Junction conditions in modified theories of gravity

PhD thesis statements

Bence Racskó

Supervisor: Dr. László Árpád Gergely

Doctoral School of Physics
University of Szeged
Faculty of Science and Informatics
Deparment of Theoretical Physics

Szeged
2025

Összefoglalás

Az értekezés a határfelületekhez kapcsolódó illesztési feltételek kérdéskörét vizsgálja az általános relativitáselméletben (GR) és módosított gravitációs elméletekben (MTG). Az ilyen feltételek akkor jelennek meg, amikor a mezőket a téridő különböző régióit elválasztó határfelületen kell összekapcsolni, például vezető-szigetelő határokon, a Schwarzschild-megoldás belső és külső tartományainak illesztésénél, csillagfejlődési modellekben vagy a bránvilág-szcenáriókban.

A GR-ben az Israel-féle formalizmus ad keretet az idő- és térszerű hiperfelületeken való illesztéshez, de a null felületek és az MTG-k esetén sok kérdés nyitott. Az értekezésben bemutatom, hogy a két hagyományos módszer – variációs elv határtagokkal (pl. Gibbons–Hawking–York) illetve disztribúciós technikák – gyakran nehezen kezelhető vagy ellentmondásos eredményekre vezetnek.

A kutatás célja ezért általános és megbízható módszer kialakítása volt. Először disztribúciós módszerekkel dolgoztam, egy egyszerűbb elméletben (Brans–Dicke) származtattam az általános (bármely kauzális típust megengedő) illesztési feltételeket, melyből kiderült hogy a null vékonyrétegek ebben az elméletben nyomásmentesek, majd bonyolultabb MTG-kben (Kinetic Gravity Braiding) vizsgáltam a null illesztési feltételeket, ahol regularizációra volt szükség. A továbbiakban, hogy a regularizáció szerepét és szükségességét megértsem, a variációs és disztribúciós eljárások közös általánosításával egy formális keretrendszert dolgoztam ki. Bevezettem a minimális evolúciós rendű Lagrange-függvény fogalmát, bizonyítottam annak általános létezését, és ebből szisztematikusan levezettem az illesztési feltételeket klasszikus térelméletek nagyon tág családjában.

Eredményeim közé tartozik: (i) az általános illesztési feltételek explicit meghatározása Brans–Dicke- és Kinetic Gravity Braiding-elméletekben, (ii) a variációs és disztribúciós módszerek általános érvényességének és ekvivalenciájának igazolása, (iii) annak kimutatása, hogy megfelelő határtagok (melyek a variációs megközelítés szerves részét képezik) mindig léteznek, valamint (iv) egy egyszerűsített eljárás kidolgozása a másodrendű rendszerekhez, amellyel hatékonyan lehet a gyakorlatban is meghatározni az illesztési feltételeket konkrét rendszerekre. A munka hozzájárul a módosított gravitációs elméletek és a kozmológiai alkalmazások matematikailag konzisztens kezeléséhez.

Scientific background

The study of boundary surfaces, impulsive phenomena and sudden phase transitions in classical field theories require modeling non-smooth behaviour at the surfaces where the transition takes place. The resulting solutions of the field equations are – in the terminology of partial differential equations – *weak solutions* in the sense that their regularity class is strictly weaker than what the field equations would require. In practice, this means that in addition to satisfying the field equations in the subdomains separated by the transition surface, the dynamical variables are also constrained by relations inposed on the surface itself. These relations are called *junction conditions*.

In electrodynamics, the junction conditions are the well-known equations relating the jump discontinuities of the electric and magnetic fields to charges and currents localized on the boundary surface, and their treatment, along with numerous practical applications, are found in many textbooks on the subject [36].

Junction conditions are also relevant in general relativity (GR), but their formulation is more complicated for a number of reasons. From a more theoretical point of view, in addition to determining boundary conditions satisfied by fields on some interior surface in some fixed space, there is also the attached problem of glueing two spacetime manifolds together along a common (hyper)surface as a prerequisite for determing the junction conditions [7]. In more practical terms, the junction conditions should be formulated in a manner that is independent of the coordinates used to describe either geometry, since these coordinates might not intersect at the junction surface in a regular manner, making comparisons difficult.

