

S² SAJÁTFÜGGVÉNYEINEK SZERKESZTÉSE
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doktori értekezés

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1. § Bevezetés

Mivel a hullámegyenlet egzakt megoldása csak néhány, meglehetősen speciális alakban felvett potenciálfüggvény esetében lehetséges, kvantumkémiail problémák megoldásánál közelítő eljárásokra vagyunk utalva.

Ezek közül a század közepén a konfigurációs kölcsönhatás módszere került előtérbe. Ennél a módszernél a nullaendű sajátfüggvényt SLATER-determinánsok lineáris kombinációjaként adjuk meg. Az energiát pedig a szokásos szekuláris egyenlet gyökei szolgáltatják. Minthogy nincs utasításunk arra vonatkozólag, hogy a SLATER-determinánsokkal megadott konfigurációk közül melyek a fontosak és melyek az elhanyagolhatóak, a szekuláris egyenlet redukálására új módszerek kidolgozása vált szükségessé, melyek közül egyik lehet a spinoperátoros eljárás.

2. § Spinoperátorok

A spektrumok multiplett szerkezetének és az anómális ZEEMANN-effektusnak elméleti alátámasztása céljából GOUDSMIT és UHLENBECK feltették, hogy az elektron, a keringésével együttjáró pályá-impulzusmomentum mellett, egy saját-impulzusmomentummal is rendelkezik. Ezt spinnek nevezték.

Kvantummechanikából ismeretes az $\mathbb{L}$ impulzusmomentum operátor sajátértékegyenlete:

$$\mathbb{L} \psi = \hbar \sqrt{l(l+1)} \psi. \quad / 2.1 /$$

Tehát az előbbieket alapján / 2.1 / a spinoperátorra is érvényes:

$$\mathbb{S} \psi = \hbar \sqrt{s(s+1)} \psi. \quad / 2.2 /$$

Az impulzusmomentum operátor felcserélési relációja alapján :

$$\begin{aligned} S_x S_y - S_y S_x &= i\hbar S_z \\ S_y S_z - S_z S_y &= i\hbar S_x \\ S_z S_x - S_x S_z &= i\hbar S_y. \end{aligned} \quad / 2.3 /$$

A spinoperátorok mátrixelőállításai a következők:

$$\begin{aligned} S_x &= \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ S_y &= \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ S_z &= \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned} \quad / 2.4 /$$

Az itt szereplő mátrixok az u.n. PAULI-féle spinmátrixok.

A spin kvantálása pedig a következő egyenletek alapján történik :

$$S^2 = \hbar^2 l_s (l_s + 1), \quad \text{ahol} \quad l_s = \frac{1}{2}$$

$$S_z = \hbar m_s, \quad \text{ahol} \quad m_s = \pm \frac{1}{2}, \quad / 2.5 /$$

m_s a spinkvantumszám. Az $m_s = +\frac{1}{2}$ -hez tartozó egyrészecske spinfüggvényt α -val, az $m_s = -\frac{1}{2}$ -hez tartozó egyrészecske spinfüggvényt β -val jelöljük. Ezek mátrixokkal a következőképpen állíthatók elő :

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad / 2.6 /$$

/ 2.4 / és / 2.6 / alapján belátható a következő egyenletek érvényessége :

$$S_x \alpha = \frac{\hbar}{2} \beta$$

$$S_y \alpha = \frac{\hbar}{2} i \beta$$

$$S_z \alpha = \frac{\hbar}{2} \alpha, \quad / 2.7 /$$

$$S_x \beta = \frac{\hbar}{2} \alpha$$

$$S_y \beta = -\frac{\hbar}{2} i \alpha$$

$$S_z \beta = -\frac{\hbar}{2} \beta. \quad / 2.8 /$$

Vezessük be a következő spinoperátorokat :

$$S^+ = S_x + i S_y \quad / 2.9 /$$

$$S^- = S_x - i S_y. \quad / 2.10 /$$

Előbbieket alapján :

$$S^+ \alpha = 0 \quad S^+ \beta = \hbar \alpha \quad / 2.11 /$$

$$S^- \alpha = \hbar \beta \quad S^- \beta = 0.$$

/ 2.11 / egyenletek miatt S^+ -t felfelé, - S^- -t pedig lefelé fordító spinoperátoroknak nevezzük. Az S^2 operátornak a fel és lefelé fordító spinoperátorokkal való összefüggését / 2.7 /, / 2.8 /, / 2.11 / alapján a következő egyenlet adja :

