The ‘gas of circles’ model and its application to tree crown extraction

Summary of the Doctoral Thesis

by

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Introduction

In this thesis, we propose a model of a 'gas of circles' (GOC), the ensemble of regions in the image domain consisting of an unknown number of circles with approximately fixed radius and short range repulsive interactions. To show its empirical success we apply the model on a forestry application, which is detecting exact tree crowns on remotely sensed images. The method is based on the 'higher order active contour' (HOAC) [RJZ03, RJZ06] framework, which incorporates long-range interactions between contour points and thereby include prior geometric information without using a template shape. This makes them ideal when looking for multiple instances of an entity in an image. For such a model to work, the circles must be stable to small perturbations of their boundaries, *i.e.* they must be local minima of the HOAC energy, for otherwise a circle would tend to 'decay' into other shapes. This is a nontrivial requirement. We impose it by performing a functional Taylor expansion of the HOAC energy around a circle, and then demanding that the first order term be zero for all perturbations, and that the second order term be positive semi-definite. These conditions allow us to fix one of the model parameters in terms of the others, and constrain the rest. The energy is minimized using gradient descent algorithm, and implemented using the level-set method [OS88]. Experiments using the HOAC energy demonstrate empirically the coherence between these theoretical considerations.

The general 'gas of circles' model has many potential applications in varied domains, but it suffers from a drawback: the local minima corresponding to circles can trap the gradient descent algorithm, thus producing phantom circles even with no supporting data. We solve the problem of phantom circles by calculating, via a Taylor expansion of the energy, parameter values that make the circles into inflection points rather than minima. In addition, we find that this constraint reduces the prior parameters one overall weight, while improving the empirical success of the model.

Although, the HOAC 'gas of circles' model is an effective tool for modeling circular shapes, there are some difficulties. It is complicated to express the space of regions in the contour representation, and consequently difficult to work with a probabilistic formulation. In addition, from the algorithmic point of view, the current model does not allow enough topological freedom, and the implementation of the HOAC model is difficult and computationally expensive. But, it is possible to create an alternative formulation of HOAC models, based on the 'phase field' framework much used in physics to model regions and interfaces. The standard phase field model is, to a good approximation, equivalent to a classical active contour model. We compute, as a function of the HOAC energy parameters, the phase field energy parameters that produce an equivalent model. This means that we can adjust the phase field parameters to ensure stable circles of a given radius also. We extend the phase field 'gas of circles' model to the case of an inflection point rather than a minimum in the circle energy at the desired radius.

We also introduce two data models. The first describes the use of only one, the most significant, infrared band of the three available bands in the colour infrared (CIR) images. The model is based on the image gradient and on Gaussian distributions, with the values at different pixels independent, and with different means and variances for tree crowns and the background. While successful, this model, even with the strong region prior, is not capable of extracting accurately the borders of all the trees. Some trees are simply too similar to the background. To solve this problem our second data model makes use of all three bands in the CIR images. We study the improvement or otherwise of the extraction results produced by modelling the three bands as independent or as correlated. As we will see, even at the level of maximum likelihood, the inclusion of 'colour' information, and in particular, interband correlations, can improve the results, and in conjunction with the region prior, the full model is considerably better than that based on one band alone.

We use the above models to extract tree crowns on aerial images, but the models are not restricted to forest management. They can also be applied to the detection of other circular objects. A few other examples: in nanotechnology to detect various particles and microarrays in electron-microscopic images; in biology to segment circular cells and molecules, or to detect pollen grains; in medical image processing; in remote sensing, in the processing of aerial and satellite images, for meteorological, military, and agricultural management.

Thesis 1: The 'gas of circles' model [HJKZ06a]

HOACs include multiple integrals over the contour. These integrals correspond to long-range interactions between tuples of contour points, and allow the incorporation of sophisticated prior geometric knowledge. Combined with length and area terms, one of the basic forms of Euclidean invariant quadratic HOAC models [RJZ03, RJZ06] is

$$E_{\mathrm{g}}(\gamma) = \lambda_C L(\gamma) + \alpha_C A(\gamma) - \frac{\beta_C}{2} \iint dp\, dp'\, \dot{\gamma}(p) \cdot \dot{\gamma}(p')\, \Psi(R(p,p')) \;, \tag{1}$$

where γ is the contour, parameterized by p; L is the length of the contour; A is its interior area; $R(p,p') = |\mathbf{r}(p,p')|$, where $\mathbf{r}(p,p') = \gamma(p) - \gamma(p')$; and Ψ is an interaction function that determines the geometric content of the model. Via the stability analysis we adjust the parameters of this model so that the low energy configurations are approximate circles of approximately fixed radius.

My contribution

The Author presented the stability analysis of the higher-order active contour model for a 'gas of circles', defined the required conditions and consequent parameter constraints.

Stability analysis

The model is of the form $E(\gamma, I) = E_{\mathrm{i}}(I,\gamma) + E_{\mathrm{g}}(\gamma)$. The likelihood energy E_{i} is described in the 4$^{\text{th}}$ thesis. We analyze the prior energy E_{g}, and show how it can be used to model a 'gas of circles'. We adjust the parameters of the model so, that configurations consisting of collections of circles of approximately a certain radius r_0 are stable, and have low energy. Stability means that, if the shape of a circle of radius r_0 is changed slightly, it will relax back into the circle. We choose parameters so that a circle of radius r_0 is a minimum of E_{g}. We expand the energy in a Taylor series to second order in perturbations around a circle of radius r_0, then adjust the parameters so that the first derivative is zero, which tells that the circle is an energy extremum, and so that the second derivative is positive definite, so the extremum is a minimum. The parameters can be further adjusted so that the energy of the minimizing circle is not too high.

