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Gravitational lensing in alternative theories of gravitation

Abstract of Ph.D. thesis

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1. Introduction

According to general relativity the energy-momentum makes the space-time curved, and the orbit of the light bends accordingly. This is taken into account by the gravitational lensing, which has become a useful tool in studying the properties of the gravitational field. The initial observations were used to verify the predictions of the theory. In the last years it has been employed to study the large scale structure of the Universe, mapping the distribution of the dark matter.

The propagation of light in the curved space-time provides a number of observable effects. These can be measured using current telescopes. The most obvious one is the production of multiple images and the angular separations of them. The lensing produces a change in brightness of the images. If the source or the lens has a time dependence, the changes in arrival times of light signals are measurable. The shape and orientation of the extended sources and the images of them differ.

If one or more of the light rays originating from the source reach the observer, then this fact is expressed by some lens equation. According to general relativity the light propagates along null geodesics. In the weak field approximation the geodesics are straight lines in zeroth order, therefore we use straight lines to describe the orbits of light. This approximation employs sections and Euclidean trigonometry. The light rays which are curved in the neighborhood of the lens can be replaced by two sections with a kink near the lens. The change in direction of the light is described by the deflection angle, which depends on the mass distribution of the lens and the impact parameter of the light.

In the case of weak lensing the small angle approximation holds for the positions of the source and the image, and for the deflection angle. In the case of strong lensing the deflection angle is close to an integer multiple of π . Usually this is an even multiple, this is called relativistic lensing. The corresponding images are called the relativistic or higher order images.

If the lens is point-like then the lensing geometry becomes axially symmetric [9]. A number of lens equations have been derived for this case [10]. Among these the Virbhadra-Ellis equation [11] is employed frequently in the literature.

If the lens is a Schwarzschild black hole then two images form (or an Einstein ring, if the source, the lens and the observer are collinear). The following classical result is important in the thesis: for image separations greater than about 2.5 times the Einstein angle, the ratio of the fluxes obeys a power law [9].

$$\frac{\mu_1}{\mu_2} \propto \left(\frac{\Delta\theta}{\theta_E} \right)^\kappa. \quad (1.1)$$

The exponent is $\kappa_{Sch} = 6.22 \pm 0.15$. Usually the lens can not be observed, thus the apparent angles can not be measured separately.

The source brightness is unknown, therefore the individual magnifications can not be measured. However the angular separation of the images can be measured, therefore the ratio of the magnifications and the image separation can be related to each other, which can be related to the law (1.1). This provides a method to determine whether the lens is a Schwarzschild black hole.

I study gravitational lensing in three selected alternative theory of gravitation in the thesis. Gravitational lensing can be used to determine which among the various gravitational theories is correct. The observations do not exclusively reflect the effects of the unknown forms of matter, but also the deviation of the dynamics of gravitation from general relativity. The Einstein equation has to be modified to explain the observations. For this one has to add some non-standard matter to the energy-momentum tensor or has to replace the dynamics of gravitation. Spherically symmetric black hole solutions of the form $ds^2 = g_{tt}(r) dt^2 + g_{rr}(r) dr^2 + r^2 (d\Theta^2 + \sin^2 \Theta d\varphi^2)$ have been derived in all the three theories examined.

Brane-world models have standard-model matter confined to a 3+1 dimensional hypersurface, and gravity acting in a higher dimensional space-time [12]. The effective Einstein equation [13] is valid instead of the Einstein equation on the brane, and it has black hole solutions. The tidal charged black hole is a static, spherically symmetric, vacuum solution [14]:

$$g_{tt}(r) = -\frac{1}{g_{rr}(r)} = -1 + \frac{2m}{r} - \frac{q}{r^2}. \quad (1.2)$$

It is characterized by two parameters: the mass m and the tidal charge q . The tidal charge arises from the Weyl curvature of the 5-dimensional space-time in which the brane is embedded. Despite the tidal charge is similar to the square of the electric charge of the Reissner–Nordström black hole in general relativity, the negative tidal charge is without classical counterpart.