$$S^2 = (S_x + i S_y)(S_x - i S_y) + S_z^2 - i (S_x S_y - S_y S_x),$$

$$S^2 = S^+ S^- + S_z^2 - \hbar S_z. \quad / 2.12 /$$

Hasonlóan adódik a következő összefüggés is :

$$S^2 = S^- S^+ + S_z^2 + \hbar S_z. \quad / 2.13 /$$

Több elektron esetén a rendszerre vonatkozó fel-, és lefelé fordító spinoperátor és az S_z operátor a következő alakú :

$$S^+ = \sum_{i=1}^n S_i^+, \quad S^- = \sum_{i=1}^n S_i^-, \quad S_z = \sum_{i=1}^n S_{iz} \quad / 2.14 /$$

ahol n az elektronok számát jelenti.

3. 6 Összefüggések az S_z operátor sajátfüggvényei között

A következőkben $\hbar$ egységekben dolgozunk. Legyen adott egy S eredő spinű rendszer, amelynek állapotát $\psi(s, m_s)$ sajátfüggvény írja le. A z tengelyre vonatkozó spinvetületek a következők :

$$m_s = s, \dots, s-i, \dots, -s. \quad / 3.1 /$$

A megfelelő sajátfüggvények pedig :

$$\psi(s, s), \dots, \psi(s, s-i), \dots, \psi(s, -s). \quad / 3.2 /$$

Ezeket rendre maximális, i-edik és minimális komponenseknek nevezzük.

VAN DER WAERDEN [4] alapján ismeretes, hogy a fel- és lefelé fordító spinoperátorok hatására bármelyik komponensből az utána következő, ill. az előtte álló komponenssel arányos sajátfüggvényt nyerjük a következő összefüggések alapján :

$$\begin{aligned} S^+ \psi(s, \mu) &= [(s-\mu)(s+\mu+1)]^{\frac{1}{2}} \psi(s, \mu+1) \\ S^- \psi(s, \mu) &= [(s+\mu)(s-\mu+1)]^{\frac{1}{2}} \psi(s, \mu-1). \end{aligned} \quad / 3.3 /$$

Bármelyik komponens kifejezhető a maximális, ill. minimális komponensből / 3.3 / alapján :

$$\psi(s, s-i) = \frac{1}{i!} \binom{2s}{i}^{-\frac{1}{2}} (S^-)^i \psi(s, s) \quad / 3.4 /$$

$$\psi(s, s+i) = \frac{1}{i!} \binom{2s}{i}^{-\frac{1}{2}} (S^+)^i \psi(s, -s). \quad / 3.5 /$$

4. § A spinösszeadás operátora

Legyen adott két rendszer : A_1 és A_2 melyeknek eredő spinjei S és S' . A megfelelő sajátfüggvények : $\psi_{A_1}(S, m_S)$ és $\psi_{A_2}(S', m_{S'})$. Ezek az S_z operátor sajátfüggvényei, azaz :

$$\begin{aligned} S_z \psi_{A_1}(S, m_S) &= m_S \psi_{A_1}(S, m_S) \\ S_z \psi_{A_2}(S', m_{S'}) &= m_{S'} \psi_{A_2}(S', m_{S'}). \end{aligned} \quad / 4.1 /$$

Egyesítsük a két rendszert. Az egyesített rendszer állapotát leíró sajátfüggvények közül a maximális komponens VAN DER WAERDEN [4] szerint a következő alakú :

$$\begin{aligned} \psi(S, M_S) &= \binom{2S+2S'-\lambda+1}{\lambda}^{\frac{1}{2}} \sum_{i=0}^{\lambda} (-1)^i \left[\binom{2S-i}{\lambda-i} \binom{2S'+i-\lambda}{i} \right]^{\frac{1}{2}} \times \\ &\times \psi_{A_1}(S, S-i) \psi_{A_2}(S', S'+i-\lambda). \end{aligned} \quad / 4.2 /$$

Két fontos speciális esetet vizsgálunk :

1./ $\lambda=0$ ekkor $S=S+S'$ és

$$\psi(S=S+S', M_S=S+S') = \psi_{A_1}(S, S) \psi_{A_2}(S', S'+i), \quad / 4.3 /$$

azaz az S spinű rendszerhez hozzáadtunk egy S' spinű rendszert, és egy $S+S'$ spinű rendszert nyertünk.