We calculate $E_{\mathrm{g}}(\gamma) = E_{\mathrm{g}}(\gamma_0 + \delta\gamma)$ to second order in $\delta\gamma$, where γ_0 is a circle of radius r_0. Since we are expanding around a circle, it is easiest to use polar coordinates (r,θ), and to choose $\theta(p) = p$ as the parameterization. Tangential changes $\delta\theta$ can be undone by a diffeomorphism and hence do not affect the energy. The radial perturbation δr can be expanded in a Fourier series on the circle:

$$\delta r(p) = \sum_k a_k e^{ikr_0 p} \qquad\qquad k = m/r_0,\, m \in \mathbb{Z} \;,$$

where $a_k \ll r_0$. The $L(\gamma)$ and the $A(\gamma)$ can be expressed to second order as:

$$L(\gamma) = 2\pi r_0 \left\{ 1 + \frac{a_0}{r_0} + \frac{1}{2} \sum_k k^2 |a_k|^2 \right\} \tag{2a}$$

$$A(\gamma) = \pi r_0^2 + 2\pi r_0 a_0 + \pi \sum_k |a_k|^2 \;. \tag{2b}$$

The expansion of the quadratic term in equation (1) is more complicated, since one have to expand $\dot{\gamma}$, R, and Ψ, but invariance with respect to translations around the circle means that the second-order term is

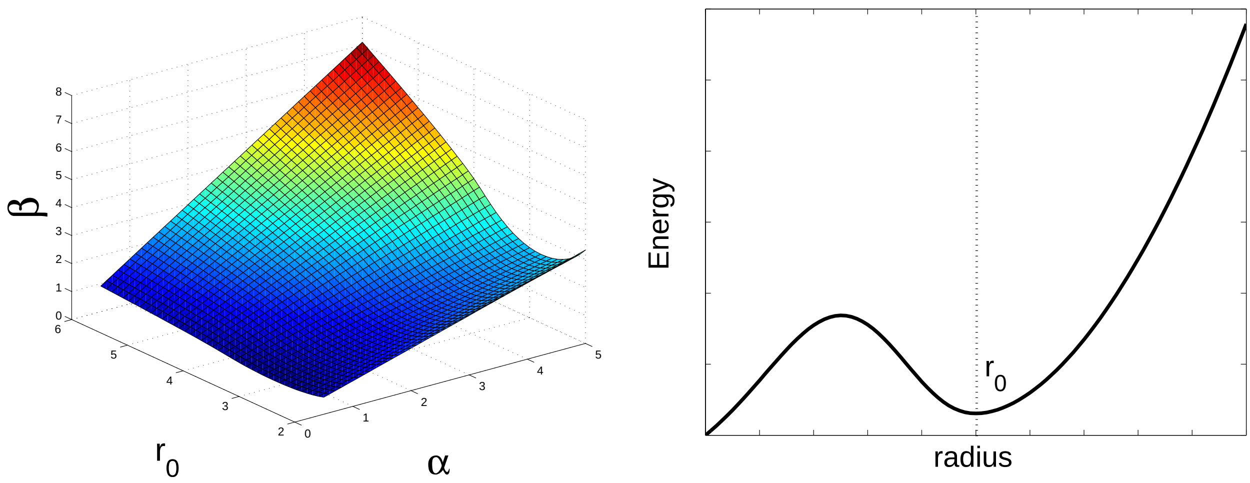

Figure 1: Left: plot of β_C against α_C and r_0 from equation (4) ($d = 4$); right: plot of E_0 versus r for $\alpha_C = 1.0$, $\beta_C = 0.96$, and $r_0 = 5.0$.

diagonal in the Fourier basis. The result, after combination with equations (2), is that the prior energy is given to second-order by:

$$E_{\mathrm{g}}(\gamma_0 + \delta\gamma) = E_0 + a_0 E_1 + \frac{1}{2}\sum_k |a_k|^2 E_2(k) \ ,$$

where

$$\begin{aligned}
E_0 &= 2\pi\lambda_C r_0 + \pi\alpha_C r_0^2 - \pi\beta_C \int_0^{2\pi} dp\ F_{00} \\
E_1 &= 2\pi\lambda_C + 2\pi\alpha_C r_0 - 2\pi\beta_C \int_0^{2\pi} dp\ F_{10} \\
E_2 &= 2\pi\lambda_C r_0 k^2 + 2\pi\alpha_C - 2\pi\beta_C \Bigg[\bigg(2\int_0^{2\pi} dp\ F_{20} + \int_0^{2\pi} dp\ F_{21} e^{ikr_0p} \bigg) \\
&\quad + k\bigg(2ir_0 \int_0^{2\pi} dp\ F_{23} e^{ikr_0p} \bigg) + k^2 \bigg(r_0^2 \int_0^{2\pi} dp\ F_{24} e^{ikr_0p} \bigg) \Bigg] \ .
\end{aligned} \tag{3}$$

The F are functions of p and r_0, and depend on Ψ.

Parameter constraints

In order to achieve an extremum, the linear term must be zero. This implies that:

$$\beta_C(\lambda_C, \alpha_C, r_0) = \frac{\lambda_C + \alpha_C r_0}{\int_0^{2\pi} dp\ F_{10}} \ , \tag{4}$$

which fixes β_C given λ_C, α_C, and r_0. One can set $\lambda_C = 1.0$ without loss of generality. Given r_0, we therefore have only two free parameters to adjust to achieve stability, *i.e.* to make E_2 positive for all k. In figure 1 left, can be seen the plot of β_C versus r_0 and α_C. A given r_0 defines a slice of the surface, the potentially stable pairs of (α_C,β_C), but only those pairs for which $E_2 \geq 0$ for all k are actually stable. Figure 1 right shows the plot of E_0 for one such pair. Note that it has a minimum at r_0, so that it is stable against radial perturbations ($k = 0$). It is also stable with respect to perturbations with $k \neq 0$. Figure 2 shows experiments using just the prior energy, starting from various initial conditions, illustrating the formation of circles of the desired radius.

