Theses of the PhD thesis

Topology preservation and skeletonization on the BCC and FCC grids

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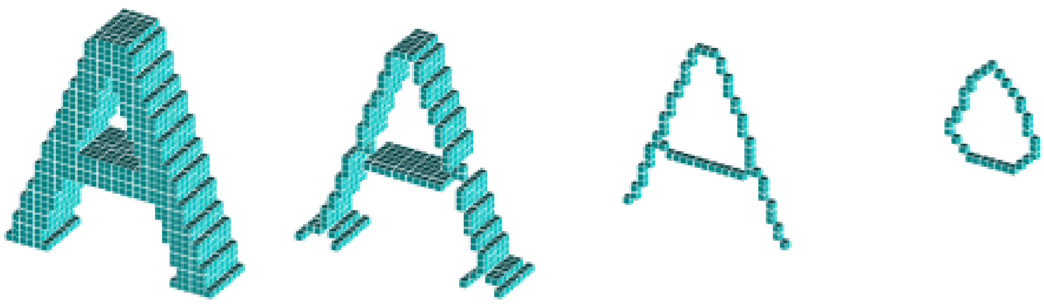
1 Introduction

Digital topology is the study of the topological properties of *binary digital pictures* (*pictures* in short) that assign a color of *black* or *white* to each element, i.e., *point* of the given grid [15]. An important requirement in digital topology is to ensure *topology preservation* of picture operators. Among the binary picture operators, our attention is restricted to *reductions*, which can only change some black points to white ones (referred to as *'deletion'*), but cannot affect the white points from the input picture [12]. *Sequential reductions* can delete only one point at a time, while *parallel reductions* may delete a set of points simultaneously [12].

In *skeletonization*, our goal is to give reduced representations of digital objects by extracting skeleton-like shape features, which should preserve their topology and shape [17]. In 3D, there are three kinds of *skeleton-like shape features*, see Fig. 1. The *medial surface* (or so-called *surface skeleton*) can both contain (2D) surface patches and (1D) line segments. In contrast, the *centerline* (or so-called *curve skeleton*) should have only line segments. Finally, the *topological kernel* is a minimal set of points that is topologically equivalent to the original object. Note that a line segment can also be called a *branch*.

Thinning is an iterative object reduction technique for producing all three skeleton-like shape features in 3D. A thinning algorithm terminates if stability is reached, i.e., there are no more 'deletable' points in the actual picture. It is the fastest skeletonization method, it can be efficiently parallelized, and topology preservation can be guaranteed. To ensure topology preservation and produce geometrically correct shape-feature, thinning methods are often combined with the *distance transform* (*DT* in short), which assigns to each point in the input binary picture the distance from its nearest white point [9]. Note that the produced *distance map* strongly depends on the applied distance.



original object — *medial surface (surface skeleton)* — *centerline (curve skeleton)* — *topological kernel*

Figure 1: *Examples for each 3D skeleton-like shape feature.*

The cubic grid is mostly preferred for sampling 3D digital pictures due to its fairly simple and easy-to-implement structure. Two of its alternative sampling schemes are the *body-centered cubic (BCC) grid* and the *face-centered cubic (FCC) grid* [15, 20], which tessellate the 3D Euclidean space into *truncated octahedra* and *rhombic dodecahedra*, respectively. It is known that they have beneficial topological and geometrical properties compared to the cubic grid [11, 15, 20]. Inspired by their advantages, in the dissertation, we put an emphasis on these two non-standard 3D grids.

The dissertation consists of four major parts. After a brief introduction, the second chapter reviews the basic concepts and existing results in the field of digital topology, topology-preserving reductions, and skeletonization. Then, in the next three chapters, the Author's new scientific results are discussed as three thesis groups.