The Hořava-Lifshitz theory is a family of field theories, in which there is a preferred foliation of the space-time, violating the Lorentz invariance [15]. The Einstein–Hilbert action is decomposed to the sum of the kinetic term $\mathcal{T} = K^{ij}K_{ij} - (\xi - 1)K^2$ and a potential term, then these terms are extended individually by adding extra terms to them [16]. The field theory obtained can be interpreted as a theory of gravitation. General relativity is recovered in the limit $\xi \rightarrow 1$. The action implies a spin-0 field in the dynamics called the scalar mode for the graviton. Applications range from cosmology, dark matter, dark energy to spherically symmetric space-times. Several versions of the theory have been proposed. The infrared-modified Hořava-Lifshitz theory is the one which is consistent with the current observational data [17]. The following static, spherically symmetric, vacuum space-time has been derived in Ref. [18]:

$$g_{tt}(r) = -\frac{1}{g_{rr}(r)} = -1 - \omega r^2 \left[1 - \left(1 + \frac{4m}{\omega r^3} \right)^{1/2} \right]. \quad (1.3)$$

ω is the Hořava-Lifshitz parameter and m is the mass of the black hole. The $\omega \rightarrow \infty$ limit is the Schwarzschild metric, the $\omega \rightarrow 0$ limit is the flat space-time.

In the $f(R)$ theories of gravitation the geometric side of the Einstein equation is modified, instead of the introduction of exotic energy-momentum tensors [19]-[20]. (In order to interpret the observations in the framework of general relativity, the dark matter and dark energy have been introduced.) The Einstein-Hilbert action is replaced by a generic function of the Ricci curvature. The field equations derived from the action can be recast as

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{1}{f'(R)} \left\{ \frac{1}{2} g_{\mu\nu} [f(R) - Rf'(R)] + \nabla_\nu \nabla_\mu f'(R) - g_{\mu\nu} g^{cd} \nabla_c \nabla_d f'(R) \right\} + \frac{T_{\mu\nu}^{(m)}}{f'(R)}. \quad (1.4)$$

The first term in Eq. (1.4) can be interpreted as an effective energy-momentum tensor of geometric origin. If $f(R) \propto R^n$ then the special theory obtained is called the R^n theory. The static, spherically symmetric, vacuum solution of the field equations in this theory is [21]

$$\begin{aligned} g_{tt}(r) &= -\frac{1}{g_{rr}(r)} = -1 - \frac{2\Phi(r)}{c^2}, \\ \Phi(r; \sigma, r_c) &= -\frac{Gm}{2r} \left[1 + \left(\frac{r}{r_c} \right)^\sigma \right]. \end{aligned} \quad (1.5)$$

$\Phi(r)$ is the gravitational potential in the distance r from the point mass m . σ is the strength of gravity parameter and r_c is a characteristic radius. The modification of the potential of the point mass influences the Galactic dynamics. The effective energy-momentum tensor describes an anisotropic curvature fluid, which violates all of the usual energy conditions.

2. New scientific results

1. I derived a new lens equation [1] by trigonometric considerations valid for point-like and spherically symmetric lenses. No power series expansions of the trigonometric functions were applied, in this sense this is an exact lens equation. The equation is more accurate than the Virbhadra-Ellis equation. It reduces however to the Virbhadra-Ellis equation in a proper limit. As for the solutions significant differences are to be expected, if the source and the observer are placed asymmetrically with respect to the lens.

I have carried on expansions in the small mass and tidal charge parameters, then in the small angles related to the positions of the source and the images. This way I obtained algebraic lens equations [1]. Among the various cases discussed the tidal charge dominated lensing has different predictions from the new lens equation and the Virbhadra-Ellis equation. This follows from that the Virbhadra-Ellis equation does not predict some of the higher order terms, or it predicts them with different coefficients.