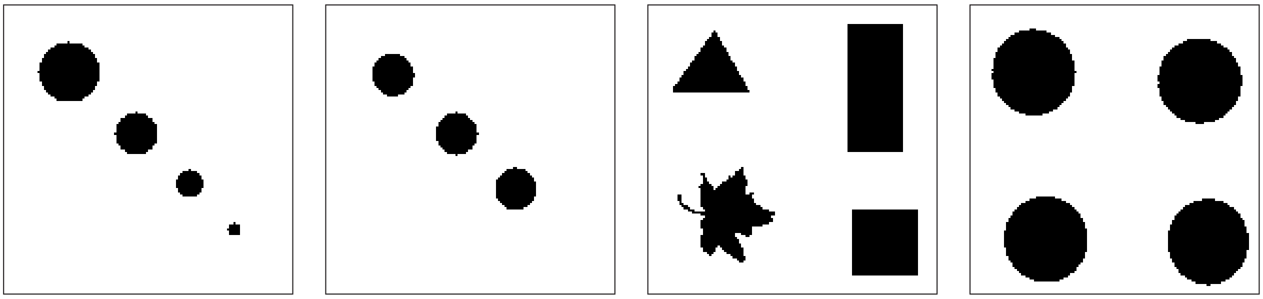

Figure 2: Formation of stable circles, second and fourth, with $r_0 = 10$ and 20 respectively, from two different initial conditions, first and third.

Thesis 2: The inflection point 'gas of circles' model [HJKZ06b]

Figure 1 right shows a plot of the energy of a circle versus radius for parameter values selected according to the 'gas of circles' criteria, it contains a local energy minimum at r_0. In the absence of supporting data, the global minimum will be the empty region, the correct behaviour. A gradient descent algorithm, however, cannot escape from these local minima, meaning that circles of radius r_0, once formed during gradient descent, cannot disappear, even if the data does not support their existence.

My contribution

The Author presented new parameter settings of the HOAC energy that creates an inflection point in the energy function rather than a minimum, so that circles vanish without supporting image data, but a small amount of image information can create an energy minimum at the given radius. This criteria fixes all parameters of the prior except one.

Inflection point

The idea we pursue is to adjust the parameters so that the minimum of the curve is replaced by a broad, approximately flat area, as shown in the three plots in figure 4. Such an energy means that in the absence of image data, a circle will shrink and disappear, whereas small amounts of image data will be sufficient to create a minimum in the flat area, thus producing a stable circle. The natural method to achieve such a broad flat region is to create an energy function that has a single inflection point. It is a nontrivial exercise to find parameter values that result in inflection points.

We still require that a circle of radius r_0 be stable to sinusoidal perturbations with $k > 0$, but now we also require that such a circle be an inflection point with respect to perturbations with $k = 0$, that is, changes of radius. We will see that these demands are sufficient to fix the prior energy E_{g} up to an overall multiplicative constant and a small range of values for d. More precisely, we still require that $E_1(r_0) = 0$ and $E_2(k, r_0) > 0$ for $k > 0$, but we now require that $E_2(0, r_0) = 0$ too. The first condition gives equation (4). The second condition, which follows from the last term of equation (3), also relates α_C and β_C:

$$\alpha_C(r_0) = \beta_C(r_0)\tilde{G}(r_0) \ , \tag{5}$$

where $\tilde{G}(r_0) = 2G_{20}(r_0) + G_{21}(0, r_0)$. We can solve equations (4) and (5) for α_C and β_C, giving

$$\alpha_C(r_0) = \frac{\lambda_C \tilde{G}(r_0)}{G_{10}(r_0) - r_0\tilde{G}(r_0)} \quad \text{and} \quad \beta_C(r_0) = \frac{\lambda_C}{G_{10}(r_0) - r_0\tilde{G}(r_0)} \ . \tag{6}$$

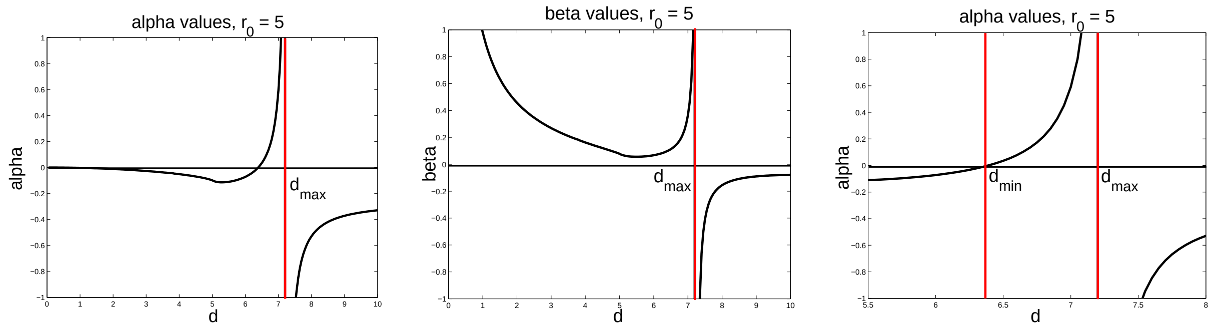

Figure 3: From left to right ($r_0 = 5.0$ throughout): α_C plotted against d; β_C plotted against d; enlarged plot of α_C near the critical domain.

These equations fix α_C and β_C as functions of r_0 and d. Since r_0 is fixed by the application, the only remaining parametric degrees of freedom are the value of d, and the overall strength of the prior term, represented by λ_C. Recall, however, that we also require α_C and β_C to be positive. The question is then how to find values of d for a given r_0 so that $\alpha_C(r_0) > 0$ and $\beta_C(r_0) > 0$.

Determination of d

To illustrate the behaviour we want to understand, figure 3 shows plots of α_C and β_C against d for fixed r_0, in this case $r_0 = 5$. There are two critical points, d_{min} and d_{max}. Only for the range $d_{\text{min}} < d < d_{\text{max}}$ are both α_C and β_C positive. Our goal is therefore to find d_{min} and d_{max} as functions of r_0.