2./ $\lambda=2S'$ ekkor $S=S-S'$ és

$$\psi(S=S-S', M_S=S-S') = \binom{2S+1}{2S'}^{\frac{1}{2}} \sum_{i=0}^{2S'} (-1)^i \binom{2S-i}{2S'-i}^{\frac{1}{2}} \psi_{A_1}(S, S-i) \psi_{A_2}(S', S'+i), \quad / 4.4 /$$

azaz az S spinű rendszerből kivontuk az S' spinű rendszert és egy $S-S'$ spinű rendszert nyertünk. A / 4.3 / és / 4.4 / egyenletek a spinösszeadás és spin kivonás egyenletei.

BERENCZ F. [1] bebizonyította, hogy a VAN DER WAERDEN-féle / 4.2 / alatti spinösszetevési egyenlet S^2 sajátfüggvényét adja $(s+s'-\lambda)(s+s'-\lambda+1)$ sajátértékkel.

Írjuk át a spinösszetevés egyenletét / 3.4 / alapján a spinoperátorokkal :

$$\Psi(S, M_S) = \binom{2s+2s'-\lambda+1}{\lambda}^{-\frac{1}{2}} \sum_{i=0}^{\lambda} (-1)^i \left[\binom{2s-i}{\lambda-i} \binom{2s'+i-\lambda}{i} \right]^{\frac{1}{2}} \times \\ \times \frac{1}{i!} \binom{2s}{i}^{-\frac{1}{2}} \frac{1}{(\lambda-i)!} \binom{2s'}{\lambda-i}^{-\frac{1}{2}} (S_{A_1}^-)^i (S_{A_2}^-)^{\lambda-i} \Psi_{A_1}(s, s) \Psi_{A_2}(s', s')$$

$$\Psi(S, M_S) = \binom{2s+2s'-\lambda+1}{\lambda}^{-\frac{1}{2}} [(2s)!(2s')!(2s-\lambda)!(2s'-\lambda)!]^{-\frac{1}{2}} \times \\ \times \sum_{i=0}^{\lambda} (-1)^i \frac{(2s-i)!(2s'+i-\lambda)!}{i!(\lambda-i)!} (S_{A_1}^-)^i (S_{A_2}^-)^{\lambda-i} \Psi_{A_1}(s, s) \Psi_{A_2}(s', s'). \quad / 4.5 /$$

/ 4.5 / alapján tehát az operátor, amellyel S^2 sajátfüggvényeit nyerjük, a következő alakú :

$$\textcircled{0} (S=s+s'-\lambda, M_S=s+s'-\lambda) = \binom{2s+2s'-\lambda+1}{\lambda}^{-\frac{1}{2}} \times \\ \times [(2s)!(2s')!(2s-\lambda)!(2s'-\lambda)!]^{-\frac{1}{2}} \times \\ \times \sum_{i=0}^{\lambda} (-1)^i \frac{(2s-i)!(2s'+i-\lambda)!}{i!(\lambda-i)!} (S_{A_1}^-)^i (S_{A_2}^-)^{\lambda-i}. \quad / 4.6 /$$

A / 3.5 / egyenletet felhasználva megadhatjuk operátorunknak azt az alternatív alakját, amelyben már az S^+ operátor is szerepel :

$$\Psi(S, M_S) = \left(\frac{2s-2s'+1}{2s+1} \right)^{\frac{1}{2}} \sum_{i=0}^{2s'} (-1)^i \frac{(2s-i)!}{(2s)! i!} \times \\ \times (S_{A_1}^- S_{A_2}^+)^i \Psi_{A_1}(s, s) \Psi_{A_2}(s', -s'). \quad / 4.7 /$$

A megfelelő operátor tehát a következő alakú :

$$\mathbb{O}(S, M_S) = \left(\frac{2S - 2S' + 1}{2S + 1} \right)^{\frac{1}{2}} \times \\ \sum_{i=0}^{2S'} (-1)^i \frac{(2S-i)!}{(2S)! i!} (S_{A_1}^- S_{A_2}^+)^i .$$