2 Basic notions and results

A $(k, \bar{k})$ binary digital picture on a grid $\mathcal{V}$ is defined as a quadruple $(\mathcal{V}, k, \bar{k}, B)$ [15], where

- $\mathcal{V}$ is the set of picture elements (i.e., points),
- $B \subseteq \mathcal{V}$ is the set of black points,
- k is the adjacency relation assigned to the black points,
- $\bar{k}$ is the adjacency relation assigned to the white points.

Since the adjacency relations are symmetric, their transitive closures form equivalence relations, and their generated equivalence classes are called *components*. In a picture, a *black component* or an *object* is a k-component, while a *white component* is a $\bar{k}$-component of the set of white points $\mathcal{V} \setminus B$. A black point is called a *border point* in a picture if it is $\bar{k}$-adjacent to at least one white point. Furthermore, a border point p is called an *isolated point* if all points in its k-adjacency are white (i.e., $\{p\}$ is a singleton object). In a finite picture, there is a unique white component that is called the *background*, while each finite white component is called a *cavity*.

The dissertation focuses on two 3D grids, which are the subsets of the cubic grid ($\mathbb{Z}^3$):

- the *body-centered cubic grid* (*BCC grid* in short) – denoted by $\mathbb{B}$ –, in which each point's coordinates have the same parity, and
- the *face-centered cubic grid* (*FCC grid* in short) – denoted by $\mathbb{F}$ –, in which the sum of each point's coordinates is even.

Hence, $\mathcal{V} \in \{\mathbb{B}, \mathbb{F}\}$. On the BCC grid, each truncated octahedron (i.e., voxel) p has 14 face-neighbors – denoted by $N_{14}(p)$ – with a Euclidean distance of at most 2. Thus, we assume $(14, 14)$ pictures on this type of grid. However, 8 of them are closer – included in $N_8(p)$ – with a (Euclidean) distance of $\sqrt{3}$. On the FCC grid, each voxel p has 12 face-neighbors – denoted by $N_{12}(p)$ –, and there are six further vertex-neighbors. All neighbors that share at least a vertex with p are included in $N_{18}(p)$. We say that elements of $N_k(p)$ are *k-adjacent* to p. Gau and Kong examined three 'reasonable' types of pictures on the FCC grid, where $(k, \bar{k}) \in \{(18, 12), (12, 18), (12, 12)\}$ [11].

Point p is the smallest element of the set Q if $p \prec q$ for each $q \in Q \setminus \{p\}$, where '$\prec$' denotes the lexicographical order of point coordinates.

A 3D reduction *preserves the topology* if

- it does not split any object (into several objects) in the input picture,
- it does not completely delete any object in the input picture,
- it does not merge any cavity with the background or another cavity in the input picture,

- it does not create a cavity where there was none in the input picture,
- it does not eliminate or merge any existing hole in the input picture, and
- it does not create new holes where there was none in the input picture.

A black point is called *simple* if its deletion is a topology-preserving reduction [15]. Furthermore, a set of black points is called a *simple set* if its elements can be arranged in a sequence such that the first point is simple, and each point is simple after deleting all of its preceding elements [14]. Additionally, a set of black points in an arbitrary picture is a *minimal non-simple (MNS) set* if it is non-simple but all of its proper subsets are simple [16]. It is showed that, on the BCC and FCC grids, the simpleness of a point can be determined by investigating its 14- and 18-adjacency, respectively [11, 21].

A *thinning algorithm* is topology-preserving if it is composed of topology-preserving reduction operations. *Sequential reductions* may delete only one black point at a time, hence for such operations, the deletion of only a simple point ensures topology preservation. In contrast, *parallel reductions* can delete a set of points simultaneously. To make sure a parallel reduction preserves the topology, we rely on Ronse's general sufficient condition: if a parallel reduction never deletes MNS sets, it is topology-preserving [16].

Thinning algorithms can be classified as *sequential* or *parallel* depending on whether they delete one or more border points at a time [12]. Additionally, parallel thinning strategies can be further distributed into subcategories. Among them, we focus on the *fully parallel* and the *subfield-based* ones.

