Aurél Gábris

Theses of the Dissertation

Multi-partite quantum optical systems for quantum information processing

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Research Institute for Solid State Physics and Optics
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Supervisor
József Janszky
Member of the
Hungarian Academy of Sciences

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Abstract

Experimental quantum optics has produced very successful solutions for preparation and observation of isolated quantum systems. In addition, the theoretical toolbox of quantum optics is also suitable for the description of the more general fundamental problems of quantum theory, such as those arising in the area of quantum information. It is therefore not surprising that quantum optics has found many applications in both experimental and theoretical aspects of quantum information processing. The great potentials of quantum information can be harnessed with multi-partite quantum systems, therefore their study is a high priority task.

In the dissertation we presented a reduced dimensional coherent state representation of entangled states. Generalised the optical Schrödinger cat states to two modes, and studied the teleportation and decoherence of coherent state quantum bits. Proposed two schemes for the realisation of universal quantum logic using atoms trapped in cavities. In these proposals, we aimed at scalability and efficient suppression of the effects of decoherence.

Overview

Quantum information science has produced revolutionary theoretical results to improve efficiency of computation, and security of communication. These results, however, remain promises until large scale experimental realizations emerge. For the implementation of quantum information processing protocols, first a right representation has to be found in the chosen physical system. In addition, specifically for quantum computation, the robust manipulation of quantum information is also necessary.

In quantum optical implementations the physical system generally consists of one or more electromagnetic modes, and possibly atoms in a cavity. These are the candidates for providing quantum bits. Presently, pure optical quantum systems usually consist of several electromagnetic modes, and the most successfully used basic components are passive linear elements (such as beam splitters), and photo detectors. In these systems quantum bits are encoded in the states of electromagnetic modes, and the transformations for quantum logic are realized using the linear elements. Certain schemes combine these with detector feedback. Since the number of degrees of freedom of the electromagnetic field is very large, the choices of representing quantum bits are equally numerous. One already has the choice to use either one, two or more modes to represent a single qubit. And even when only a single mode is selected, we have the choice of working in a discrete or a continuous representation, and the dimensionality of the subspace designated to the representation is also at our choice.

In cavity QED implementations, the atoms in the cavity can also be used to represent quantum bits. In this case, in addition to the freedom of choice regarding the electromagnetic field, also the relevant electronic energy level structure of the atoms can be varied by choosing which kind of atoms and which of their spectral resonances are used. We see a wealth of additional possibilities to unfold when, in order to provide a basis for manipulation of quantum information, the atom–photon interaction inside the cavity is included in the description. From the theoretical point of view, this system allows immense number of configurations by choosing the atomic level structure, the polarisations and frequencies of the near-resonant cavity modes, and external pump lasers. Research on cavity QED quantum computing is concerned with finding out of these configurations, the most suitable under the various conditions.

Very important ones of these conditions, are decoherence phenomena present in such systems. These phenomena pose great obstacle to experimental realizations, and therefore must be considered in every theoretical proposal in as much detail as possible. Generally, decoherence comes from unwanted interaction of quantum bits with the environment or each other, and ultimately can lead to the loss of all the information initially encoded in the qubits. The typical decoherence channels in cavity QED systems are photon loss, and spontaneous decay of the excited atomic states.

Objectives

The objectives of the Dissertation can be summarised as follows:

- Development of coherent state representation of general states of the two-mode electromagnetic field and of quantum bits, with a focus on the decoherence of the latter.
- Theoretical proposal for realisation of universal quantum logic using trapped atoms and cavity systems, aiming for scalability and fault tolerance.

Methods

Chapter 2 and Chapter 3 of the Dissertation can be identified as employing somewhat different systems and interactions. We shall introduce their methods separately in the following two subsections.

Coherent state quantum bits

Coherent states are an interesting class of quantum states of the electromagnetic field for several reasons. These are important from the point of quantum state representation, since these form a complete continuous basis, and are also the natural basis for discussing standard decoherence effects of light. Additionally, coherent states can be produced by ideal lasers, and therefore these states constitute a readily available resource. Our results related to coherent state quantum bits has been recollected in Chapter 2 of the Dissertation.

Coherent states are labelled by a complex variable that corresponds to the classical amplitude and phase. Their important feature from the aspect of quantum state representation is that coherent state basis is over complete, therefore the expansion into coherent states is not unique, furthermore, all coherent states overlap. This property also implies that the In the one-dimensional coherent state representation, every quantum state of the electromagnetic field is expanded using only a set of coherent states that lie along a single continuous curve in phase space. It is called one-dimensional, because a curve can be parametrised using one real variable. This representation works because of the over completeness of the coherent basis. A special case of the one-dimensional coherent state representation is when the chosen curve is a straight line across the origin. In this case a discrete basis can be expressed using Hermite polynomials.

