Theses of Doctoral (Ph.D.) Dissertation

Design and Development of Global Optimization Methods with Applications

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Introduction

Global optimization is a vast field of expertise. Based on the attributes of the objective function and the search space, particular optimization methods better fit a problem than others, selecting the most appropriate algorithm or theoretical concept to solve a problem can be challenging. Optimization algorithms can be separated into classes depending on multiple attributes. We categorized them based on the application of random variables into three main classes, deterministic, stochastic, and hybrid algorithms.

In my dissertation, I discussed the design and development of solutions to global optimization problems to demonstrate the application of the above three main approaches, combined with interval arithmetic when mathematical rigorousness was required, furthermore I also presented the improvement of an existing optimization algorithm.

Circle covering with fixed centers

We aimed to determine the optimal cover of arbitrary polygons by open circles with fixed centers but variable radii using reliable, rigorous numerical tools, therefore we applied interval arithmetics with computer representable numbers in our algorithms. Optimality means that the sum of squares of radii is minimal. Formally, we have to find the minimum of the following objective function.

$$f(r_1, r_2, \ldots, r_n) = \sum_{i=1}^{n} r_i^2,$$

where $r_1, r_2, \ldots, r_n$ are the radii when we try to cover a polygon with n circles.

A solution to this type of circle covering problem can be used to optimize the required power of different towers in a network that broadcasts terrestrial signal for telecommunication.

Cover verification

First, we introduced an algorithm to verify whether a set of open circles with given centers and radii completely covers a polygon. Our approach is a branch-and-bound algorithm that systematically partitions the interval inclusion of the polygon into subintervals until all of them is covered by at least one of the circles, or a subinterval is found that is smaller than the predefined margin of error, and whose cover cannot be determined. See Figure 1 for illustration.

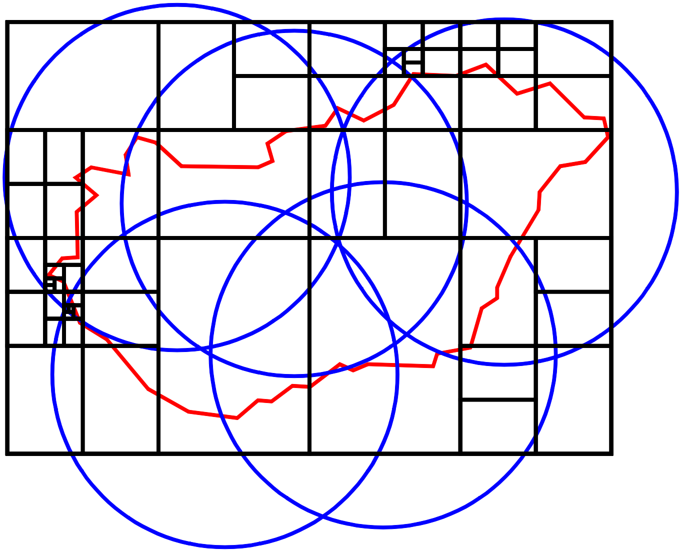
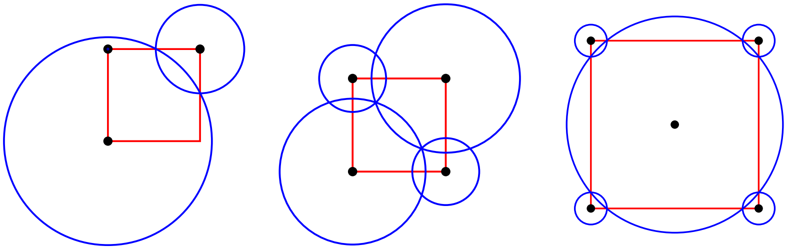


Figure 1: A set of circles covering the polygon approximation of the land of Hungary including the final partition that the algorithm generated.

When the margin of error is set to zero, the algorithm is guaranteed to terminate and recognize complete covers, but it will fail to stop for partial covers.