- In *fully parallel* algorithms, each iteration step requires exactly one parallel reduction with a global deletion rule
- In the *subfield-based* strategy, the set of all picture elements $\mathcal{V}$ is partitioned into $t \geq 2$ subfields. In each subiteration, only one subfield is activated, and the set of points corresponding to the currently activated subfield are taken into consideration for possible deletion.

3 Sufficient conditions for topology-preserving parallel reductions

Based on the characterizations of MNS sets in the considered picture types [11, 13], in Chapter 3 of the PhD thesis, we present some novel sufficient conditions for topology-preserving reduction operations for $(14, 14)$ pictures of the BCC grid, and all three types of pictures on the FCC grid. They can be separated into two categories: *configuration-based*, and *point-based* ones. In addition, the generated reductions from the point-based conditions are also introduced.

3.1 Configuration-based and point-based conditions

We established one configuration-based condition for topology-preserving reduction operations applied on the BCC grid, and further seven ones for reduction operations acting on the FCC grid. The last one of them is a unified condition for the FCC grid, which makes easier to understand the difference between the three considered picture types.

The previous conditions just provide a method of verifying that a formerly constructed parallel reduction preserves the topology, rather than a methodology for constructing topology-preserving reductions. That is why we proposed point-based conditions that directly provide deletion rules of topology-preserving reductions, and allow us to develop topology-preserving thinning algorithms.

Two of these point-based conditions are established for the BCC grid and eight for the FCC grid. Like in the case of the configuration-based conditions, the last two of them are unified ones for the FCC grid. The proposed point-based conditions can be classified into two groups:

- The *symmetric* conditions claim that any point belonging to at least one MNS set is classified as non-deletable. Hence, they do not make a distinction between the elements of the examined sets.

- The *asymmetric* variants say that only the smallest element of each MNS set is non-deletable.

The result of the point-based sufficient conditions is that they take the deletion of individual points into account (instead of point-configurations). The proofs of all theorems can be found in the Author's corresponding publications.

3.2 Generating topology-preserving parallel reductions

The point-based conditions cannot be applied just for the validation of thinning algorithms' topology-preserving behaviour, but we can develop such new algorithms with them, whose topology preservation is guaranteed. For this purpose, the first step is establishing new parallel reductions based on all of these conditions.

Since only $(14, 14)$ pictures are considered on the BCC grid, we proposed one symmetric and one asymmetric operation for this type of pictures. For the FCC grid, we generated one symmetric and one asymmetric reduction for each of the three types of pictures.

The *support* of an image operator is the minimal set of points whose values determine whether a point is changed [12]. On the BCC grid, the support of the symmetric parallel reduction contains 64 points, and the count of points in the support of the asymmetric parallel reduction is 47. On the FCC grid, all generated symmetric and asymmetric reductions have a support with 84 and 64 elements, respectively.

Since all of these reductions are proved to be topology-preserving, they ensure a safe technique for designing topology-preserving parallel thinning algorithms.

4 Parallel kernel-thinning algorithms

With the help of the previously generated asymmetric reductions, in Chapter 4 of the PhD thesis, we derived fully-parallel kernel-thinning algorithms. Furthermore, two subfield-based methods are proposed in order to extract the topological kernels of objects sampled on the BCC grid.

4.1 Fully parallel algorithms

We constructed fully parallel kernel-thinning algorithms by iteratively applying the asymmetric reductions described in Chapter 3 of the dissertation. With this technique, we established four methods in total: one for the BCC grid, and three for each picture type of the FCC grid. We showed that all of these algorithms are capable of producing topological kernels.

It is important to note that we could also establish thinning algorithms from the symmetric reductions. However, the symmetric reductions are not suitable for kernel-thinning, since they cannot produce topological kernels. For example, if an object is formed by two black points, they will leave it unchanged instead of shrinking it to a isolated point.