Applications of junction conditions in gravitation are also numerous. In the study of physically meaningful exact solutions, it is inevitable that solutions comprised of qualitatively different domains would need to be obtained separately and glued together at a common boundary. Such is the case for example with the outer (vacuum) and inner (matter modelled as perfect fluid) Schwarzschild solutions [28]. Dynamic stellar evolution models, such as the well-known Oppenheimer–Snyder collapse [22], which was the first indicator that black hole formation is not only possible, but inevitable in realistic situations, also requires matching an interior and exterior solution.

Spherically symmetric thin shells of matter provide valuable toy models, whose equations of motion are also given by the junction conditions. In brane world models, the junction formalism provides embedding conditions that the brane must satisfy [25, 26]. Of particular note are junction surfaces which are null (lightlike) that can be used to model impulsive gravitational waves [2]. Null junction surfaces require extra care [7, 4, 20] as the standard formalism due to Israel [14] is restricted to timelike or spacelike boundaries, and many other techniques and results developed in the literature also assume timelike or spacelike signature.

Developments in cosmology, particularly the discovery of dark matter and dark energy lead to modified theories of gravity (MTG), which seek to replace the unknown, only gravitationally interactive forms of matter with geometric effects due to large scale modifications of GR [5]. These are alternative gravitational field theories which must reduce to GR in the appropriate limit (as GR had withstood a plethora of experimental and observational verifications especially in the Solar System scale) but can carry additional degrees of freedom on galactic and cosmological scales. The development of some MTGs precede these cosmological observations and were instead motivated by other factors such as a desire to unify gravitation with other interactions [33, 15, 16].

The zoo of MTGs is quite formidable with many different categories and interrelations between them. Of particular interest are scalar-tensor theories in which in addition to the metric tensor one or more scalar fields also take part in the gravitational interaction. Scalar-tensor theories can lead to cosmologies that are both compatible with observations and carry a richer dynamical structure than the ΛCDM model in GR. Many other MTGs such as higher derivative theories turn to be dynamically equivalent to some scalar-tensor theory, which also amplifies their universality [5].

The most general family of scalar-tensor theories in a four dimensional spacetime with a single scalar field and second-order field equations has been described by G. W. Horndeski [13] in the 70s, as essentially a generalization of the well-known Lovelock theorem [17, 18, 19] to scalar-tensor gravity. It was rediscovered later in a different form as a generalization of covariant Galileon theory [9], often called the DGSZ reformulation of Horndeski theory. Its Lagrangian contains four arbitrary functions of the scalar field and its kinetic term that can be used to tune the dynamical system itself. As such, almost all scalar-tensor theories studied in the literature (with the exception of some multi-scalar and higher-derivative theories) belongs to this class.

Due to their numerous applications, especially regarding cosmology and brane world scenarios, the formulation of junction conditions in MTGs is an active research area. In particular, Padilla and Sivanesan have determined [23] the junction conditions for timelike and spacelike junction surfaces in the full Horndeski class, which has been used by, among others, Nishi, Kobayashi *et al.* [21] to study bounce scenarios and galilean genesis. However, much less attention has been given in the literature to null junction conditions, despite many applications.

Another, unrelated issue, that is nonetheless amplified by the problem of null junction conditions, is that there does not seem to be any consistent and reliable method to determining the junction conditions themselves in a MTG. Padilla and Sivanesan have used a variational principle which however requires the extension of the action with certain surface terms similar to the Gibbons–Hawking–York term [11, 35] in GR. This method is frequently used in the brane world literature [6, 8]. Other authors try to represent the Euler–Lagrange equations as a relation between distributions [4, 20], rather than smooth tensor fields, such that the junction conditions are given by the singular part of the field equations.

Both approaches are fraught with difficulties, for example there is no general theorem that Gibbons–Hawking–York-like surface terms exist at all, and even in GR, this surface term is well-defined only for timelike and spacelike boundaries. Such surface terms are also known to be non-unique, but the effect on the junction condition of changing the surface term has not been investigated. The use of distributions can also lead to difficulties. In GR, the field equations are linear in the second derivatives of the metric with coefficients that involve the metric only algebraically, so in this case the interpretation of the Euler–Lagrange equations as a singular distribution is well-defined, but MTGs can have field equations that are not quasilinear, or quasilinear equations whose coefficients depend on derivatives, higher order field equations where the suitable regularity class on the junction surface is disputed [27]. Since distributions are inherently linear objects, their use in the characteristically non-linear field equations of MTGs is problematic and can lead easily to invalid expressions and false results. Correspondingly, various schemes have emerged to regularize the distributions that appear in these expressions, and other ad-hoc approaches (see comments in [8, 27]). Even when seemingly no issues arise, it is unclear whether the variational and distributional approaches would inevitably lead to the same junction conditions.