I have analysed how the image separations and the flux ratios are modified as compared to the Schwarzschild lensing, by the perturbations arising from second order mass and first order tidal charge contributions. The most apparent modification appears in the flux ratio, this is presented on Fig. 3. of Ref. [1]. Depending on the sign of the quantity $q - 5m^2$ the flux ratio can be either increased or decreased compared to the Schwarzschild lensing.

In the case of mass-dominated weak lensing the positions of the images are similar [1] to the Reissner-Nordström black hole lensing [22]. In the case of the tidal charge-dominated lensing the effect of the lens with positive tidal charge resembles [1] the lensing properties of a negative mass Schwarzschild lens [23]. In the case of a dominant negative tidal charge, similarly to the positive mass Schwarzschild lens, one positive and one negative image emerge. The location of the images is different, this is shown on Fig. 4. of Ref. [1].

2. I have demonstrated that the image separation and the ratio of the fluxes obey the power law (1.1) for image separations greater than about 2.5 times the Einstein angle. Fig. 6. of Ref. [1] presents the ratio of the magnifications of the images as function of the image separation normalized to the Einstein angle, on the log-log scale. According to the curve presented on the figure the exponent in the space-time (1.2) is $\kappa_q = 2.85 \pm 0.25$. Since this value differs from the Schwarzschild value, measuring the fluxes and the angular separation of the images provides a distinction between the Schwarzschild space-time and the (1.2) space-time.

3. I derived a formula for the radius of the first relativistic Einstein ring in the tidal charged black hole space-time [2]. The Einstein angle is a function of the lens mass, the tidal charge and the impact parameter. The tidal charge modifies both weak and strong lensing characteristics of the black hole. Even if strong lensing measurements are in agreement with the Schwarzschild lens model, the margin of error of the detecting instrument (the designed GRAVITY interferometer [24]) allows for a certain tidal charge. The study of the angular radius of the first relativistic Einstein ring led to the constraint $q \in [-1.815, 0.524] \times 10^{20} \text{ m}^2$ for the supermassive black hole in the Galactic Center [2].

4a. I have demonstrated that for every value of the Hořava-Lifshitz parameter there exists a maximal deflection angle δ_{max} , occurring at the corresponding distance of minimal approach r_{crit} . All the rays passing the lens both above or below r_{crit} will experience less deflection, than the one passing through r_{crit} . This effect is explained by Figs. 4. and 6. of Ref. [3].

The existence of the maximal deflection angle δ_{max} implies that for any mass and lensing geometry there is an ω for which only the positive image forms. Since rays forming the negative images have larger deflection angle, than the ones responsible for the positive images. For each mass and lensing geometry there is an ω , such that the corresponding δ_{max} will not be sufficient to deflect any of the rays passing below the lens to the observer. Therefore the negative image does not form. This feature does not exist in the Schwarzschild lensing.