4.2 Subfield-based algorithms on the BCC grid

We partitioned the BCC grid into four and eight subfields such that each point belongs to a different subfield than any of its 14-adjacent ones.

Due to the fact that there are not any two adjacent points in the same subfield, we can simplify our configuration-based sufficient condition: if only simple points are deleted, their parallel deletion is a topology-preserving reduction.

We reported two subfield-based parallel kernel-thinning algorithms with this deletion rule. The thinning process is terminated if no points are deleted within an iteration step). Note that some black points may become simple directly after a subiteration process, which may overdeform an object, and may cause that the resulting skeletal points are not centered of the input object. To prevent this phenomenon, we must ensure that only those black points are visited which were border points at the beginning of the actual iteration. That is why the border points are collected into a set before the first subiteration. We showed that these two methods have a similar computational cost due to the fact that any border point is examined exactly once in each iteration.

5 Distance-oriented sequential thinning

Distance transform is often applied in thinning in order to ensure topology preservation and produce geometrically correct skeleton-like features, especially medial surfaces and centerlines. These methods can be classified as follows:

- *Anchor-based thinning*: First, the set of CMBs is detected as safe skeletal (anchor) points. Then thinning is applied to connect CMBs and determining the set of further skeletal points.
- *Distance-driven thinning*: Distance mapping is followed by a thinning process. In the i-th iteration, only object points with distance value i are taken into consideration for possible deletion.
- *Hybrid approach*: It combines the anchor-based and distance-driven approaches.

Point p is called a *local maximum* if for any point $q \in N_i(p)$, $DT(p) \geq DT(q)$, where i is the considered neighborhood. In a Chamfer distance map, point p is a *center of maximal* *ball* (*CMB*, for short) [18] if $DT(q) < DT(p) + w$ for any point q concerned in the Chamfer mask, where w is the weight that corresponds to q in the Chamfer mask. Note that local maxima coincide with CMBs in case of neighbor-based distances.

It is showed that the original object can be completely reconstructed from the skeleton if all the CMBs are included in the produced medial surface [22].

In Chapter 5 of the PhD thesis, our attention is focused on sequential thinning combined with the distance transform to produce the medial surface and the centerline of objects. We used three types of distance functions: neighbor-based, weighted (Chamfer) [10], and the non-errorfree Euclidean distance [19].

5.1 Surface-thinning

Strand proposed the first – and, to the best of our knowledge, the only existing – surface-thinning algorithms for $(14, 14)$ pictures of the BCC grid and $(12, 12)$ pictures of the FCC grid. Both can be categorized as 'hybrid', since they combine the anchor-based and distance-driven approches. His methods rely on neighbor-based distances, and consider CMBs as safe skeleton points. As a consequence, the original objects are fully reconstructible from their medial surface. He introduced a surface edge point condition in order to preserve surface patches. The thinning algorithms consist of two thinning phases. Forward thinning reduces the input objects to 2–3 voxel thick surface patches, which are peeled further in the backward thinning. The unusual aspect of this process is that during backward thinning, object points are visited in descending order of their distance value. The sequential thinning is rather sensitive to the visiting order of border points. As a result, the final skeleton usually has some unwanted branches or surface segments. We showed that Strand's methods have nonlinear time complexity.

We established two modified versions of Strand's methods – an anchor-based and a distance-driven one - for each of the two considered picture types. In order to construct linear time surface-thinning methods, we merged the thinning phases in Strand's algorithms and simplified the organization of the thinning iterations. Our deletion rule also preserves non-simple points and surface edge points. We showed that our modified variants have linear time complexity by giving an upper limit to the visiting number of all black points during the thinning phase. Furthermore, we also determined that they are less sensitive to the visiting order of border points due to the fact that, in each iteration, the border points are visited in lexicographical order.