The need for a two mode generalisation of the one-dimensional coherent state representation arises from the need to describe entangled states. We have made a generalisation based on two-mode squeezing. This work has been presented in Sec. 2.1, where we have first given a coherent state expansion of the two mode squeezed vacuum in terms of two-mode coherent states such as $|\alpha\rangle|\alpha^*\rangle$, using only a single complex variable. The weight function turned out to be a Gaussian, and we have proceeded showing that a discrete basis can be defined in terms of Laguerre-2D polynomials. We have proven the completeness of the thus defined one-complex plane representation using the completeness property of the Laguerre-2D basis.

The one-complex plane representation is especially useful for discussing two-mode squeezed states and describing the effects of decoherence. These advantages had later been exploited in the study of continuous variable teleportation both in the ideal and in lossy case.

In Sec. 2.2 we have introduced two-mode Schrödinger cat states, starting from the one-complex plane representation. These states are generalisations of the usual optical Schrödinger cat states that exhibit up to two ebits of entanglement. We have developed a formalism to calculate the entanglement of a pure quantum state given in a non-orthogonal basis. We have calculated the entanglement of four different type of two-mode Schrödinger cats using this formalism. We identified which kind can be considered to be the best resource of reliable entanglement, and used it as the quantum channel in a teleportation protocol. We calculated explicitly what is the required Bell projection, and theoretically constructed an optical scheme that may be used to obtain successful teleportation with probability 1/8.

In Sec. 2.3 we have considered the decoherence problem of coherent state quantum bits using a model often used to describe decoherence in optics. In this model the interaction with the environment is represented by a beam splitter, with the signal and the environment mode entering on its two input ports. The state of the environment after the interaction is discarded by partial tracing. Under natural conditions the environment of an electromagnetic field is a thermal state very close to the vacuum, however, in certain cases the immediate environment of the observed system could be prepared in an alternative state.

We have studied a read-out problem of a deterministic quantum decision problem algorithm, under the effect of the above mentioned decoherence mechanism. The speciality of such an algorithm is that its output can only be one of the basis states, and cannot be a superposition. Examples to these are the Deutsch and the Deutsch–Jozsa algorithms. Therefore, in case of a coherent state based implementation, the read-out problem involves the discrimination of the two optical Schrödinger cat states. We have used the quantum version of the Sanov theorem to quantify distinguishablity. We could therefore give the probability of successfully distinguishing the two outcomes using optimal measurements, by calculating quantum relative entropies.

We have studied a case when the environment can be prepared in a squeezed vacuum state, and searched for an optimal value of the squeezing parameter that yields the best distinguishablity for a given representation and decoherence. We have found that a single variable is sufficient to incorporate effect of phase relations on the distinguishablity. This phase variable additively contains the phase of the Schrödinger cats, the squeezed state of the environment, and the phases of the beam splitter. Strong numerical evidence suggests that the optimal value for the thus defined phase variable is zero. We have studied the dependence of distinguishablity on the beam splitter transmission coefficient, the intensity of the coherent states in the Schrödinger cat states, and the magnitude of squeezing. Our numerical studies have shown that for every value of transmittance and intensity, there is an optimal value of squeezing. The results also indicate that the optimal relative improvement, due to the squeezing of the environment, increases with intensity of the coherent states in the basis states.

Quantum logic in atom–cavity systems

In Chapter 3 we have presented two proposals for cavity quantum electrodynamics quantum computing. In both schemes we have assumed that we had well localised, neutral atoms strongly coupled to a cavity field. Since the atoms are neutral there is no direct interaction between them, and the cavity field may act as the only significant mediator.

In the proposal of Sec. 3.1 we have considered a scheme where the atoms have two ground states and two excited states, the cavity has two modes with opposite circular polarisation nearly resonant with the atomic transition. When the cavity resonance frequencies are far detuned from the atomic transitions, and we start from an initial state where the cavity contains only one excitation and the atoms are in a superposition of the two ground states, the Hamiltonian of the system can be greatly simplified using certain approximations. Under these conditions the Hamiltonian takes the form of a spin z–z interaction with a star shaped topology, where a central spin is pair-wise coupled to all the other spins, and the rest are not coupled to each other. In this analogy, the central spin corresponds to the polarised photon in the cavity, and the other spins to the two-level atoms.

To formulate quantum logic, we have associated one quantum bit to each atom, and an additional one to the polarised photon in the cavity. For N atoms this yielded $N+1$ quantum bits, the photonic quantum bit playing a distinguished role. Using the refocussing technique from nuclear magnetic resonance quantum computing, we have derived effective two qubit gates based based on the above-mentioned spin z–z coupling. To perform refocussing, it is sufficient to realise single qubit operations on the atoms, and such operations can be carried out using resonant Raman pulses. We have suggested that to realize a CNOT gate, a sufficient set of single qubit operations could be realized, in addition to the Raman pulses, by injecting a two level Rydberg atom into the cavity, and varying its transition frequency using a static external field. We have pointed out that with CNOT gates between the cavity qubit and any atomic qubit, any quantum operation can be constructed, with the photonic qubit acting as an "optical bus."

In this proposal we have described a system where there are no theoretical limitations to scalability since N can be a number of any order. Also, since only the ground states of the atoms have been used in the proposal, the impact of atomic decay rate is weak. Related to the decoherence of the system, the only significant issue is cavity loss, that therefore places challenging requirements on the quality of the cavity.