From equations (6), it can be seen that d_{max} arises from a zero in the denominator, while d_{min} arises from a zero in the numerator. It is therefore sufficient to find these zeros in order to find d_{min} and d_{max}. To proceed, we note a scaling property of G_{00}. The function G_{00} is given by the following integral:

$$G_{00}(r) = \int_{-\pi}^{\pi} dp\ \cos(p)\, r^2\, \Psi\left(2r\left|\sin\frac{p}{2}\right|\right) . \tag{7}$$

Since $\Psi(z)$ is a function of z/d only, by pulling d^2 out of the integral one can write G_{00} as $G_{00}(r) = d^2\hat{G}_{00}(r/d)$. Now recall that $G_{10} = \frac{1}{2}\partial_r G_{00}$ and $\tilde{G} = \partial_r G_{10}$. Then find that:

$$\tilde{G}(r_0) = \hat{\tilde{G}}(r_0/d) \qquad \text{and} \qquad G_{10}(r_0) - r_0\tilde{G}(r_0) = d\big(\hat{G}_{10}(r_0/d) - \frac{r_0}{d}\hat{\tilde{G}}(r_0/d)\big) ,$$

where $\hat{G}_{10}(z) = \frac{1}{2}\partial_z\hat{G}_{00}(z)$ and $\hat{\tilde{G}}(z) = \partial_z\hat{G}_{10}(z)$. Thus both numerator and denominator of equations (6) can be written, up to multiplication by positive coefficients, as functions of r_0/d. Now, $f(r,d) = \hat{f}(r/d)$ and $f(r,d_0) = 0$ imply $f(ar, ad_0) = 0$ for all $a \in \mathbb{R}$; thus if we determine d_{min} and d_{max} for one value of r_0,their values are known for any r_0.

To determine d_{min} and d_{max} while avoiding iterative numerical procedures to find these points, we use a polynomial approximation to G_{00}:

$$G_{00}(r) = \sum_{n=0}^{\infty} b_n r^n .$$

It is easy to show that:

$$b_m = \begin{cases} 0 & m < 2 , \\ \frac{1}{(m-2)!}\int_{-\pi}^{\pi} dp\ \cos(p)\, Y^{(m-2)}(0) & m \geq 2 , \end{cases} \tag{8}$$

where $Y(r) = \Psi(2r|\sin(p/2)|)$. The derivatives of Y evaluated at zero are:

$$\frac{Y^{(m)}(0)}{(2|\sin(p/2)|)^m} = \Psi^{(m)}(0) = \begin{cases} 1 & m = 0 , \\ 0 & m = 1 \text{ or } m \text{ even} , \\ (-1)^{\frac{m-1}{2}}\frac{1}{2d}\left(\frac{\pi}{d}\right)^{m-1} & m \geq 3 \text{ and } m \text{ odd} . \end{cases}$$

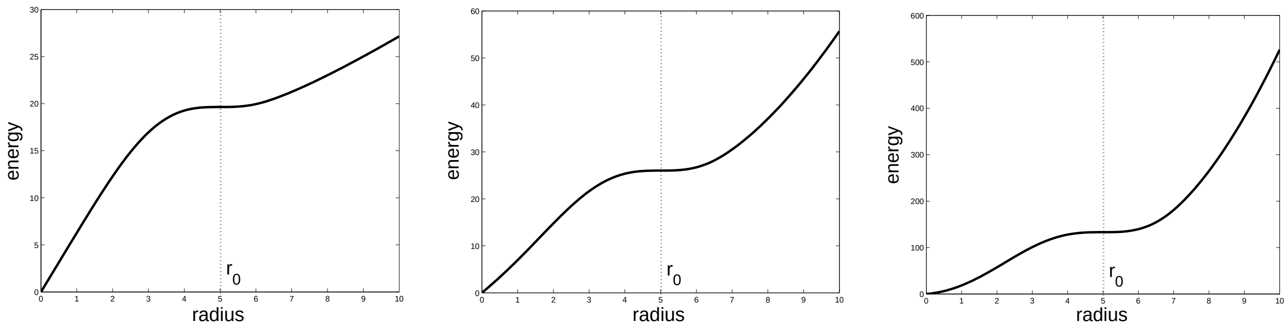

Figure 4: Plot of E_0 against r for $r_0 = 5.0$ with α and β determined by equations (6). First, $d = 6.4$; second, $d = 6.8$; third, $d = 7.2$. For this value of r_0, $d_{\text{min}} = 6.3880$, $d_{\text{max}} = 7.2495$.

Substituting into equation (8) gives b_m:

$$b_m = \begin{cases} 0 & m < 5 \text{ or } m \text{ even} , \\ (-1)^{\frac{m-1}{2}} \frac{4(2\pi)^{m-3}}{m!!(m-4)!!} \frac{1}{d^{m-2}} & m \geq 5 \text{ and } m \text{ odd} . \end{cases}$$

One can then derive expressions for $\tilde{G}$ and $G_{10} - r\tilde{G}$:

$$\tilde{G}(r) = 2 \sum_{\substack{m \geq 3 \\ m \text{ odd}}} \frac{(-1)^{\frac{m+1}{2}} (2\pi)^{m-1}(m+1)}{m!!(m-2)!!} \left(\frac{r}{d}\right)^m$$

$$G_{10}(r) - r\tilde{G}(r) = 2d \sum_{\substack{m \geq 4 \\ m \text{ even}}} \frac{(-1)^{\frac{m-2}{2}} (2\pi)^{m-2}}{[(m-3)!!]^2} \left(\frac{r}{d}\right)^m .$$

We computed the roots of these polynomials including terms up to $m = 49$. The smallest positive roots furnish the values of d_{min} and d_{max}. The result is that $d_{\text{min}} \simeq 1.2776 r_0$ and $d_{\text{max}} \simeq 1.4499 r_0$. The graphs in figure 4 show plots of E_0 against r for $r_0 = 5$, with d values chosen from the domain $d_{\text{min}} < d < d_{\text{max}}$.