5.2 Curve-thinning

We established two, distance-driven curve skeletonization methods. One of them works on $(14, 14)$ pictures of the BCC grid, and the other one considers $(12, 12)$ pictures of the FCC grid. For this purpose, instead of surface edge points we retain either of two types of curve endpoints. We consider a border point deletable if it is simple but not an endpoint. We reorganised the thinning iterations: the non-deleted black points must be visited again during the further iterations in order to shrink the surface patches until line branches are only left. The sufficient number of iterations depends on the structure of the objects in the input picture. Therefore, it is impossible to set a constant upper limit to the visiting number of object points. As a result, the extraction of the curve skeleton has nonlinear time complexity. To ensure distance-driven thinning, all distinct distance values are collected into a set right after the distance transform. As a consequence, the total number of iterations is equal to the number of (positive) unique distance values, which depends on the shape of objects and the chosen distance function.

It is important to note that anchor point condition should not be used because the detected ridge points belong to the surface skeleton, so surface segments will probably be also generated.

We measured the reconstructibility of the original objects from their produced centerline on the FCC grid. We determined that the reconstructed object from a more compact representation usually have significantly less black points because of the loss of details of its shape.

On the FCC grid, we also measured the required time for the curve-thinning method to process. We can state that the better the approximation to the Euclidean distance is, the more time the thinning takes. This happens due to the growing number of distinct values, which implies that the number of thinning iterations is increasing as well. Additionally, the Euclidean DT takes more raster scans than chamfer ones through the picture, which makes it computationally even more expensive. We can also state that thinning with the stricter endpoint condition takes less time because it leaves fewer border points to visit for possible deletion in the remaining iterations.

6 Contributions of the thesis

In the **first thesis group**, my contributions are related to constructing sufficient conditions for topology-preserving parallel reductions. Detailed discussion can be found in Chapter 3 of the PhD thesis.

I/1. I verified the proposed configuration-based sufficient conditions for topology-preserving parallel reductions on $(14, 14)$ pictures of the BCC grid, and $(18, 12)$, $(12, 18)$, and $(12, 12)$ types of pictures of the FCC grid.

I/2. I verified the proposed symmetric and asymmetric point-based sufficient conditions for topology-preserving parallel reductions on $(14, 14)$ pictures of the BCC grid, and all three types of pictures of the FCC grid.

I/3. I implemented the two topology-preserving parallel reductions derived from the symmetric and asymmetric point-based conditions on $(14, 14)$ pictures of the BCC grid, and all three types of pictures of the FCC grid, and identified their support.

In the **second thesis group**, my contributions are related to constructing parallel kernel-thinning algorithms. Detailed discussion can be found in Chapter 4 of the PhD thesis.

II/1. I implemented the (topology-preserving) fully parallel kernel-thinning algorithm on $(14, 14)$ pictures of the BCC grid, and proved its capability of producing topological kernels.

II/2. I implemented the (topology-preserving) fully parallel kernel-thinning algorithms on all three types of pictures of the FCC grid.

II/3. I implemented and evaluated the proposed kernel-thinning algorithms working with 4 and 8 subfields on $(14, 14)$ pictures of the BCC grid.

In the **third thesis group**, my contributions are related to constructing computationally efficient sequential thinning methods to extract medial surfaces and centerlines. Detailed discussion can be found in Chapter 5 of the PhD thesis.

III/1. I reproduced Strand's sequential surface-thinning algorithms combined with neighbor-based distance information on $(14, 14)$ pictures of the BCC grid and $(12, 12)$ pictures of the FCC grid, and proved their runtime complexity.

III/2. I modified Strand's methods to produce surface skeletons in linear-time on both considered types of pictures.

III/3. I constructed a distance-driven sequential thinning method with two line endpoint criteria to extract curve skeletons directly from the input object on $(14, 14)$ pictures of the BCC grid.

III/4. I constructed a distance-driven sequential thinning method with two line endpoint criteria to extract curve skeletons directly from the input object on $(18, 12)$ pictures of the FCC grid. I measured this algorithm's execution time and the reconstructibility of the original objects from their produced centerline.

