditions for topology preservation [3, 7, 17, 21] considered point-configurations and the ensurance of their correctness leads to long and complex proofs (that is failed in some cases [8, 9]).

Existing methods to evaluate skeleton approximations [5, 6, 20] did not give reliable solutions even for 2D cases. The applied similarity measure does not take the original elongated shapes and their "ideal" skeleton into account.

2 Results of the dissertation

The results of the dissertation are summarized in the following three thesis points.

2.1 New sufficient conditions for topology-preservation of reduction operations

Chapter 3 of the dissertation presents some novel sufficient conditions for topology-preserving reduction operations for $(8, 4)$ and $(26, 6)$ images that consider individual points instead of point-configurations. Here I resume two theorems discussed in the dissertation that take the lexicographical order of the point coordinates into account.

Theorem 1. (Theorem 3.1.6.)
A parallel reduction operation $\mathcal{T}$ is topology-preserving for picture $(\mathbb{Z}^2, 8, 4, B)$ if all of the following conditions hold for any point $p \in B$ deleted by $\mathcal{T}$:

1. The point p is simple in picture $(\mathbb{Z}^2, 8, 4, B)$.

2. For any two 4-adjacent points p and q that are deleted by $\mathcal{T}$, p is simple after deletion of q, q is simple after p is deleted, or $q \prec p$.

3. The point p is not an element of the objects depicted in Fig. 4.

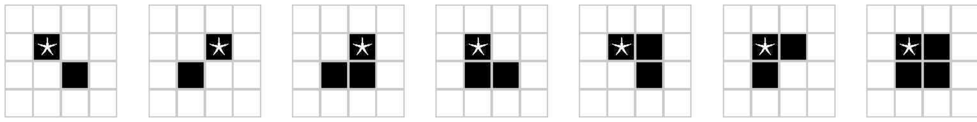


Figure 4: The set of critical objects of Condition 3 of Theorem 1. The symbol $\star$ denotes the smallest elment of the objects.

Theorem 2. (Theorem 3.2.4.)
Let $\mathcal{T}$ be a parallel reduction operator. Let p be any black point in any picture $(\mathbb{Z}^3, 26, 6, X)$ such that point p is deleted by $\mathcal{T}$. Operator $\mathcal{T}$ is topology preserving for (26,6) pictures if all of the following conditions hold:

1. For any $Q \subseteq B$ composed of simple points, in which point p is the smallest element and the elements of Q are mutually 18-adjacent to each other, point p is simple in picture $(\mathbb{Z}^3, 26, 6, (B \setminus Q) \cup \{p\})$.
2. The point p is not the smallest element of such an object that is composed of mutually 26-adjacent points and it contains a pair non 18-adjacent points.

There are 9 theorems (with proofs) for topology-preserving reduction operations applied in $(8, 4)$ pictures and further 15 theorems for reduction operations in $(26, 6)$ pictures. The result of the new sufficient conditions is that they take the deletion of individual points into account (instead of points-configurations). The new results can not be applied just for the validation of the three types of thinning algorithms (i.e., the fully parallel, the directional, and the subfield-based [2]), but we can develop such new algorithms with them, whose topology preservation is guaranteed.

2.2 Designing thinning algorithms

The topological correctness of thinning algorithms can be validated with the help of some sufficient conditions for topology preserving reductions [3, 7, 17, 21].

The conventional design scheme of the topologically correct thinning algorithms is depicted in Fig. 5(a). First the deletion conditions have to be designed, then one has to prove that they fulfill the sufficient conditions for topology preservation of reduction operations. Since the former sufficient conditions investigate point-configurations, their verifications lead to long and complex (and sometimes failed [8, 9]) proofs. The drawback of this method is that if a reduction operation fails one of the sufficient conditions for topology preservation, then the deletion conditions have to be redesigned.

Since the conventional designing scheme is complex and risky, we introduced a novel method based on the sufficient conditions for topology-preserving reductions detailed in Chapter 3 of the thesis. The novel designing scheme is depicted in Fig. 5(b). The common concept of the nine 2D and fifteen 3D theorems is that they consider individual points (instead of point-configurations), hence they may lead to the construction of deletion conditions and not just to validate reduction operations. The only thing that we have to do is to simplify the conditions based on the considered geometric constraints (e.g., prohibited deletion of the endpoints), then we get some deletion conditions, for which the proof of the topology preservation is not necessary and the thinning algoritms composed of them are topologically correct as well.