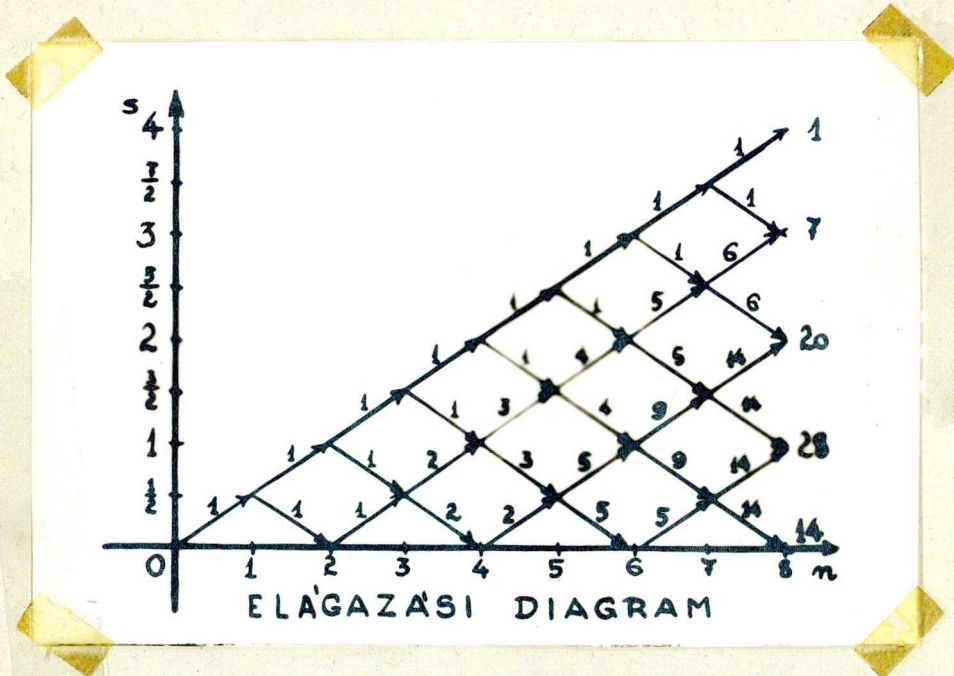
/ 4.8 /

5. § A spinösszetevés operátora SLATER- determinán-
sokkal megadott sajátfüggvények esetében

Egy n elektronból álló rendszer lineárisan füg-
getlen spinállapotainak a számát S eredő spin esetén
CORSON [2] szerint a következőképpen számíthatjuk ki :

$$N(S) = \binom{\frac{n}{2} - S}{S} - \binom{\frac{n}{2} - S - 1}{S} \quad / 5.1 /$$

Est a számot az elágazási diagramból is leolvas-
hatjuk :



/ A diagramban feltüntetett számok a vonalak multiplici-
tását jelentik. / Az elágazási diagram a spinállapotok
szerkesztési módját mutatja. Eszerint egy n elektron-
ból álló rendszer S eredő spinű állapota oly módon konst-
ruálható meg, hogy x_1 számú, párhuzamos α spinű elektron-
ból álló rendszerhez hozzáadunk x_2 számú, párhuzamos β

spinű elektronból álló rendszert; majd az egyesített rendszerhez hozzáadjuk az x_3 számú α spinű elektronból álló rendszert stb. Ezt az eljárást addig folytatjuk, míg az összes elektront figyelembe nem vettük. Ha a részrendszerek állapotát SLATER-determinánsokkal írtuk le, akkor / 4.6 / és / 4.8 / alkalmazásával S^2 sajátfüggvényeit kapjuk.

A továbbiakban / 4.6 / és / 4.8 / egyenleteknek SLATER-determinánsokra vonatkozó alakját adjuk meg, majd alkalmazásként nyolc elektronból álló rendszer különböző multiplicitású spinállapotait számítjuk ki. Ezt azért tehetjük, mert BERENCZ P. [1] bebizonyította, hogy / 3.3 / egyenletek speciálisan SLATER-determináns alakban felvett sajátfüggvényekre is érvényesek.