Configuration optimization

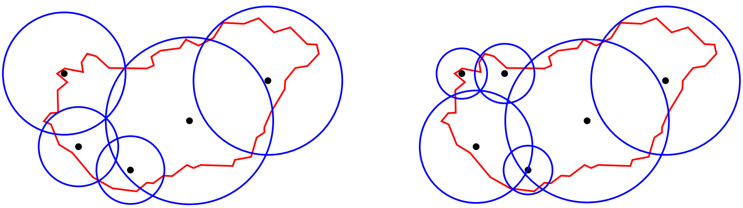
The optimization solution is also a branch-and-bound algorithm that searches the space of allowed radii until it finds an optimal complete cover. If some criteria are met by the search space, that can be easily ensured, the algorithm is guaranteed to terminate and return a configuration of radii with which the circles cover the target polygon and the value of objective function of this configuration is within the predefined precision compared to any global optima. For examples, see the Figure 2.



(a) Optimal covering from three corners.

(b) Optimal covering from all corners

(c) Optimal covering from the corners and the center of the square.



(d) Optimal covering by five circles.

(e) Optimal covering by six circles.

Figure 2: Example scenarios when we aim to optimally cover the unit square and the polygon approximation of the land of Hungary. Black dots denote the centers of the circles. The precision parameter of optimization was set to 1%.

Designing LED based streetlights

We were searching for an answer to a practical question, how to design streetlights with LED technology that provide better lighting than incandescent streetlights or other LED streetlights already available in the market. As the intersection of the target surface and a light cone of a LED light is an ellipse and the intensities of different light sources simply add up, we can interpret this designing problem as covering a rectangle shaped area with ellipses while overlapping is allowed. This is a multi-objective, global optimization problem with high dimensionality. Our solution was part of designing tool that we created for Wemont Kft, and it became part of an international patent about LED streetlight construction, for details see the patent [8].

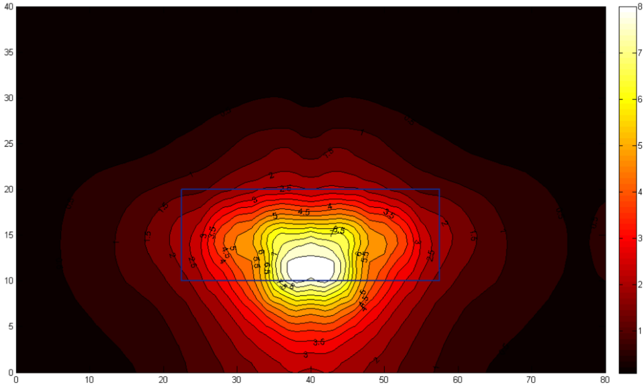


Figure 3: The light pattern of a regular incandescent streetlight. The rectangle represents the street surface that should be lighted. Colors denote the different levels of illuminance measured in LUX. Such street lights lack the necessary control to only light what should be visible at night and leave in the dark what should not be.

Light pattern calculation

We had to determine the light pattern of designs in order to be able to evaluate them. We developed a grid-based, custom light pattern calculation method for this purpose that determined the pattern using the geometry of the streetlight and the street, and the direction, position, and lighting characteristics of the applied LED packages in the lamp. The algorithm took advantage of any symmetries present in the pattern in order to reduce the number of grid vertices that must be handled.

Genetic algorithm

Our approach was a stochastic solution, we constructed a genetic algorithm whose essence was a geometric crossover concept being able to combine partially good parts of different designs based on the light pattern. First, we generate a random rectangle on the target surface and determine which LED lights' direction vectors point into the rectangle in both parent designs. The offspring designs will contain the LED lights that point into the rectangle from one parent, and the LED lights that point outside of the rectangle from the other parent. We swap light pattern parts instead of candidate parts this way as it is illustrated on Figure 4.