Thesis 3: **The phase field 'gas of circles' model [HJ07a, HJ07b]**

A phase field ϕ is a real-valued function on the image domain Ω. Given a threshold z, a phase field determines a region by the map $\zeta_z(\phi) = \{x \in \Omega : \phi(x) > z\}$. Thus phase fields are a level set representation. The difference with the usual distance function level set representation is that the functions are not constrained: the set of possible ϕ, Φ, is a linear space. The simplest phase field energy is [RJZ05]:

$$E_0(\phi) = \int_\Omega dx \left\{ \frac{D}{2} \partial\phi \cdot \partial\phi + \lambda(\frac{1}{4}\phi^4 - \frac{1}{2}\phi^2) + \alpha(\phi - \frac{1}{3}\phi^3) \right\} .$$

With $D = 0$, $\phi_R \triangleq \arg\min_{\phi:\, \zeta_z(\phi) = R} E_0(\phi)$, *i.e.* the minimizing phase field for a given fixed region R, would take the value 1 inside R and -1 outside. The effect of $D \neq 0$ is to smooth ϕ_R so that it has an interface of finite width around ∂R. The phase field model is approximately equivalent to a classical active contour [RJZ05] in the sense that:

$$E_0(\phi_R) \approx \lambda_C L(\partial R) + \alpha_C A(R) , \tag{10}$$

The contour parameters are given by [RJZ05]:

$$\alpha_C = 4\alpha/3 , \qquad \lambda_C^2 = 16 D\lambda K/15 , \qquad K = 1 + 5(\alpha/\lambda)^2 , \qquad w^2 = 15D/\lambda K .$$

To approximate the quadratic HOAC energy, following nonlocal term was added to E_0 by Rochery *et al.* [RJZ05]:

$$E_{NL}(\phi) = -\frac{\beta}{2} \int_{\Omega^2} dx\, dx'\, \partial\phi(x) \cdot \partial\phi(x')\, G(x - x') \;, \tag{11}$$

where $G(x - x') = \Psi(|x - x'|/d)$. With $\beta_C = 4\beta$, $E_{\text{g}} = E_0 + E_{NL}$ is equivalent (as usual, to a good approximation) to the HOAC model, and can be used in its place, thus allowing the incorporation of non-trivial prior knowledge about region geometry while still profiting from all the advantages of the phase field framework.

My contribution

The Author showed how to transform the previously determined 'gas of circles' parameters into the phase field model so that it creates stable circles with the desired radius and determined the constraints on the parameter conversion. The Author presented the steps of the transformation and the constraints converting the inflection point 'gas of circles' parameters to the phase field framework.

The 'gas of circles' higher-order phase field model

Using the relations between the phase field and contour parameters, we create a phase field 'gas of circles' model equivalent to the HOAC model described in Thesis 1 (units are chosen so that $d = 1$, $\tilde{\cdot}$ denotes the parameters normalized by λ_C):

1. Choose w. It cannot be too small, or a subpixel discretization will be needed, and it cannot be too large or the phase field model will not be a good approximation to the HOAC model.

2. Choose $\tilde{\alpha}_C \leq \sqrt{5}/(2w)$.

3. Determine the $\tilde{\beta}_C$ parameter corresponding to r_0 and $\tilde{\alpha}_C$ using the method defined in Thesis 1.

4. Set $\tilde{\lambda} = \frac{15}{8w}[1 + \sqrt{1 - 4\tilde{\alpha}_C^2 w^2/5}]$, $\tilde{\alpha} = 3\tilde{\alpha}_C/4$, $\tilde{\beta} = \tilde{\beta}_C/4$, and $\tilde{D} = w/4$.

5. Choose λ_C and multiply $\tilde{D}$, $\tilde{\lambda}$, $\tilde{\alpha}$, and $\tilde{\beta}$ by it to get D, λ, α, and β.

The phase field inflection point 'gas of circles' model

We combine the phase field 'gas of circles' model with the constraint that the circle energy have an inflection point rather than a minimum at the desired radius. This is a non-trivial requirement. The relations between the phase field and contour parameters were derived using an approximate ansatz for ϕ_R by Rochery *et al.* [RJZ05]. For the 'gas of circles' model, the approximations are not expected to be important, since small errors in the parameters will produce small changes in behaviour. However, it is important to see whether these approximate parameter relations preserve the inflection point behaviour.

In the previous subsection, we chose $\tilde{\alpha}_C$ according to the constraint $\tilde{\alpha}_C \leq \sqrt{5}/(2w)$. In Thesis 2, we described how a given value of r_0 and d fixes $\tilde{\alpha}_C$ and $\tilde{\beta}_C$. To satisfy both these constraints one need to choose $w \leq \sqrt{5}/2\tilde{\alpha}_C$. To determine the parameters of the new phase field inflection point 'gas of circles' model, we therefore take the following steps:

1. Choose a d value satisfying the inflection point criteria given in Thesis 2. This fixes $\tilde{\alpha}_C$ and $\tilde{\beta}_C$.

2. Choose w using the above criterion.

3. Determine $\tilde{\lambda}$, $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{D}$ as before.

4. Multiply these parameters by λ_C.

Thesis 4: Data terms for HOAC and phase field models [HJKZ06a, HJ07a, Hor07]

My contribution

The Author introduced two different data models. First, the HOAC model was completed with a term based on the image gradient and Gaussian-based intensity terms coming from a single-spectral image. Second, the previous intensity term was extended to multispectral images using a multi-dimensional Gaussian model.

The simplest likelihood energy, uses single spectral images. We model the image in R, and in the background $\bar{R}$, using Gaussian distributions. We add a term that predicts high gradients along the boundary ∂R:

$$\begin{aligned} E_{C,\mathrm{I}}(I,R) =& \lambda_i \int dp\, \mathbf{n}(p) \cdot \partial I(\gamma(p)) \\ &+ \alpha_i \left[\int_R d^2x\, \frac{(I(x)-\mu_{\text{in}})^2}{2\sigma_{\text{in}}^2} + \int_{\bar{R}} d^2x\, \frac{(I(x)-\mu_{\text{out}})^2}{2\sigma_{\text{out}}^2} \right] , \end{aligned}$$

where $\mathbf{n}$ is the (unnormalized) outward facing normal. Note that to facilitate comparison of parameters in the prior energy, we set $\lambda = 1$ in E_{g} and introduce a weight α_i in E_{i}. The parameters μ_{in}, σ_{in}, μ_{out}, and σ_{out} are learned from examples using maximum likelihood, and then fixed.