Objective

The original motivation behind the present work was to determine junction conditions in Horndeski theory that are valid for null or generic (i.e. no assumption on causal type, allowing even signature change) junction conditions, which are absent from the current literature, as well as investigate possible applications. Both the field equations and the timelike/spacelike junction conditions that have been determined by Padilla and Sivanesan are extremely complicated. Thus, at first I studied simpler scalar-tensor theories such as Brans–Dicke theory, where the existing method of tensor distributions was applicable.

Then I considered a more general theory which contains a nontrivial G_3 term (in regards to the DGSZ reformulation of Horndeski's action functional) and belongs to the class of so-called Kinetic Gravity Braiding theories. Unlike Brans–Dicke theory that has been mostly ruled out based on the measurements carried out by the Cassini probe, this MTG is compatible with the results of the recent gravitational wave measurements, and the nontrivial G_3 term admits the Vainshtein screening mechanism. The null junction conditions have been determined through the use of tensor distributions, where however a number of problematic terms had appeared. Considering the junction surface as an infinitesimally thin limit of "thick shells", I have interpreted these terms as containing mean values of discontinuous quantities at the junction surface, a perspective which seems to be consistent with the existing literature.

Further examination of the obtained junction conditions however cast doubt on the veracity of either the obtained results themselves, or the consistency and reliability of the computation processes of junction conditions. For example, in the obtained junction conditions, mean values of discontinuous quantities had appeared, however in the junction conditions obtained by Padilla and Sivanesan for the full Horndeski theory, the conditions only involve jumps. A direct comparison was not possible, since the latter were valid for timelike or spacelike junction surfaces, and the former for null surfaces only. In the particular situation, all mean values were possible to be made vanish by the appropriate choice of some auxiliary gauge condition, but analogous calculations made in an even more general theory showed that this is not a generic feature since there this was no longer possible. This seemed to imply that either the distributional method is not suitable for use in sufficiently complicated MTGs (conditions to ensure that no problematic terms appear would have made the junction conditions themselves trivial), or that the results of the distributional and variational methods need not agree. It was also unknown whether the variational methods can be generalized to null junction surfaces [29] and computations of null or generic junction conditions through this method were completely absent from the literature.

The objective behind the present work then necessarily shifted from practical computation to theoretical aspects, namely to obtain a reasonably complete understanding of junction conditions in general systems. Specifically, I wanted to answer the following questions:

- Is there a well-defined junction formalism in any field theory? If not, what are the conditions that need to be imposed on the system to ensure this is the case.
- Do the somewhat ad-hoc distributional and variational methods that have been frequently used in GR and MTGs to obtain the junction conditions have some sort of general rule behind them (for example, similar to how the Noether theorem can be used to obtain conservation laws from infinitesimal symmetries of *any* Lagrangian, regardless of its concrete properties)?
- If the answer to the preceding question is affirmative, do the junction conditions obtained by these methods necessarily agree?
- Do Gibbons–Hawking–York-like surface extensions of an action always exist (when needed)? Can they be extended to boundary surfaces of any causal type? Are the junction conditions independent of the choice of surface term, assuming that the surface term is "proper" in a suitable sense?
- If there is a general, well-defined distributional method for the computation of junction conditions, why were the results obtained for the Kinetic Gravity Braiding theory mentioned above so different qualitatively from the junction conditions of Padilla and Sivanesan? If the use of distributions requires some subtleties, how to perform the computations for a general MTG?
- Assuming positive answers to the above questions (where relevant), do these methods generalize to higher order theories?
- Especially for variational approaches, determining the variation of the action extended by surface terms in a suitable form to read off the junction conditions can be extremely laborous. Depending

on the answers for the preceding questions, what are the most efficient computational methods?

The research has been carried out through the use of tensor distributions and currents [10] as well as some advanced results from the calculus of variations that have been developed independently by Vinogradov [32], Takens [30] and Tulczyjew [31] in a more global setting. These results allow one to find integrability conditions and inversion formulae (homotopy operators) for the various complicated differential operators that appear in the calculus of variations, which have been instrumental for the completion of the present work.