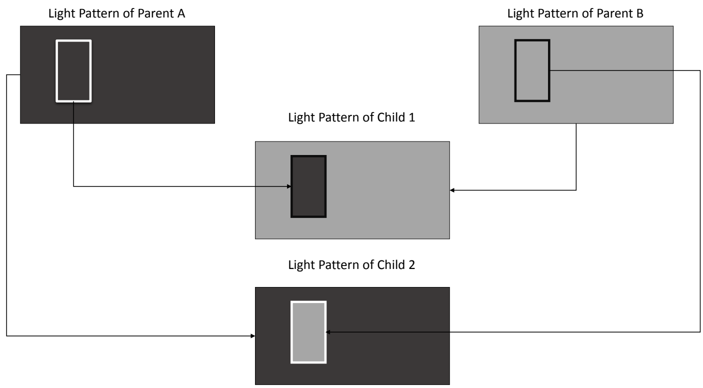


Figure 4: The recombination of two designs based on light patterns.

We used a configurable fitness function that allowed us to set the relative importance for the optimization of the different requirements measuring the cost and energy consumption of the LED packages, and the quality of the light pattern based on various statistics.

Before the partner firm would have designed streetlights for production, we tested our solution on synthetic lighting scenarios and created designs with light patterns of significantly better quality while the whole process, including the parametrization of the optimization, decreased to at least one fourth of the time that the engineers had previously needed for the same task. You can see deployed streetlights designed by our algorithm in Figure 5.

Figure 5: The genetic algorithm designed streetlights that light only the targeted public area. The light pattern is almost perfectly cut off along the edges. Source: `http://www.wemont.hu/activity/led-es-kozvilagitas`, accessed 19 May 2017.

Searching chaotic trajectories

We studied the forced damped pendulum, a mass point hung with a weightless solid rod whose other end point is fixed as illustrated in Figure 6.

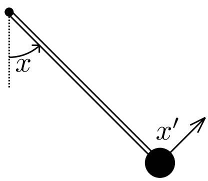


Figure 6: Illustration of the forced damped pendulum. x denotes the angle of the pendulum, and x' is angle velocity.

The pendulum is affected by the gravitation, a dampening friction, and an external periodic force. The system can be described by the Poincaré maps, a system of differential equation

$$x_1^{'}(t) = x_2(t),$$
$$x_2^{'}(t) = \sin(x_1(t)) - 0,1x_2(t) + \cos(t),$$

where x_1 is the angle and x_2 is the angle velocity.

Formally, a trajectory is the set of all solutions of this system on the initial conditions of $x_1(0) = \hat{x_1}$ and $x_2(0) = \hat{x_2}$ where $(\hat{x_1}, \hat{x_2})$ is the initial state of the pendulum.

If we study the trajectories of the pendulum divided into 2π long time intervals, the chaotic trajectories will be the bi-infinite series of three types of motions happening during these time intervals, the pendulum goes through the lower equilibrium point clockwise exactly once, it goes through the lower equilibrium point counterclockwise exactly once, or it does not go through the lower equilibrium point at all.

We created a method to locate regions of initial states from which the forced damped pendulum will execute a given sequence of the above motions.

Searching algorithm

We designed an objective function expressing the measure of difference between trajectories and the motion prescriptions based on the concept of Hausdorff distance, and we located initial states by optimizing this function with GLOBAL, a stochastic algorithm based on clustering. We used interval arithmetic to ensure reliable numeric results. Theoretically, our approach can find trajectories for arbitrary but fixed length series of motions, but in practice, the implementation of numeric representation limits the manageable length of such series.

Results

We were able to reveal regions of trajectories for all possible series of prescribed motions of length 3 and for a few other trajectories that we studied to demonstrate the capabilities of our solution. See Table 1 for examples.

Prescribed motions	Example initial state	Found trajectories	Function evaluations	Run time
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\oplus\oplus$	(3.5145566; 1.1854134)	3	2 305	666
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\oplus\otimes$	(3.541253; 1.1780008)	1	3 356	965
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\oplus\ominus$	(4.1354217; 1.1146838)	9	1 489	431
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\otimes\oplus$	(3.4500625; 1.2046848)	1	3 057	862
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\otimes\otimes$	(3.6355882; 1.1519576)	1	7 229	2 089
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\otimes\ominus$	(4.1873482; 1.1159454)	2	2 477	723
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\ominus\oplus$	(4.3271325; 1.1040739)	3	2 968	858
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\ominus\otimes$	(3.9656183; 1.0787189)	2	3 212	931
$\varepsilon_0, \varepsilon_1, \varepsilon_2 = \oplus\ominus\ominus$	(3.7628911; 1.096835)	7	1 863	540

Table 1: The optimization results for combinations of three prescribed motions. ε_i denotes the i^{th} 2π long time interval, $\ominus$, $\oplus$, and $\otimes$ represent the three motion types, going through counterclockwise, clockwise, and not going through the lower equilibrium point.