A SLATER-determinánsokat a következőképpen jelöljük:

$$\varphi(S, m_s) = |\alpha\alpha\beta \dots \alpha|, \quad / 5.2 /$$

Legyen x_1 elektronunk párhuzamos α spinnel. Egyesítsük ezeket $S = \frac{x_1}{2}$ eredő spinű rendszerre / A_1 /, az A_1 rendszer maximális komponense / 4.3 / szerint

$$\varphi_{A_1}(S = \frac{x_1}{2}, m_s = \frac{x_1}{2}) = |\alpha^1 \alpha^2 \dots \alpha^{x_1}|. \quad / 5.3 /$$

Legyen x_2 elektronunk párhuzamos β spinnel. Egyesítsük ezeket $S' = \frac{x_2}{2}$ eredő spinű rendszerre / A_2 /, az A_2 rendszer minimális komponense :

$$\varphi_{A_2}(S' = \frac{x_2}{2}, m_{S'} = -\frac{x_2}{2}) = |\beta^1 \beta^2 \dots \beta^{x_2}|. \quad / 5.4 /$$

Egyesítsük az A_1 és A_2 rendszert $S = S' = \frac{x_1}{2} - \frac{x_2}{2}$ eredő spinnel, akkor / 4.7 / szerint az egyesített rendszer maximális komponense a következő alakú :

$$\varphi_{A_1 A_2}(s, m_s) = \left(\frac{x_1 - x_2 + 1}{x_1 - 1} \right)^{\frac{1}{2}} \sum_{i=0}^{x_2} (-1)^i \frac{(x_1 - i)!}{x_1! i!} (S_{A_1}^- S_{A_2}^+)^i |\alpha^{1, \dots, x_1, 1, \dots, x_2} \beta \dots \beta|. \quad / 5.5 /$$

A megfelelő operátor tehát :

$$\mathbb{O}_{A_1 A_2} = \left(\frac{x_1 - x_2 + 1}{x_1 + 1} \right)^{\frac{1}{2}} \sum_{i=0}^{x_2} (-1)^i \frac{(x_1 - i)!}{x_1! i!} (S_{A_1}^- S_{A_2}^+)^i. \quad / 5.6 /$$

As $A_1 A_2$ rendszerhez adjuk hozzá az A_3 rendszert oly módon, hogy az eredő spin $\frac{x_1 - x_2 + x_3}{2}$ legyen. Ekkor

$$\varphi_{A_1 A_2 A_3} = \mathbb{O}_{A_1 A_2} |\alpha^{1, \dots, x_1, 1, \dots, x_2, 1, \dots, x_3} \alpha \dots \alpha|. \quad / 5.7 /$$

Adjunk hozzá $A_1 A_2 A_3$ rendszerhez A_4 rendszert oly módon, hogy az eredő spin $\frac{x_1 - x_2 + x_3 - x_4}{2}$ legyen. Az egyesített rendszer maximális komponense :

$$\begin{aligned} \varphi_{A_1 A_2 A_3 A_4} &= \left(\frac{x_1 - x_2 + x_3 - x_4 + 1}{x_1 - x_2 + x_3 + 1} \right)^{\frac{1}{2}} \left(\frac{x_1 - x_2 + 1}{x_1 + 1} \right)^{\frac{1}{2}} \sum_{i=0}^{x_4} (-1)^i \times \\ &\times \frac{(x_1 - x_2 + x_3 - i)!}{(x_1 + x_2 + x_3)! i!} (S_{A_1 A_2 A_3}^- S_{A_4}^+)^i \sum_{j=0}^{x_2} (-1)^j \frac{(x_1 - j)!}{x_1! j!} \times \\ &\times (S_{A_1}^- S_{A_2}^+)^j |\alpha^{1, \dots, x_1, 1, \dots, x_2, 1, \dots, x_3, 1, \dots, x_4} \beta \dots \beta|. \end{aligned} \quad / 5.8 /$$

A megfelelő operátor :

$$\begin{aligned} \mathbb{O}_{A_1 A_2 A_3 A_4} &= \left(\frac{x_1 - x_2 + x_3 - x_4 + 1}{x_1 - x_2 + x_3 + 1} \right)^{\frac{1}{2}} \left(\frac{x_1 - x_2 + 1}{x_1 + 1} \right)^{\frac{1}{2}} \sum_{i=0}^{x_4} (-1)^i \times \\ &\times \frac{(x_1 - x_2 + x_3 - i)!}{(x_1 - x_2 + x_3)! i!} (S_{A_1 A_2 A_3}^- S_{A_4}^+)^i \sum_{j=0}^{x_2} (-1)^j \frac{(x_1 - j)!}{x_1! j!} (S_{A_1}^- S_{A_2}^+)^j. \end{aligned} \quad / 5.9 /$$

Az előbb látott módon adjunk hozzá az $A_1 A_2 A_3 A_4$ rendszerhez további $A_5, A_6, \dots$ stb. rendszereket, amíg valamennyi elektront figyelembe nem vettünk.