The advantage of the new designing scheme is that applying different geometric constraints (e.g., different kind of endpoints) leads to new thinning algorithms automatically.

The sufficient conditions presented in my thesis can be classified into two groups, i.e., the symmetric and the asymmetric ones. By this way we can create symmetric and asymmetric thinning algorithms with the help of the new designing scheme.

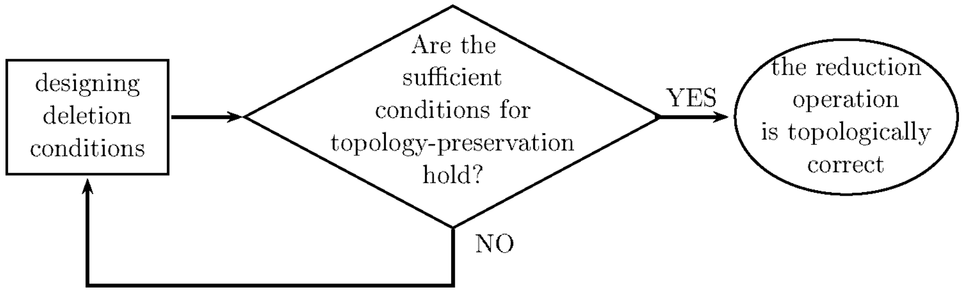
Chapter 4 of my dissertation describes the further four results:

- We introduced a new directional strategy for directional 2D thinning algorithms using $2 \cdot 2$ deletion directions where the deletion direction sequence is $\langle NE, SW, NW, SE \rangle$. The $2 \cdot 2$-directional algorithms reached the best places from the 92 types of 2D thinning algorithms ranked in Chapter 6 of my thesis.
- We introduced the iteration-level border checking strategy for 2D and 3D subfield-based thinning, which reduces the number of the unwanted side skeletal branches.
- The author constructed and implemented numerous 2D and 3D thinning algorithms, where the sufficient conditions are combined with isthmuses (as geometric constraint).
- A general method is proposed for efficient implementation of (sequential and parallel) thinning algorithms in Section 4.3 of the thesis.

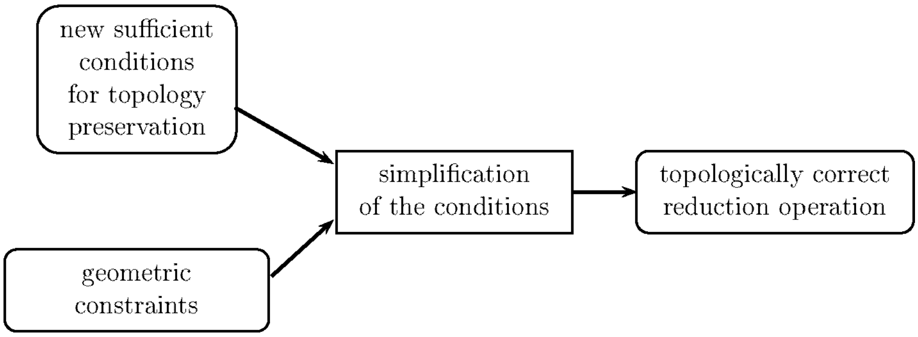
2.3 Quantitative evaluation and comparison of skeletonization algorithms

Chapter 5 of my dissertation presents a novel method for quantitative evaluation of 2D skeletonization algorithms. The two key components of our method are the image database and a skeleton-specific similarity measure. The image database contains 55 image pairs of the reference images and their ideal skeletons. On the other hand we introduced the normalized distance maps that are derived from the Euclidean distance maps computed from the white points of the original image and from the considered skeletal points. With the help of the normalized distance maps we proposed four new skeleton-specific similarity measures (that are not used for general point sets).

In order to validate the new similarity measures, we introduced a new kind of skeleton, that is called sequence-skeleton, by combining the generalized morphological skeletons with neighbourhood sequences. If a neighbourhood sequence is better



(a) The conventional designing scheme.



(b) The proposed designing scheme.

Figure 5: Designing schemes of thinning algorithms.

than an another one, then the sequence-skeleton resulted by the first neighbourhood squence is better than the sequnce-skeleton given by the second one. By this way, we have some skeleton approximations where the goodness of the skeleton is known. With the help of the sequence-skeletons we have shown that the proposed similarity measure AA fulfills the requirements of the reliable similarity measures.