Results

T1. (*Thesis statement based on* **[R1])** I have computed the junction conditions for the Brans–Dicke scalar-tensor theory in the Jordan frame (choice of dynamical variables where the scalar field couples non-minimally to the metric but does not couple to matter) valid for junction surfaces of any fixed (timelike, spacelike or null) signature. The calculation has been carried out by the method of tensor distributions. I obtained that in this theory null shells are necessarily pressureless. This agrees with earlier results by Barrabès and Bressange [3] for Brans–Dicke-like multiscalar MTGs, which have however been carried out in the Einstein frame (choice of dynamical variables where the scalar field couples to matter but only minimally couples to the metric). The physical equivalence of the Jordan and Einstein frames in scalar-tensor theory is strongly contested (which appears to be related to interpretational issues regarding the meaning of "equivalence" in this context), with some evidence that although mathematical duality is present allowing results to be transformed between the frames, the frames themselves describe distinct physics. It is thus interesting that this particular condition on the energy-momentum tensor is frame-independent.

T2. (*Thesis statement based on* **[R2]**) I have computed the junction conditions by distributional techniques along a null hypersurface in a class of kinetic gravity braiding theories which are compatible with the then-recent measurement of gravitational waves. This class has a nontrivial G_3 term (in the sense of the DGSZ reformulation [9] of Horndeski's action), which significantly complicates the field equations. Due to discontinuous functions appearing in the calculations as coefficients of the Dirac delta supported on the hypersurface, which are ill-defined in distribution theory, I used a regularization scheme based on regarding the junction surface as the limit of a shell of finite thickness, which is consistent with the existing literature. This result is important in the sense that it signals the need for universal and consistent mathematical techniques to compute junction conditions in theories of similarly complicated field equations, without having to use various regularization schemes.

T3/A. (*Thesis statement based on* **[R3]**) In the framework of GR, I have successfully computed the generic junction conditions using a variational principle, which was not accomplished until now, showing the viability of this approach even when the junction surface is non-Riemannian. The variational problem was formulated as the Einstein–Hilbert action extended by a generalization of the Gibbons–Hawking–York surface term due to Parattu *et al.* [24] These junction conditions coincide with the ones derived by Mars and Senovilla [20] through the use of distributions.

T3/B. (*Thesis statement based on* **[R3]**) I have investigated the dependence of the calculation on surface terms. It turns out that if a given surface term extending the Einstein–Hilbert action is "proper" in the sense that the variation of the extended action involves only variations of the metric at the boundary to order 0 (i.e. no transversal derivatives), then the correct junction conditions are obtained, irrespective of which surface term is used, resolving a known ambiguity [29] in the formalism.

T3/C. (*Thesis statement based on* **[R3]**) I have also obtained the correct junction conditions through the use of a first-order alternative to the Einstein–Hilbert action, without needing to use surface terms, since then the boundary part of the action's variation already involves the metric variation only algebraically. The first-order action really just differs from the usual Einstein–Hilbert action by a total divergence, which when integrated, becomes a Gibbons–Hawking–York like surface term on the boundary, so by the preceding result, junction conditions can be computed from a first-order action without having to add surface terms by hand. I have shown through explicit calculations that the junction conditions obtained this way does indeed agree with those in the preceding thesis statement, and made comparisons regarding the relative difficulties of the computations.

T3/D. (*Thesis statement based on* **[R3]**) I have also generalized an approach by Hajícek and Kijowski [12] from the timelike to the generic case. In this approach, surface terms are not needed to be added to the interior boundary by hand, since they appear naturally when the Lagrangian is interpreted as a distribution. However by appealing to the particularly nice functional structure of the Einstein–Hibert Lagrangian, I have drawn the conclusion that this method does not generalize to more complicated cases.

T4/A. (*Thesis statement based on* **[R4]**) A minimal order Lagrangian for a differential system is a Lagrangian function whose order is half of that of the system (rounded upwards if the system has odd order). It is known [1] that variational field equations do not in general admit minimal order Lagrangians. I have proven that (up to possible global obstructions), a minimal *evolutionary* order Lagrangian always exists, i.e. one which satisfies the minimal order property with respect to derivatives along one distinguished coordinate only (called the evolutionary coordinate). The proof of this theorem is constructive, the Lagrangian can be explicitly found by quadratures. This theorem is important for the rest of the results, but I also expect that it could generate independent interest outside the context of this work, as it also concerns the computation of "correct" canonical momenta for higher order Hamiltonian dynamics, and the existence of variational problems for evolution equations where the boundary conditions coming from the variational problem are consistent with the initial conditions (Cauchy problem) of the system.