Table 1 summarizes the relation between the thesis points and the corresponding publications.

Table 1: *Correspondence between the thesis points and my publications.*

Publication	**Thesis point**									
	I/1	I/2	I/3	II/1	II/2	II/3	III/1	III/2	III/3	III/4
[1]							•	•	•	
[2]	•	•	•		•					
[3]							•	•		
[4]						•				
[5]										•
[6]	•	•	•	•						
[7]	•	•	•		•					
[8]					•					

The author’s publications on the subjects of the thesis

Journal publications

[1] **Karai, G.**, Kardos, P.: Distance-Based Skeletonization on the BCC Grid. *Acta Cybernetica* vol. 25(2), 351–367, 2021

[2] **Karai, G.**, Kardos, P., Palágyi, K.: Sufficient Conditions for Topology-Preserving Parallel Reductions on the Face-Centered Cubic Grid. *J. Math. Imaging Vis.* vol. 66, 271–292, 2024

Full papers in conference proceedings

[3] **Karai, G.**: Distance-Oriented Surface Skeletonization on the Face-Centered Cubic Grid. In *Proc. 1st Int. Joint Conf. on Discrete Geometry and Mathematical Morphology – DGMM 2021*, LNCS vol. 12708, 177–188, Springer, 2021

[4] **Karai, G.**, Kardos, P., Palágyi, K.: Subfield-based Parallel Kernel-thinning Algorithms on the BCC Grid. In *Proc. 11th Int. Conf. on Patt. Rec. Applications and Methods – ICPRAM 2022*, 288–295, SciTePress, 2022

[5] **Karai, G.**: Distance-Driven Curve-Thinning on the Face-Centered Cubic Grid. In *Proc. 2nd Int. Joint Conf. on Discrete Geometry and Mathematical Morphology – DGMM 2022*, LNCS vol. 13493, 354–365, Springer, 2022

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7 Összefoglalás

Az értekezés a digitális topológia és vázkijelölés területén mutat be új ismereteket két alternatív 3D rácsra, a tércentrált (BCC) kockarácsra és a lapcentrált (FCC) kockarácsra összpontosítva.

A munka három fő témakörből áll. Az első részben a párhuzamos redukciók topológia-megőrző tulajdonságára adtunk elegendő feltételeket, a másodikban az új feltételeinket használtuk fel zsugorító algoritmusok tervezésére, míg a harmadik részben hatékony szekvenciális vázkijelölő módszereket ismertettünk az objektumok középfelszínének és középvonalának kijelölésére.

A ,,Sufficient conditions for topology-preserving parallel reductions" című fejezetben konfiguráció-alapú és pont-alapú feltételeket fogalmaztunk meg a topológia megőrzésére párhuzamos redukciók számára a BCC rács $(14, 14)$, valamint az FCC rács $(18, 12)$, $(12, 18)$ és $(12, 12)$ képeire. Közös jellemzőjük, hogy a vizsgált képtípusokon előforduló összes lehetséges minimálisan nem egyszerű (MNS) halmazok törlésének tiltását követelik meg. A pont-alapú feltételek között megkülönböztetjük a szimmetrikusokat és az aszimmetrikusokat. Továbbá az aszimmetrikus pont-alapú feltételekből topológia-megőrző párhuzamos redukciókat származtattunk.

A ,,Parallel kernel-thinning algorithms" című fejezetben teljesen párhuzamos zsugorító algoritmusokat származtattunk az általunk konstruált redukciókból azok iteratív végrehajtásával mind a négy figyelembe vett képtípusra. Bebizonyítottuk, hogy nem marad egyszerű pont az erdményképeken, vagyis mindegyikük az objektumok topológiai magját állítja elő. Ezenkívül felosztottuk a BCC rácsot négy, illetve nyolc almezőre úgy, hogy minden pont különböző almezőbe tartozzon, mint bármely szomszédja. Ezt kihasználva olyan almező-alapú zsugorító algoritmusokat konstruáltunk, melyek egy-egy aliterációban az azonos almezőbe tartozó összes egyszerű pontot törlik. Ezáltal a többelemű MNS halmazok vizsgálata nélkül is garantáltan topológia-megőrző eljárásokhoz jutottunk.