The equivalent phase field data term can be defined as:

$$E_{\text{i}}(I,R) = \int_\Omega dx \left\{ \lambda_i\, \partial I \cdot \partial\phi + \alpha_i \left[\frac{(I-\mu_{\text{in}})^2}{2\sigma_{\text{in}}^2}\phi_+ + \frac{(I-\mu_{\text{out}})^2}{2\sigma_{\text{out}}^2}\phi_- \right] \right\} ,$$

where $\phi_\pm = (1 \pm \phi)/2$.

In some of the cases not only single-spectral images but color or multispectral images are also available. To use the opportunity of the other channels we developed a multispectral data term for the HOAC GOC model based on Gaussian distributions with full covariance matrices. The parameters of E_{i} are learnt from samples of each class using maximum likelihood, and then fixed. We denote the mean vectors inside and outside as M_{in} and M_{out} and the covariance matrices Σ_{in} and Σ_{out}. We define the energy as:

$$\begin{aligned} E_{C,\mathrm{I}}(I,R) = &-\int_R dp\, \ln\left[\det{}^{-1/2}(\Sigma_{\text{in}}/2\pi) e^{-\frac{1}{2}(I(p)-M_{\text{in}})^T \Sigma_{\text{in}}^{-1} (I(p)-M_{\text{in}})} \right] \\ &- \int_{\bar{R}} dp\, \ln\left[\det{}^{-1/2}(\Sigma_{\text{out}}/2\pi) e^{-\frac{1}{2}(I(p)-M_{\text{out}})^T \Sigma_{\text{out}}^{-1} (I(p)-M_{\text{out}})} \right] . \end{aligned}$$

The energy is minimized using gradient descent.

Experimental results

In this section we present experimental results were obtained on aerial images provided by the Hungarian Central Agricultural Office, Forestry Administration (CAO, FA) and French Forest Inventory (IFN) . The

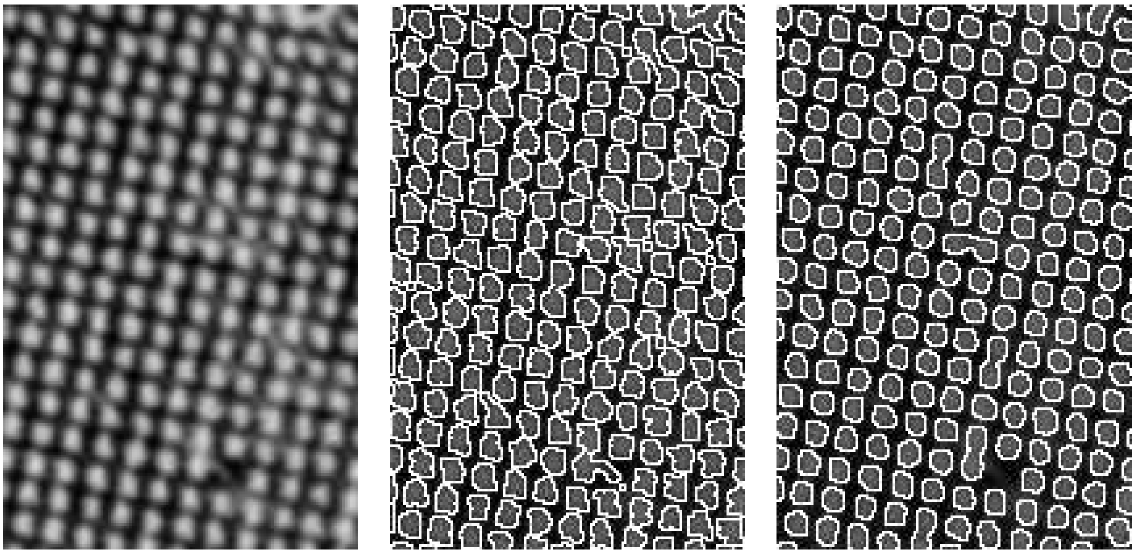

Figure 5: From left to right: image of poplars ©IFN; the best result with a classical active contour; result with the 'gas of circles' model .

exact parameter values and the details of the segmentation can be found in the experimental results part of the thesis.

In figure 5 the data is on the left, the best result obtained with a classical active contour model is in the middle, and the result with the HOAC 'gas of circles' model is on the right. The trees are close together, using the classical active contour, the result is that the tree crown boundaries touch in the majority of cases, despite their separation in the image. The HOAC model produces more clearly delineated tree crowns, but there are still some joined trees.

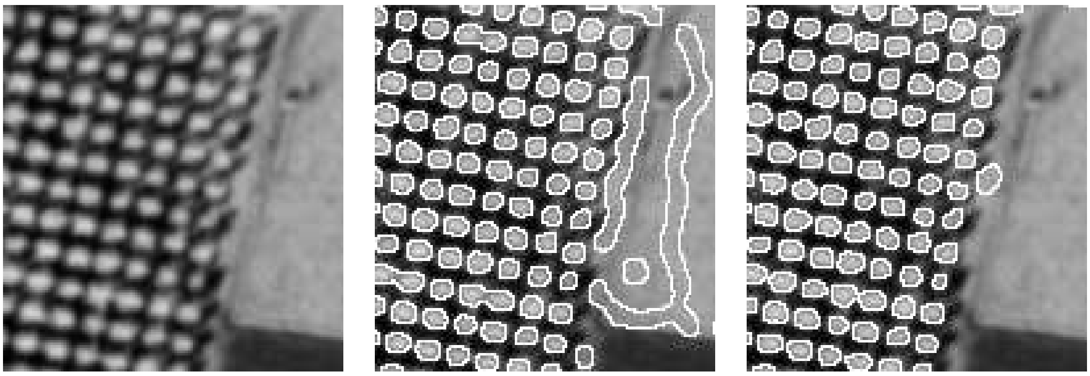

Figure 6: From left to right: an image of regularly planted poplars with different fields in the right part © IFN ; result with the phase field 'gas of circles' model; result with the inflection point GOC model.