Akkor pl. nyolc rendszer egyesítése esetén a következő operátort nyerjük :

$$\begin{aligned}
 \textcircled{1} \mathcal{O}_{A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8} &= \left(\frac{x_1 - x_2 + x_3 - x_4 + x_5 - x_6 + x_7 - x_8 + 1}{x_1 - x_2 + x_3 - x_4 + x_5 - x_6 + x_7 + 1} \right)^{\frac{1}{2}} \times \\
 &\times \left(\frac{x_1 - x_2 + x_3 - x_4 + x_5 - x_6 + 1}{x_1 - x_2 + x_3 - x_4 + x_5 + 1} \right)^{\frac{1}{2}} \left(\frac{x_1 - x_2 + x_3 - x_4 + 1}{x_1 - x_2 + x_3 + 1} \right)^{\frac{1}{2}} \left(\frac{x_1 - x_2 + 1}{x_1 + 1} \right)^{\frac{1}{2}} \times \\
 &\times \sum_{i=0}^{x_8} (-1)^i \frac{(x_1 - x_2 + x_3 - x_4 + x_5 - x_6 + x_7 - i)!}{(x_1 - x_2 + x_3 - x_4 + x_5 - x_6 + x_7)! i!} (\mathcal{S}_{A_1 A_2 A_3 A_4 A_5 A_6 A_7}^- \mathcal{S}_{A_8}^+)^i \times \\
 &\times \sum_{j=0}^{x_6} (-1)^j \frac{(x_1 - x_2 + x_3 - x_4 + x_5 - j)!}{(x_1 - x_2 + x_3 - x_4 + x_5)! j!} (\mathcal{S}_{A_1 A_2 A_3 A_4 A_5}^- \mathcal{S}_{A_6}^+)^j \times \quad / 5.10 / \\
 &\times \sum_{k=0}^{x_4} (-1)^k \frac{(x_1 - x_2 + x_3 - k)!}{(x_1 - x_2 + x_3)! k!} (\mathcal{S}_{A_1 A_2 A_3}^- \mathcal{S}_{A_4}^+)^k \times \\
 &\times \sum_{l=0}^{x_2} (-1)^l \frac{(x_1 - l)!}{x_1! l!} (\mathcal{S}_{A_1}^- \mathcal{S}_{A_2}^+)^l.
 \end{aligned}$$

6. § Nyolc elektronos multiplettek

Ismeretes, hogy egy spinállapot multiplicitását a $2s+1$ szám adja meg, ahol s a spineredőrt jelenti.
A következő spinállapotok lehetségesek :

s	$2s+1$	A SPINÁLLAPOT NEVE
0	1	SZINGULETT
$\frac{1}{2}$	2	DOUBLETT
1	3	TRIPLETT
$\frac{3}{2}$	4	QUARTETT
2	5	QUINTETT
$\frac{5}{2}$	6	SZEXTETT
3	7	SZEPTETT
$\frac{7}{2}$	8	OKTETT
4	9	NONETT

Nyolc elektronból álló rendszer esetén a SLATER-determinánsok száma $2^8 = 256$. Ennek megfelelően az elágazási diagram szerint megszerkeszthetünk 14 szingulett, 28 triplett, 20 quintett, 7 szeptett és 1 nonett spinállapotot. / $14 \cdot 1 + 28 \cdot 3 + 20 \cdot 5 + 7 \cdot 7 + 1 \cdot 9 = 256$ /
A továbbiakban a különböző multiplicitású spinállapotok sajátfüggvényeit adjuk meg.

A./ Szingulettek

A 0 eredő spinhez tartozó SLATER- determinánsokat a következőképpen jelöljük :