A ,,Distance-oriented sequential thinning" című fejezetben kielemeztük Strand távolságtérképpel kombinált, középfelszínre vékonyító algoritumsait a BCC rács $(14, 14)$ és az FCC rács $(12, 12)$ képeire vonatkozóan. Megállapítottuk, hogy egyik sem lineáris futási komplexitású. Kidolgoztuk Strand eljárásainak két-két, bizonyítottan lineáris futási idejű módosított változatát. Ezenkívül a BCC rács $(14, 14)$ és az FCC rács $(18, 12)$ képeire javasoltunk középvonalra vékonyító eljárásokat. Ennek érdekében két-két vonalvégpont feltételt vezettünk be. Az FCC rácson értelmezett változatnál megállapítottuk, hogy a kompaktabb középvonalból rekonstruált objektumok általában jelentősen kevesebb fekete pontot tartalmaznak, mivel az eredeti alakjuk részletei elvesznek a tömörebb reprezentáció miatt.

Nyilatkozat

Karai Gábor „Topology preservation and skeletonization on the BCC and FCC grids" című PhD disszertációjában a következő eredményekben Karai Gábor hozzájárulása volt a meghatározó:

I/1 A topológia-megőrző párhuzamos redukciókra javasolt konfiguráció-alapú elegendő feltételek ellenőrzése a BCC rács *(14,14)* képeire, valamint az FCC rács mindhárom képtípusára, vagyis a *(18,12)*, *(12,18)* és *(12,12)* képekre.

I/2 A topológia-megőrző párhuzamos redukciókra javasolt szimmetrikus és aszimmetrikus pont-alapú elegendő feltételek ellenőrzése a BCC rács *(14,14)* képeire, valamint az FCC rács mindhárom képtípusára.

I/3 A topológia-megőrző párhuzamos redukciókra javasolt szimmetrikus és aszimmetrikus pont-alapú elegendő feltételekből származtatott (garantáltan topológia-megőrző) párhuzamos redukciók lokális környezeteinek meghatározása és implementálása a BCC rács *(14,14)* képeire, valamint az FCC rács mindhárom képtípusára.

II/1 Egy teljesen párhuzamos zsugorító algoritmus implementálása a BCC rács *(14,14)* képeire, továbbá annak bizonyítása, hogy az algoritmus topológiai magot állít elő.

II/2 Teljesen párhuzamos (topológia-megőrző) zsugorító algoritmusok implementálása az FCC rács mindhárom képtípusára.

II/3 4- és 8-almezős (topológia-megőrző) zsugorító algoritmusok implementálása és kiértékelése a BCC rács *(14,14)* képeire.

III/1 Strand algoritmusainak implementálása a BCC rács *(14,14)* képeire, valamint az FCC rács *(12,12)* képeire, valamint a futási komplexitásuk bizonyítása.

III/2 Strand algoritmusainak két, bizonyítottan lineáris időigényű módosított változatának kidolgozása mindkét vizsgált képtípuson.

III/3 Egy távolság-rendezett bejárási sorrendet követő, középvonalra vékonyító eljárás konstruálása két vonalvégpont-kritériummal a BCC rács *(14,14)* képeire.

III/4 Egy távolság-rendezett bejárási sorrendet követő, középvonalra vékonyító eljárás konstruálása két vonalvégpont-kritériummal az FCC rács *(18,12)* képeire. Az eljárás futási idejének és az eredeti objektumok rekonstruálhatóságának vizsgálata a kinyert középvonalukból.

Ezek az eredmények Karai Gábor PhD disszertációján kívül más tudományos fokozat megszerzésére nem használhatók fel.

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Doktori Iskola vezetője