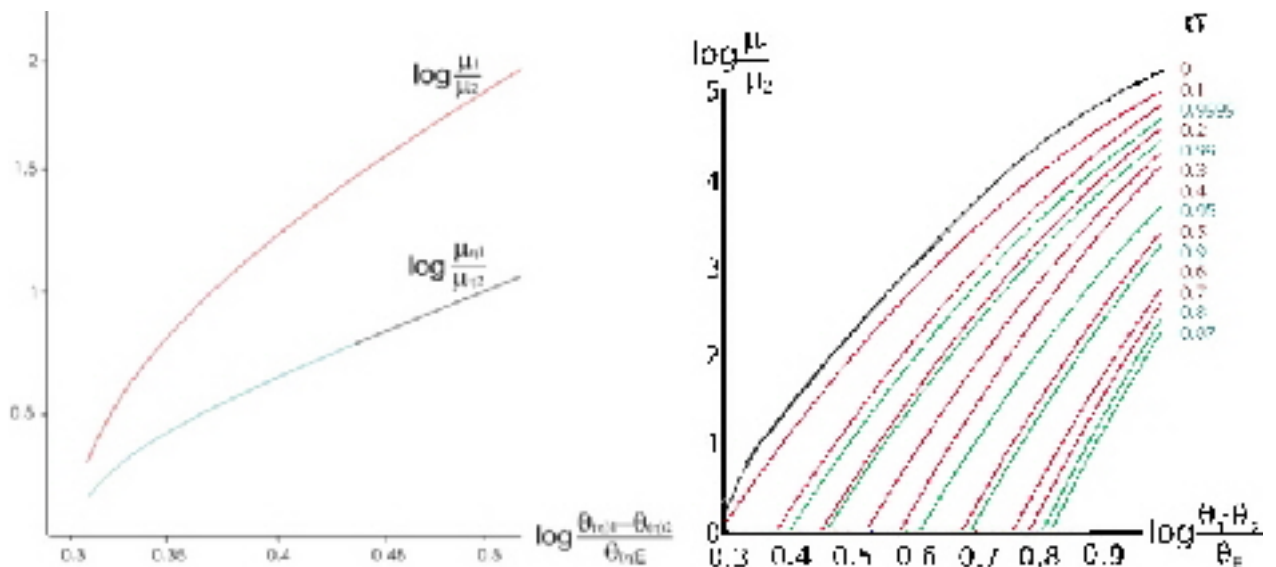
4b. I have determined the order of the Hořava-Lifshitz parameter compatible with the observations from weak lensing. The dimensionless Hořava-Lifshitz parameter normalized with the square of the mass is of order 10^{-16} . The results are presented in the tables I. and II. of Ref. [3]. I discussed also the first and second relativistic Einstein rings emerging in strong lensing. I compared the constraints with related results in the literature [25], and concluded that the constraint presented in the table III. of Ref. [3] is the strongest one up to date.

5. I have found that the characteristic radius r_c of the compact object with metric (1.5) divides the space-time into two regions according to the strength of the gravitational potential. Gravity is weakened in the region $r < r_c$, and strengthened in the region $r_c < r$ as compared to the Newtonian potential.

I have computed the image positions for $\sigma = 0.25$ and $\sigma = 0.75$ [4]. For the larger value of σ the image separation grows faster with an increase in the mass and grows more slowly as the source moves away from the optical axis. For fixed β the magnification of the images increase with σ , especially the magnification of the positive image. The most apparent increase is found in the flux ratio μ_1/μ_2 .

6. I have demonstrated that the image separation and the ratio of the fluxes obey the power law (1.1) for image separations greater than about 2.5 times the Einstein angle. Fig. 8. of Ref. [4] presents the ratio of the magnifications of the images as function of the image separation normalized to the Einstein angle, on the log-log scale. Based on the curves presented, the dependence of the exponent κ on σ in the space-time (1.5) is shown in the table I. of Ref. [4]. The function $\kappa(\sigma)$ has a double degeneracy, except a small neighborhood of the general relativity limit $\sigma = 0$. Consequently the future measurements of the slope κ of the curves characterizing the ratio of the magnifications should be able to constrain the parameter σ . The observations can either support or falsify the R^n theory.

It is a common feature of the tidal charged black hole and of the lens in the R^n theory that the ratio of the fluxes of the images as a function of the separation of the images obeys a power law, which differs from the power law of the Schwarzschild black hole. For the tidal charged black hole the exponent κ is smaller than the exponent for the Schwarzschild lens. For the lens in the R^n theory the exponent κ is larger than in the Schwarzschild case (for every nonzero σ).



The logarithm of the ratio of the magnifications of the images as a function of the logarithm of the image separation divided by the Einstein angle. The tidal charge dominated lens is presented by the lower curve on the left figure. The right figure shows the R^n black hole lensing for a series of σ . The upper curve represents the Schwarzschild lensing on both figures.

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