Improvement of a stochastic global optimization method

We reimplemented and algorithmically improved GLOBAL, a stochastic optimization method aiming to solve non-linear, constrained optimization problems. It is a versatile tool proven to be competitive in multiple comparisons, recently in [5]. The algorithm randomly generates sample points and starts local searches from them, but it tries to avoid the case when two local searches lead to the same optimum point. To achieve that, GLOBAL clusters the generated points to already found local optima, and it starts local searches exclusively from the points that cannot be added to any of the existing clusters.

Algorithmic improvements

The general idea behind our improvements is that comparing points computationally costs much less than evaluating the objective function, therefore a more thorough clustering can reduce the number of local searches and thus the objective function evaluations that will result in a shorter runtime for the whole optimization.

We kept the basic approach of single-linkage clustering but modified it to be recursive. In each iteration, the original algorithm attempted to add a point to any of the clusters only once and started local searches from the unclustered points. With our modification, GLOBAL retries clustering points whenever any of the clusters gained a new element in the previous attempt because the adding a point to a cluster depends on the point by point comparisons of the unclustered point with the elements of the clusters. In addition, the renewed GLOBAL also launches a clustering after each local search as they are guaranteed to change an existing cluster or create a new one.

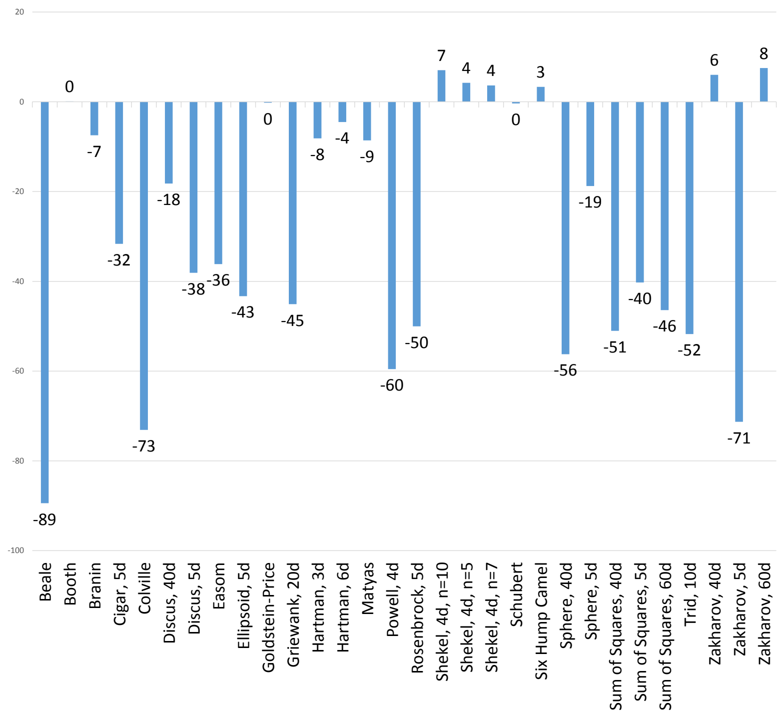


Figure 7: The relative change measured in percent in the average of executed objective function evaluations by the improved GLOBAL compared to the original algorithm.

We also decided to hold onto all the clustering information during the whole optimization, that was partially dropped in each iteration of the algorithm previously. The portion of points having worse objective function values are mainly located far from the local optima that act like cluster centers. Before this modification, clusters became more dense over the iterations because only the points closer to the centers were carried over from one iteration to the next one, and new clusters might have appeared in the place where discarded points were located previously. This is the exploration of the same region of the search space multiple times that is redundant and thus inefficient.