Chapter 6 of my thesis introduced three kinds of ranking methods where 92 types of 2D thinning algorithms are compared based on the AA similarity measure. As a result of the comparison it can be stated that the $2 \cdot 2$-directional thinning algorithms described in Chapter 4 of the dissertation won all the three ranking contrary to the existing and often referred 12 thinning algorithms.

3 Summary of the thesis points

3.1 Novel sufficient conditions for topology-preserving reduction operations

Results belonging to the first thesis point are: 9 theorems are given for $(8, 4)$ pictures and further 15 theorems are about reduction operations for $(26, 6)$ pictures, where each theorem describes some sufficient conditions for topology-preservations.

1.1. The new sufficient conditions consider individual points (instead of point-configurations).

1.2. Using new sufficient conditions, the verification of the topological correctness is even simpler and topology-preserving reduction operations can be derived from them.

1.3. We gave sufficient conditions for all the three reduction strategies.

1.4. Based on the lexicographical order of the point coordinates some non-simple sets can be reduced, since some object points may be deletable from them. The sufficient conditions using the ordered points and the reduction operations derived from them are called asymmetric, hence we can distinguish symmetric and asymmetric family of algorithms.

1.5. The deletion rules of the reduction operations derived from the asymmetric sufficient condtions are simpler than the symmetric ones and their supports contain fewer points. By this way the asymmetric thinning algorithms can be implemented more easily and they are faster than the symmertic ones.

The following publications yield the basis of this thesis point. The results concerning $(8, 4)$ pictures are described in two journal papers [12, 15], while the ones for $(26, 6)$ pictures are detailed in a book chapter [19] and in four conference papers [10, 11, 13, 18].

3.2 Designing thinning algorithms

Results concenrning to the second thesis point are listed as follows:

2.1. We proposed a novel designing scheme for thinning algorithms that combines the sufficient conditions presented in the first thesis point with some geometric constraints.

2.2. This novel strategy yields such thinnig algorithms, whose topology-preservation is guaranteed. No complex or long proof is neccessary.

2.3. The sufficient conditions combined with new geometric constraints lead to the constuction of new thinning algorithms.

2.4. For the 2D directional thinning, we introduced the $2 \cdot 2$-directional strategy that applies the direction sequence $\langle NE, SW, NW, SE \rangle$.

2.5. The iteration-level border checking strategy strategy is a new approach for subfield-based parallel thinning.

2.6. The isthmus-based thinning algorithms accumulate skeleton-like feature points during the thinning process. Moreover, the 3D surface-thinning algorithms consider such isthmuses that produce curve segments for tubular parts.

2.7. The thesis presents a general scheme to implement (sequential and parallel) thinning algorithms.

3.3 Quantitative evaluation and comparison of skeletonization algorithms

Results of the third thesis point are listed as follows:

3.1. A new method is proposed for quantitative evaluation of 2D skeletonization algorithms. The two key components of the method are the 55 image pairs composed of reference images with their reference skeletons and a new skeleton-specific similarity measure.

3.2. The introduced normalized distance map combines two Euclidean distance maps, where one is computed from the white points of the reference image and the other one is computed from the reference skeletons. Based on the normalized distance maps the author with his co-authors proposed four similarity measures.

3.3. In order to validate the new similarity measures we introduced the sequence-skeletons, the generalized morphological skeletons combined with neigbourhood sequences and we gave the criteria of the reliable similarity measures, that is hold by similarity measure AA.

3.4. Ninety-two 2D thinning algorithms have been evaluated and compared with three different ranking methods. Eighty of the thinning algorithms under comparison were presented in Chapter 4 of the dissertation. We can state that the new $2 \cdot 2$-directional ones reached the best places and ranked all the 12 existing and often referred thinning algorithms.

Author's publications related to the theses

The following table shows the relation between the publications and the thesis points. This table contains only those publications that are accepted by the PhD School in Computer Science, Univesity of Szeged.

Publications	Thesis points			Type	Number of authors	Score
	1.	2.	3.			
[19]	•	•		Book chapter	3	0.60
[12]	•	•		Journal paper	3	0.60
[15]	•	•		Journal paper[1]	2	0.75
[16]		•		Conference paper	2	0.75
[14]		•		Conference paper	2	0.75
[13]	•	•		Conference paper	3	0.60
[11]	•	•		Conference paper	3	0.60
[10]	•	•		Conference paper	3	0.60
[18]	•	•		Conference paper	2	0.75
[1]			•	Conference paper	4	0.48
Total:						6.48

[1]Impact Factor: 0.684 (2010)

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