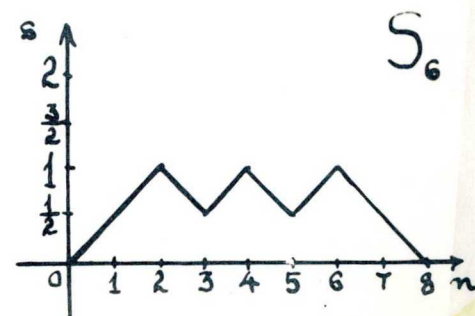
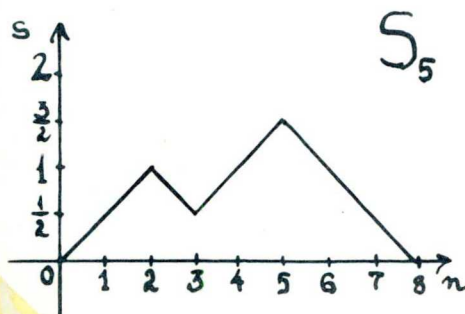
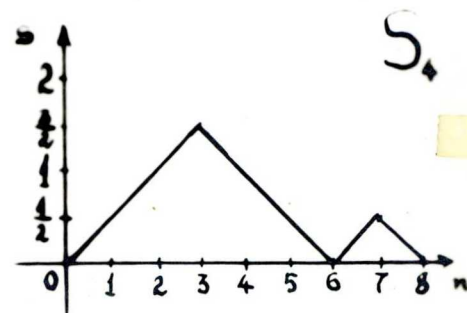
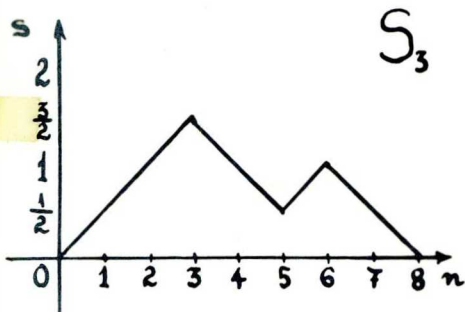
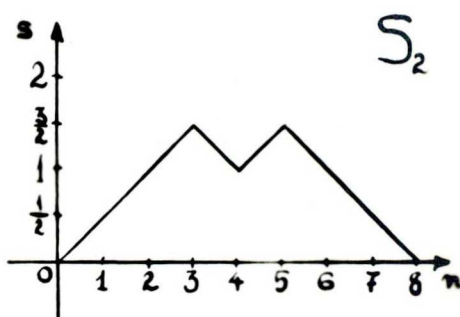
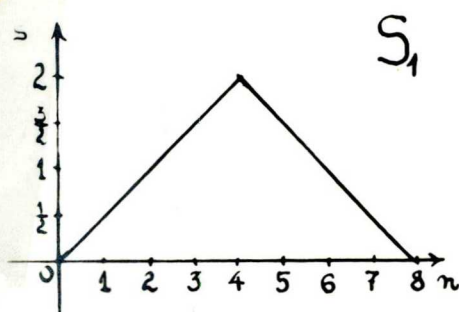
$$\begin{aligned}
 \varphi_1^0 &= |\alpha \alpha \alpha \alpha \beta \beta \beta \beta| \\
 \varphi_2^0 &= |\alpha \alpha \alpha \beta \alpha \beta \beta \beta| \\
 \varphi_3^0 &= |\alpha \alpha \alpha \beta \beta \alpha \beta \beta| \\
 \varphi_4^0 &= |\alpha \alpha \alpha \beta \beta \beta \alpha \beta| \\
 \varphi_5^0 &= |\alpha \alpha \alpha \beta \beta \beta \beta \alpha| \\
 \varphi_6^0 &= |\alpha \alpha \beta \alpha \alpha \beta \beta \beta| \\
 \varphi_7^0 &= |\alpha \alpha \beta \alpha \beta \alpha \beta \beta| \\
 \varphi_8^0 &= |\alpha \alpha \beta \alpha \beta \beta \alpha \beta| \\
 \varphi_9^0 &= |\alpha \alpha \beta \alpha \beta \beta \beta \alpha| \\
 \varphi_{10}^0 &= |\alpha \alpha \beta \beta \alpha \alpha \beta \beta| \\
 \varphi_{11}^0 &= |\alpha \alpha \beta \beta \alpha \beta \alpha \beta| \\
 \varphi_{12}^0 &= |\alpha \alpha \beta \beta \alpha \beta \beta \alpha| \\
 \varphi_{13}^0 &= |\alpha \alpha \beta \beta \beta \alpha \alpha \beta| \\
 \varphi_{14}^0 &= |\alpha \alpha \beta \beta \beta \alpha \beta \alpha| \\
 \varphi_{15}^0 &= |\alpha \alpha \beta \beta \beta \beta \alpha \alpha| \\
 \varphi_{16}^0 &= |\alpha \beta \alpha \alpha \alpha \beta \beta \beta| \\
 \varphi_{17}^0 &= |\alpha \beta \alpha \alpha \beta \alpha \beta \beta| \\
 \varphi_{18}^0 &= |\alpha \beta \alpha \alpha \beta \beta \alpha \beta| \\
 \varphi_{19}^0 &= |\alpha \beta \alpha \alpha \beta \beta \beta \alpha| \\
 \varphi_{20}^0 &= |\alpha \beta \alpha \beta \alpha \alpha \beta \beta| \\
 \varphi_{21}^0 &= |\alpha \beta \alpha \beta \alpha \beta \alpha \beta| \\
 \varphi_{22}^0 &= |\alpha \beta \alpha \beta \alpha \beta \beta \alpha| \\
 \varphi_{23}^0 &= |\alpha \beta \alpha \beta \beta \alpha \alpha \beta| \\
 \varphi_{24}^0 &= |\alpha \beta \alpha \beta \beta \alpha \beta \alpha| \\
 \varphi_{25}^0 &= |\alpha \beta \alpha \beta \beta \beta \alpha \alpha| \\
 \varphi_{26}^0 &= |\alpha \beta \beta \alpha \alpha \alpha \beta \beta| \\
 \varphi_{27}^0 &= |\alpha \beta \beta \alpha \alpha \beta \alpha \beta| \\
 \varphi_{28}^0 &= |\alpha \beta \beta \alpha \alpha \beta \beta \alpha| \\
 \varphi_{29}^0 &= |\alpha \beta \beta \alpha \beta \alpha \alpha \beta| \\
 \varphi_{30}^0 &= |\alpha \beta \beta \alpha \beta \alpha \beta \alpha|
 \end{aligned}$$