In Sec. 3.2 we have considered an other configuration where the atoms have only two levels, and they interact dispersively with a single mode of the cavity electromagnetic field. Due to Stark shifts, an effective interaction between the atoms arises, that under certain conditions induces an approximately unitary time evolution regarding even only the atomic system. The Hamiltonian of this evolution is quadratic in terms of the collective spin operators, and diagonal in the Dicke state basis.

We have pointed out that this Hamiltonian can serve as a basis for universal quantum computing together with local unitary control for any number of atoms. In this system complete local unitary control can be achieved by selectively addressing the atoms by resonant laser pulses. Up to $N = 3$ atoms we have given an explicit recipe for exact construction of quantum logic. We have given the respective decompositions of the universal CNOT gate in terms of applications of single qubit operations and the collective time evolution.

For the $N = 3$ atom system, in first step we have reduced the three qubit interaction to a two qubit gate using a technique similar to that of refocussing in nuclear magnetic resonance. In the second step, we have used this two qubit gate to construct various CNOT gates, with the aid of the invariant method. To underline the computational universality of this interaction for this system, we have discussed the construction of a simplified version of the three qubit Toffoli gate.

A peculiarity of the last proposal for realisation of quantum logic is that although its operation relies heavily on the presence of the cavity, it is very robust against cavity losses. A higher photonic decay rate only reduces strength of the effective coupling between the qubits, but it does not increase decoherence. The major source of decoherence remains the spontaneous decay of the excited atomic state. However, due to the simplicity of the atomic structure in the proposal, with considerations to the actual experimental circumstances, the scheme can easily be modified in a way that the impact of this effect can be almost completely eliminated.

Theses

1. We have introduced a representation of states of the two mode electromagnetic field, that uses functions depending on a single complex variable. The representation is based on continuous superposition of two-mode coherent states of the form $|\alpha\rangle|\alpha^*\rangle$. For these reasons, the representation has therefore been termed one-complex plane coherent state representation. We have shown that there exists a one-to-one mapping between two-mode squeezed Fock states and Laguerre-2D polynomials. We have used this result to demonstrate that every state of the two-mode electromagnetic field can be represented by a function of a single complex variable. [1, 2]
2. We have proposed a two mode generalisation of optical Schrödinger cat states. We have developed a formalism to calculate bipartite entanglement of pure states in a non-orthogonal basis, and using this formalism we have shown that these two mode Schrödinger cat states are generally entangled. As an illustration we have devised a quantum teleportation protocol where the quantum channel is made up by a two-mode Schrödinger cat state. We have made a proposal that can serve as a basis for a linear optical implementation of the partial Bell detection, and shown that it simultaneously allows conditional teleportation of two quantum bits, with a probability 1/8. [3]
3. Taking a general approach to measurement, based on the quantum version of Sanov theorem we have shown that distinguishablity of optical Schrödinger cat states can be better conserved by preparing their immediate environment in a squeezed state. A typical example where this distinguishablity problem arises is the read-out stage of a deterministic decision problem quantum algorithm. Based on numerical results, we have concluded that there is a unique optimal setting of the phase parameters in the system, and for every selection of Schrödinger cat states and strength of decoherence there is an optimal magnitude of squeezing for the environment. [4]
4. We have proposed a potentially scalable cavity QED scheme for quantum logic. The proposed setup consists of N four-level atoms localised in a bimodal cavity, such that each atom and the cavity photon realises a quantum bit. We have shown that universal quantum logic can be realized along the same lines as for NMR quantum computing. The

single qubit gates required for operation could be performed on the atomic quantum bits by resonant Raman pulses, and by an auxiliary atom on the photonic quantum bit. Since the photonic quantum bit is the only quantum bit in the scheme that interacts with the others, it plays the role of an optical bus. [5]

5. We have proposed a scheme for quantum logic on two-level atoms in a dispersive cavity. We have pointed out the computational universality of the system for any number of atoms. In the two atom and the three atom case we have derived CNOT gates exactly. The multi-qubit gates have been constructed using single qubit gates realised by resonant external laser pulses, and the collective interaction of the atoms induced by the cavity. The proposed system is particularly robust against cavity losses, and these losses are generally the main source of decoherence in cavity QED schemes. This robustness manifests itself in features, that the state of the cavity can be any thermal distribution, and an increase in the loss rate does not increase decoherence, it merely reduces the effective coupling between the atoms. [6]

Conclusions

We have demonstrated that coherent states can find further useful applications in quantum information. We have presented a reduced dimensional representation that is suitable for the description of entangled states, and also served as a basis for generalising optical Schrödinger cat states. We have also analysed a method based on squeezing the environment for the purpose of suppressing decoherence of coherent state quantum bits.

In two proposals, we have demonstrated the applicability of cavity QED systems for realisation of universal quantum logic. The first proposal has good scalability properties, while the second very successfully addresses the problem of cavity losses.

Related publications

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