T4/B. (*Thesis statement based on* **[R4]**) I have constructed well-defined junction conditions for an arbitrary variational field theory through distributional means. The junction conditions themselves relate the jumps of the canonical momenta (in the higher order cases, these are Ostrogradski-momenta [34]) to singular sources supported on the junction surface. The canonical momenta are computed from a minimal evolutionary order Lagrangian, with respect to the slicing of the manifold by the evolutionary coordinate. Ill-defined distributional operations (and thus the need for any regularization scheme) are avoided through a series of integration by parts revealed by the particular structure of total derivatives in the representation of the field equations as Euler–Lagrange equations of a minimal evolutionary order Lagrangian.

T4/C. (*Thesis statement based on* **[R4]**) I have also formulated junction conditions for an arbitrary variational field theory via a variational principle, and have shown its equivalence to the distributional methods. The action is the integral of any minimal evolutionary order Lagrangian, and the junction conditions arise as natural boundary conditions on the junction surface regarded as an interior boundary. I have given the interpretation of the imposed regularity conditions on the field variables and the need to define the action in terms of minimal evolutionary order Lagrangians in terms of the well-posedness of the variational principle and its consistency with the Cauchy problem for the equation.

T4/D. (*Thesis statement based on* **[R4]**) I have proven a general (although local) existence theorem for Gibbons–Hawking–York-like surface terms. A non-minimal order Lagrangian differs from an equivalent minimal evolutionary order one in total derivatives, which become boundary terms when integrated, so this result follows from the existence of minimal evolutionary order Lagrangians. The junction conditions can thus also be computed from a non-minimal order Lagrangian if corresponding surface terms are added to the junction surface from both sides. This gives a theoretical justification for this common technique.

T5/A. (*Thesis statement based on* **[R4]**) I have derived a useful technique for the practical computation of junction conditions for systems of evolutionary order two (the most common case), which is generally less laborous than directly computing a minimal evolutionary order Lagrangian and its canonical momenta. It involves computing the distributional form of the field equations by essentially treating singular distributions as if they were functions. This often leads to ill-defined products of the Dirac delta with functions whose discontinuity intersect its support, however the technique then allows one to systematically transform these coefficients of the Dirac delta, interpreted as purely formal expressions, into a valid canonical momentum for the system. Although the calculation process itself goes through potentially ill-defined quantities, the proof shows that the final result is correct and also explains why the calculation can be performed this way. The method then provides a computational shortcut that is useful for explicit derivations of junction conditions. I have also shown that if the coefficients of the Dirac delta are actually continuous (does not depend on derivatives of the dynamical variables) along the junction surface, then the process does not modify the coefficients, which is why in these situations the straightforward distributional computations will produce the same results (this is the case in GR and Brans–Dicke theory). The technique has been illustrated on a simple scalar-field system.

T5/B. (*Thesis statement based on* **[R4]**) I have calculated the general junction conditions in the Horndeski-class theory with nontrivial G_3 term that I have investigated earlier via the naive distributional methods. Since the system is second order, I have carried out the calculations via the simplified technique disseminated in **T5/A**. The computations thus required no regularization schemes and remained unambigous. This illustrated the usefulness to practical computation of the studies presented in this thesis. Due to the mathematically rigorous theory behind the computational technique which avoids ill-defined distributional operations, the lack of need for ad-hoc regularizations, and since in "simple" systems where the prior techniques do not lead to ill-defined terms the results coincide with those obtained by the prior techniques, it stands to reason that the junction conditions should be computed by the methods developed in this work.

Publications

MTMT identifier: 10064174

Publictions related to the thesis

[R1] **B. Racskó**, L. Á. Gergely, Light-Like Shockwaves in Scalar-Tensor Theories, *Universe*, **4**, 44 (2018)
DOI: 10.3390/universe4030044
Impact factor: 2.165 (2018)

[R2] **B. Racskó**, L. Á. Gergely, The Lanczos Equation on Light-Like Hypersurfaces in a Cosmologically Viable Class of Kinetic Gravity Braiding Theories, *Symmetry*, **11**, 616 (2019)
DOI: 10.3390/sym11050616
Impact factor: 2.645 (2019)

[R3] **B. Racskó**, Variational formalism for generic shells in general relativity, *Class. Quantum Grav.* **39**, 015004 (2022)
DOI: 10.1088/1361-6382/ac38d2
Impact factor: 3.7 (5-year)