Figure 6 left shows a difficult image to analyse. It has two fields with different intensities on the right. The result with the phase field 'gas of circles' model is shown in figure 6 middle. This result clearly demonstrate the disadvantage of the non-inflection point model: phantom objects are created in the homogenous areas. Figure 6 right shows the result with the inflection point 'gas of circles' phase field model. The result is very good, with only one false positive. Both results were obtained in less than 1 minute.

Finally figure 7 left shows a different type of CIR image, of isolated trees in fields. The result with the multispectral GOC model, shown in figure 7 middle, is correct, ignoring the field, for example. The result with the model in Thesis 2, in the right image, is not as good, with one large false positive, and smaller errors on each of the detected trees.

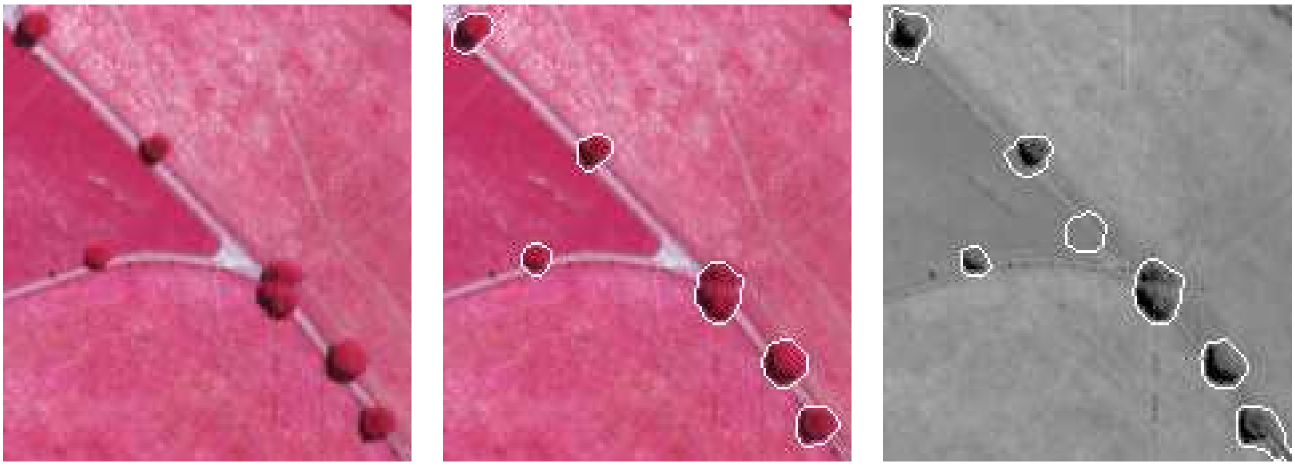

Figure 7: From left to right: a CIR image © IFN; result with multispectral GOC model; result with model in Thesis 2 using only the infrared spectral band;

	[HJKZ06a]	[HJKZ06b]	[HJ07a]	[HJ07b]	[Hor07]
1	*				
2		*			
3			*	*	
4	*		*		*

Table 1: Connection between the theses and the Author's publications

Summary of the Author's contributions

We summarize the results of the Author arranging them into four theses. Table 1 illustrates how the thesis points are covered by the publications of the Author.

1. The Author presented the stability analysis of the higher-order active contour model for a 'gas of circles', defined the required conditions and consequent parameter constraints.

2. The Author presented new parameter settings of the HOAC energy that creates an inflection point in the energy function rather than a minimum, so that circles vanish without supporting image data, but a small amount of image information can create an energy minimum at the given radius. This criteria fixes all parameters of the prior except one.

3. The Author showed how to transform the previously determined 'gas of circles' parameters into the phase field model so that it creates stable circles with the desired radius and determined the constraints on the parameter conversion. The Author presented the steps of the transformation and the constraints converting the inflection point 'gas of circles' parameters to the phase field framework.

4. The Author introduced two different data models. First, the HOAC model was completed with a term based on the image gradient and Gaussian-based intensity terms coming from a single-spectral image. Second, the previous intensity term was extended to multispectral images using a multi-dimensional Gaussian model.

Publications of the Author

[HJ07a] P. Horváth and I. H. Jermyn. A 'gas of circles' phase field model and its application to tree crown extraction. In *Proc. European Signal Processing Conference (EUSIPCO)*, Poznan, Poland, September 2007.

[HJ07b] P. Horváth and I. H. Jermyn. A new phase field model of a 'gas of circles' for tree crown extraction from aerial images. In *Proc. International Conference on Computer Analysis of Images and Patterns (CAIP)*, Lecture Notes in Computer Science, Vienna, Austria, August 2007.

[Hor07] P. Horváth. A multispectral data model for higher-order active contours and its application to tree crown extraction. In *Proc. Advanced Concepts for Intelligent Vision Systems*, Lecture Notes in Computer Science, Delft, Netherlands, August 2007.

[HJKZ06b] P. Horváth, I. H. Jermyn, Z. Kato, and J. Zerubia. An improved 'gas of circles' higher-order active contour model and its application to tree crown extraction. In *Proc. Indian Conference on Vision, Graphics and Image Processing (ICVGIP)*, Lecture Notes in Computer Science, Madurai, India, December 2006.

[HJKZ06a] P. Horváth, I. H. Jermyn, Z. Kato, and J. Zerubia. A higher-order active contour model for tree detection. In *Proc. International Conference on Pattern Recognition (ICPR)*, Hong Kong, China, August 2006.

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[RJZ03] M. Rochery, I.H. Jermyn, and J. Zerubia. Higher order active contours and their application to the detection of line networks in satellite imagery. In *Proc. IEEE Workshop Variational, Geometric and Level Set Methods in Computer Vision*, at ICCV, Nice, France, October 2003.

[RJZ05] M. Rochery, I.H. Jermyn, and J. Zerubia. Phase field models and higher-order active contours. In *Proc. IEEE International Conference on Computer Vision (ICCV)*, Beijing, China, October 2005.

[RJZ06] M. Rochery, I.H. Jermyn, and J. Zerubia. Higher order active contours. *International Journal of Computer Vision*, 69(1):27–42, August 2006.

