Detailed abstract of the dissertation

The main topics of my dissertation are *extremal graph theory*, and *bootstrap percolation*.

Extremal graph theory is a classical, well studied part of combinatorics. A central question of extremal graph theory concerns the maximum number of edges in a graph (network) on n vertices, containing no subgraph isomorphic to a fixed smaller graph. Extremal graph theory was one of the favorite research topics of Paul Erdős, and from the 1950s, many of the best known graph theorists worked in this area. In the last ten years much research has been devoted to *global* questions concerning *induced subgraphs*, going beyond the study of specific problems. In applications, such a question might appear in the following way: how can a computer network be built, if certain substructures cannot occur (for example it cannot contain five computers with each of the ten pairs connected). A property in this sense is a set of possible computer networks, which satisfy some restrictions. The size of a graph property is the cardinality of graphs in the property. Our goal, which is a living important topic in recent mathematical research, was to give estimates for the size of these sets. This type of question has suddenly become very important in the age of the internet. The significance of our results is that we show for some important classes of graph properties that their sizes fall into one of a number of narrow ranges.

In the last fifteen years, Erdős, Frankl, Rödl, Prömel, Steger, Bollobás, Thomason and many others studied these questions and obtained numerous substantial results. With Bollobás and Weinreich, we investigated the possible size and structure of a hereditary property, when the possible size is not very large, i.e., $2^{o(n^2)}$, solving three conjectures of Scheinerman and Zito. In particular, we gave a full description of the hereditary properties in the range below n^n, and we classified all possible sizes. Erdős, Frankl and Rödl proved various monotone results for properties having size in the higher ranges, and their work was continued by Prömel and Steger. These results imply that for every monotone property there is a natural number k such that the size of the property is $2^{(1-1/k+o(1))n^2}$. Further, in a series of papers, Prömel and Steger showed that the error term can be sharpened for some natural properties. Recently, with Bollobás and Simonovits, we managed to extend these results to all monotone properties. In particular we have given essentially sharp estimates: for all monotone properties, there is an integer k and a positive constant c such that the cardinality of a property is between $2^{(1-1/k)\binom{n}{2}}$ and $2^{(1-1/k)\binom{n}{2}+n^{2-c}}$. Our results are proved by applying techniques of probabilistic combinatorics, exploiting properties of the automorphism groups of graphs and hypergraphs, and making use of a number of classical combinatorial theorems including the famous Szemerédi's Regularity Lemma.

As a graduate student in Hungary, I lectured on combinatorics to undergraduates. With one of them, Pete, I developed a geometric approach to *bootstrap percolation* which, when used with probabilistic tools, gives some of the main results in this area. Bootstrap percolation originated in the 1990s in statistical physics and biology, where it was used as a model of cellular automata. With our new method, Pete and I could simplify the proof of a fundamental result of Aizenman and Lebowitz in *finite size* scaling bootstrap percolation. In Memphis, I continued my work on these finite models of bootstrap percolation (which tend to be less amenable to standard techniques than the infinite models). The speed of the transition is always an important question, and with Bollobás we have proved the existence of sharp threshold functions. Furthermore, we have given good bounds on the threshold function of the bootstrap percolation on the hypercube.

I am interested in looking at several questions left open in my dissertation on hereditary and monotone graph properties and on bootstrap percolation. There is no doubt that the two main areas of my research so far, the theory of hereditary graph properties and bootstrap percolation, are far from being exhausted and will remain fertile areas for a long time. In fact, it is very likely that answering a number of major questions would transform our understanding of these fields.