[R4] **B. Racskó**, Junction conditions in a general field theory, *Class. Quantum Grav.* **41**, 015020 (2024)
DOI: 10.1088/1361-6382/ad0fb6
Impact factor: 3.7 (5-year)

Other publications

- L. Á. Gergely, A. Fóris, **B. Racskó**, Static and radiative cylindrically symmetric spacetimes, Proceedings of Science,g 463, 214 (2024)
  DOI: 10.22323/1.463.0214
- **B. Racskó**, L. Á. Gergely, Geometrical and physical interpretation of the Levi-Civita spacetime in terms of the Komar mass density, *Eur. Phys. J. Plus* **138**, 413 (2023)
  DOI: 10.1140/epjp/s13360-023-04027-9
  Impact factor: 2.8 (2023)
- **B. Racskó**, L. Á. Gergely, Null shells in kinetic gravity braiding scalar-tensor theories, *Romanian Astron. J.* **30**, 1, 25–34 (2020)
- B. Gilicze M. Moczok, D. Madarász, N. Juhász, **B. Racskó**, L. Nánai, Periodic surface structure creation by UV femtosecond pulses on silicon, *AIP Conf. Proc.* 1796, 030001 (2017)
  DOI: 10.1063/1.4972366

Conference talks

- Null shells in kinetic gravity braiding scalar-tensor theories. Recent Developments in Astronomy, Astrophysics, Space and Planetary Sciences, Cluj-Napoca, Romania (27-29 May 2019)
- Null junction conditions in kinetic braiding theories. IberiCOS 2019, Bilbao, Spain (15-17 April, 2019)
- Thin shells and shockwaves in generalized Brans-Dicke theories. FUture Gravitational Alternatives Meeting, Valencia, Spain (3-4 October 2018)

- Lightlike shockwaves in scalar-tensor theories. BGL 17: 10th Bolyai-Gauss-Lobachevsky Conference on Non-Euclidean Geometry and its Applications, Gyöngyös, Hungary (20-26 August 2017)
- Surface Structure Creation Induced By FS - Laser Pulses Investigated Using Fractal Dimension Analysis Methods. TIM 15-16 Physics Conference, Timisoara, Romania (26-28 May 2016)

References

[1] I. M. Anderson, T. Duchamp, On the Existence of Global Variational Principles, *American Journal of Mathematics*, **102**(5), 781-868 (1980)

[2] C. Barrabès and P. A. Hogan, *Singular Null Hypersurfaces in General Relativity*, World Scientific (2003)

[3] C. Barrabès, G. F. Bressange, Singular hypersurfaces in scalar-tensor theories of gravity. Class. Quantum Gravity, **14**, 805–824 (1997)

[4] C. Barrabès, W. Israel, Thin shells in general relativity and cosmology: The lightlike limit, *Phys. Rev. D* **43**, 1129–42 (1991)

[5] S. Capoziello, V. Faraoni, *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics*, Springer (2011)

[6] H. A. Chamblin, H. S. Reall, Dynamic dilatonic domain walls, *Nucl. Phys. B* **562**, 133–57 (1999)

[7] C. J. S. Clarke, T. Dray, Junction conditions for null hypersurfaces, *Class. Quantum Grav.*, **4**, 265–275 (1987)

[8] S. C. Davis, Generalized Israel junction conditions for a Gauss-Bonnet brane world, *Phys. Rev. D* **67**, 024030 (2003)

[9] C. Deffayet, X. Gao, D. A. Steer, G. Zahariade, From k-essence to generalised Galileons, *Phys. Rev. D* **84**, 064039 (2011)

[10] G. de Rham, *Differentiable Manifolds: Forms, Currents, Harmonic Forms*, Springer (1984)

[11] G. W. Gibbons, S. W. Hawking, Action integrals and partition functions in quantum gravity, *Phys. Rev. D*, **15**, 2752 (1977)

[12] P. Hájíček, J. Kijowski, Lagrangian and Hamiltonian formalism for discontinuous fluid and gravitational field, *Phys. Rev. D* **57**, 914 (1998)

[13] G. W. Horndeski, Second-order scalar-tensor field equations in a four-dimensional space, *Int. J. Theor. Phys.* **10**, 363-384 (1974)

[14] W. Israel, Singular hypersurfaces and thin shells in general relativity, *Nouvo Cim. B*, **44**, 1-14 (1966)