Coauthor's declaration

June 27, 2007

I hereby certify that I am familiar with the following publications of the applicant Mr Péter Horváth:

1. Peter Horvath, ***Ian Jermyn***, Zoltan Kato, and Josiane Zerubia. **A Higher-Order Active Contour Model for Tree Detection**. In Proceedings of the *International Conference on Pattern Recognition (ICPR'06)*. Hong Kong, China, August 2006. IAPR, IEEE.

2. Peter Horvath, ***Ian Jermyn***, Zoltan Kato, and Josiane Zerubia. **An improved 'gas of circles' higher-order active contour model and its application to tree crown extraction**. In Proceedings of the *Indian Conference on Vision, Graphics and Image Processing (ICVGIP'06)*. Madurai, India, December 2006. LNCS.

3. Peter Horvath, ***Ian Jermyn***. **A new phase field model of a 'gas of circles' for tree crown extraction from aerial images**. In Proceedings of the *12th International Conference on Computer Analysis of Images and Patterns (CAIP'07)*, Vienna, Austria, August 2007. LNCS.

4. Peter Horvath, ***Ian Jermyn***. **A 'gas of circles' phase field model and its application to tree crown extraction.** In Proceedings of the *European Signal Processing Conference (EUSIPCO'07)*, Poznan, Poland, September 2007. EURASIP

Regarding to our joint results referred to in these articles, the following ones were obtained as the result of joint contribution by the applicant and myself:

1. The introduction and state-of the art. Conclusion.

2. The Introduction and the Conclusion. Fixing the parameters giving an inflection point. Determination of the interaction function parameter providing non-negative parameters.

3. Using the recent inflection point 'gas of circles' model parameters with the phase field framework.

4. The application of higher-order phase field model instead of the higher-order active contour model, for the description of multiple circles with similar radii.

The applicant's contribution was prominent in obtaining the following results:

1. Stability analysis and conditions, and consequent parameter constraints. Addition of the gradient term to data energy. The synthetic justification of the model and the experiments and analysis of results on aerial images.

2. The idea to use an inflection point instead of a minimum in the circle energy, thereby solving the problem of phantom circles. The experiments and analysis of results.

3. Converting the inflection point 'gas of circles' parameters, to the phase field framework, determining the steps of the transformation and the constraints. The synthetic and real experiments and analysis of results.

4. Determining the steps and the constraints of the parameter conversion from the higher-order active contours to the phase field framework. Synthetic and real experiments and analysis of results.

My own contribution was prominent in obtaining the following results:

1. Using the higher-order active contour model to describe circular regions. The introduction of the data term using the pixel intensities.

2. Using Taylor polynomials to approximate the interaction function's parameter.

3. –

4. Embedding recent higher-order active contour data model into the phase field framework.

Sophia Antipolis, 27 June 2007.

I. H. Jermyn

Dr. Ian H. Jermyn

Coauthor's declaration

July 31, 2007

I hereby certify that I am familiar with the following publications of the applicant Mr Péter Horváth:

1. Peter Horvath, Ian Jermyn, ***Zoltan Kato***, and Josiane Zerubia. **A Higher-Order Active Contour Model for Tree Detection**. In Proceedings of the *International Conference on Pattern Recognition (ICPR'06)*. Hong Kong, China, August 2006. IAPR, IEEE.

2. Peter Horvath, Ian Jermyn, ***Zoltan Kato***, and Josiane Zerubia. **An improved 'gas of circles' higher-order active contour model and its application to tree crown extraction**. In Proceedings of the *Indian Conference on Vision, Graphics and Image Processing (ICVGIP'06)*. Madurai, India, December 2006. LNCS.

Regarding to our joint results referred to in these articles, the following ones were obtained as the result of joint contribution by the applicant and myself:

1. publication: The introduction and overview on the state-of the art. Conclusion.

2. publication The introductory and the conclusion part. Fixing the parameters so that using the derivative constraints giving inflection point. Determination of the interaction function parameter providing non-negative parameters.

The applicant's contribution was prominent in obtaining the following results:

1. publication: The expression of the quadratic term, stability conditions. The synthetic justification of the model and the experimental results on aerial images.

2. publication: The idea using an inflection point instead of energy minimum at the circle energy, solving the problem of phantom circles. The experimental results part.

My own contribution was prominent in obtaining the following results:

1. publication: Tests on real data.
2. publication: Validation of the experimental results.

Szeged, 31 July, 2007.

Zoltán Kató

Coauthor's declaration

June 27, 2007

I hereby certify that I am familiar with the following publications of the applicant Mr Péter Horváth:

1. Peter Horvath, Ian Jermyn, Zoltan Kato, and ***Josiane Zerubia***. **A Higher-Order Active Contour Model for Tree Detection**. In Proceedings of the *International Conference on Pattern Recognition (ICPR'06)*. Hong Kong, China, August 2006. IAPR, IEEE.

2. Peter Horvath, Ian Jermyn, Zoltan Kato, and ***Josiane Zerubia***. **An improved 'gas of circles' higher-order active contour model and its application to tree crown extraction**. In Proceedings of the *Indian Conference on Vision, Graphics and Image Processing (ICVGIP'06)*. Madurai, India, December 2006. LNCS.

Regarding to our joint results referred to in these articles, the following ones were obtained as the result of joint contribution by the applicant and myself:

1. The introduction and state-of the art. Conclusion.

2. The Introduction and the Conclusion. Fixing the parameters giving an inflection point. Determination of the interaction function parameter providing non-negative parameters.

The applicant's contribution was prominent in obtaining the following results:

1. Stability analysis and conditions, and consequent parameter constraints. Addition of the gradient term to data energy. The synthetic justification of the model and the experiments and analysis of results on aerial images.

2. The idea to use an inflection point instead of a minimum in the circle energy, thereby solving the problem of phantom circles. The experiments and analysis of results.

My own contribution was prominent in obtaining the following results:

1. – Tests on real data
2. – Validation with the French Forest Institute

Sophia Antipolis, 27 June 2007.

Dr. Josiane Zerubia

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