We compared the modified GLOBAL with the original one and experienced a significant improvement in the number of objective function evaluations to find the global optimum on a wide range of optimization test functions, see Figure 7 for details.

Redesigned implementation

We created a new modularized JAVA implementation of the improved GLOBAL that provides an easy way to customize not just the applied local search method but the clustering algorithm as well. By default, the implementation uses the new single-linkage clustering algorithm and UNIRANDI, a random walk method that executes line searches, as the local search method.

Summary of contributions

I summarized my thesis points and corresponding publications in four groups.

1. I developed a deterministic solution to find optimal covers of arbitrary polygons by circles with fixed centers but variable radii using interval arithmetic to ensure numerical reliability. Optimality means that the sum of the squares of the radii is minimal. The method and the numerical results have been published in the paper [4].

    (a) First, I constructed a branch-and-bound algorithm using interval arithmetic to verify within a predefined precision whether a polygon is covered by a given configuration, pairs of centers and radii, of open circles. I presented the theoretical analysis of the algorithm that proves its correctness, examples to illustrate the operation of the algorithm were also provided.

(b) I constructed a second branch-and-bound algorithm, also using interval arithmetic, to find the radii configuration of circles with fixed centers that optimally covers a given polygon within a predefined precision. I provided a proof the correctness of the algorithm, and I also participated in the compilation of results and their numerical analysis of optimality.

2. I created a stochastic solution to design LED based streetlights that can have better light pattern, energy consumption, and production cost compared to what other streetlights can offer. The solution has been published in the book [7]. I similarly approached another high complexity problem of designing failure detection systems that offered the same level of challenge in the paper [3].

   (a) I constructed an algorithm to determine the light pattern of a LED streetlight. The result of the algorithm are illuminance values in 2-dimensional grid over the target street. The way of calculation considers the effect of neighboring streetlights. The algorithm leverages any axis of symmetry present in the lighting scenario and only handles the required minimal number of vertices to be able to construct the complete light pattern. I implemented the algorithm both on the CPU and on the GPU.

   (b) I developed a genetic algorithm that optimizes designs for given lighting scenarios. The fitness function is a configurable multi-objective function measuring the goodness of the light pattern, and the cost and consumption of the LED packages in the design. The essence of the algorithm is the geometric crossover operator that combines designs based on their light patterns. I implemented the algorithm and participated in the creation of numerical test results on synthetic lighting scenarios.

3. To study the chaotic trajectories of the forced damped pendulum, I developed a numerically reliable method to be able to find initial states of this system from which it will execute a given series of prescribed motions of arbitrary but fixed length. The method and numeric results has been published in the paper [6].

(a) I created an objective function that measures the deviation of a trajectory from a given series of prescribed motions. The function compares the interval inclusions of the trajectory and the regions of states corresponding to the prescribed motions.

(b) I participated in the integration of the objective function with GLOBAL, a stochastic optimization algorithm, thus obtaining a method capable of finding initial states of the forced damped pendulum from which it will execute a given series of prescribed motions of fixed length. Theoretically, the length of the series can be arbitrary long, but it is limited in practice depending on the implementation. I participated in the creation of numerical results, and I found regions of initial states belonging to various series of prescribed motions.

4. I added algorithmic improvements to GLOBAL, a stochastic global optimization method, and created a new implementation of the algorithm with these modifications. The improved algorithm and the comparison of the new and the previous implementations have been published in the book [1] and in the paper [2].

   (a) I modified the original single-linkage clustering strategy to use the available cluster membership information more efficiently to prevent the initiation of local searches that would probably lead to already known local optima. I also altered GLOBAL to keep the cluster membership information when the algorithm periodically drops a portion of the generated sample points that further prevents unnecessary objective function evaluations at the cost of increased memory usage. I participated in the numerical comparison of the modified and the original algorithm.

   (b) I created a modularized implementation of GLOBAL in JAVA with my algorithmic improvements that provides the option of customizing the local search method and the clustering algorithm as well.

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