[15] T. Kaluza, Zum Unitätsproblem in der Physik, *Sitzungsber. Preuss. Akad. Wiss. Berlin.* 966–972 (1921)

[16] O. Klein, Quantentheorie und fünfdimensionale Relativitätstheorie, *Z. Phys A* **37** (12), 895–906 (1926)

[17] D. Lovelock, Divergence-free tensorial concomitants, *Aequat. Math.* **4**, 127–138 (1970)

[18] D. Lovelock, The Einstein tensor and its generalizations, *J. Math. Phys.* **12**, 498–501 (1971)

[19] D. Lovelock, The four dimensionality of space and the Einstein tensor, *J. Math. Phys.* **13**, 874–876 (1972)

[20] M. Mars, J. M. M. Senovilla, Geometry of general hypersurfaces in spacetime: junction conditions, *Class. Quantum Grav.* **10**, 1865 (1993)

[21] S. Nishi, T. Kobayashi, N. Tanahashi, M. Yamaguchi, Cosmological matching conditions and galilean genesis in Horndeski's theory, JCAP, **03**, 008 (2014)

[22] J. R. Oppenheimer, H. Snyder, On Continued Gravitational Contraction, *Phys. Rev.* **56**, 455 (1939)

[23] A. Padilla, V. Sivanesan, Boundary terms and junction conditions for generalized scalar-tensor theories, *J. High Energ. Phys.* **2012**, 122 (2012)

[24] K. Parattu, S. Chakraborty, T. Padmanabhan, Variational principle for gravity with null and non-null boundaries: a unified boundary counter-term, *Eur. Phys. J. C* **76**, 129 (2016)

[25] L. Randall and R. Sundrum, An Alternative to Compactification, *Phys. Rev. Lett.* **83**, 4690 (1999)

[26] L. Randall and R. Sundrum, A Large Mass Hierarchy from a Small Extra Dimension, *Phys. Rev. Lett.* **83**, 3370 (1999)

[27] B. Reina, J. M. M. Senovilla, R. Vera, Junction conditions in quadratic gravity: thin shells and double layers, *Class. Quantum Grav.* 33, 105008 (2016)

[28] K. Schwarzschild, Über das Gravitationsfeld einer Kugel aus inkompressibler Flüssigkeit nach der Einsteinschen Theorie, *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, Phys.-Math. Klasse*, 424-434 (1916)

[29] J. M. M. Senovilla, Equations for general shells, *J. High Energy Phys.* JHE11(2018)134 (2018)

[30] F. Takens, A global version of the inverse problem of the calculus of variations, *J. Differential Geom.* **14,** 543-562 (1979)

[31] W. M. Tulczyjew, *The Euler-Lagrange resolution*, Lecture Notes in Mathematics **836** 22–48, Springer-Verlag, New York (1980)

[32] A. M. Vinogradov, The $\mathcal{C}$-spectral sequence, Lagrangian formalism and conservation laws II. The non-linear theory, *J. Math. Anal.* **100** (1), 41–129 (1984)

[33] H. Weyl, Gravitation und Elektrizität, *Sitz. Kön. Preuss. Akad. Wiss.* 465-480 (1918)

[34] R. P. Woodard, Avoiding Dark Energy with 1/R Modifications of Gravity, *Lect. Notes Phys.* **720**, 403 (2007)

[35] J. W. York, Role of Conformal Three-Geometry in the Dynamics of Gravitation, *Phys. Rev. Lett.*, **28**, 1082 (1972)

[36] A. Zangwill, *Modern Electrodynamics*, Cambridge University Press (2012)

Declaration of co-authorship

I, **Dr. László Árpád Gergely**, hereby declare that in achieving the results presented in the publications co-authored with **Bence Racskó**, listed below and corresponding to thesis statements **T1** and **T2** of his doctoral dissertation entitled "*Junction conditions in modified theories of gravity*", Bence Racskó's contribution was decisive. I confirm that I have not used these results to obtain any academic degree, nor will I do so in the future.

[T1] B. Racskó, L. Á. Gergely, Light-Like Shockwaves in Scalar-Tensor Theories, *Universe*, **4**, 44 (2018)

[T2] B. Racskó, L. Á. Gergely, The Lanczos Equation on Light-Like Hypersurfaces in a Cosmologically Viable Class of Kinetic Gravity Braiding Theories, *Symmetry*, **11**, 616 (2019)

12.09.2025 SZEGED

Date and place

Signature