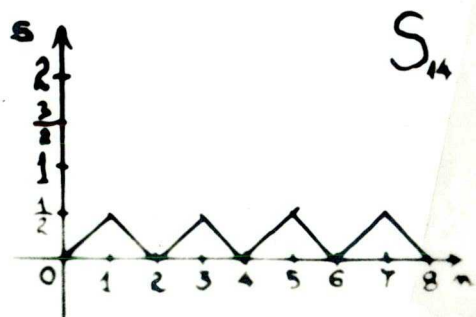
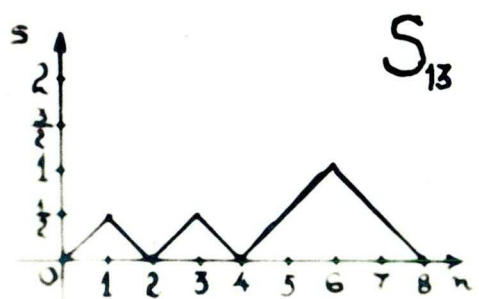
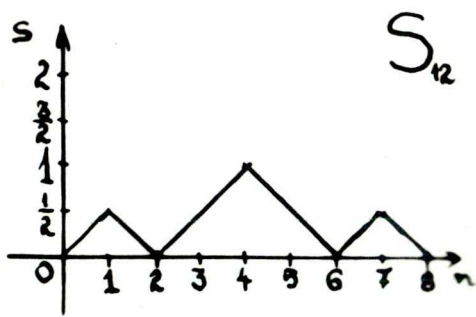
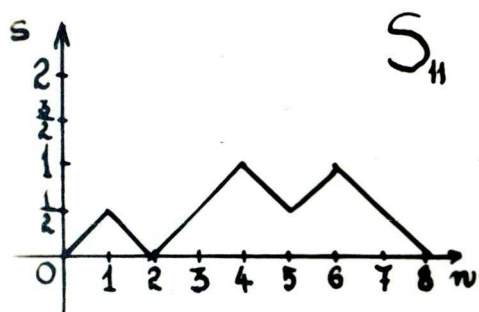
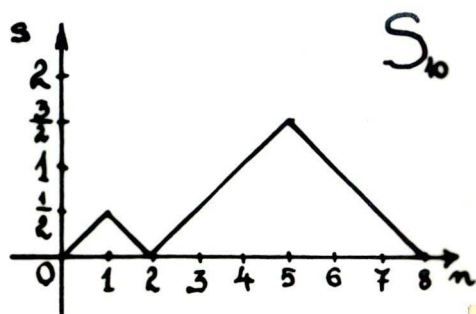
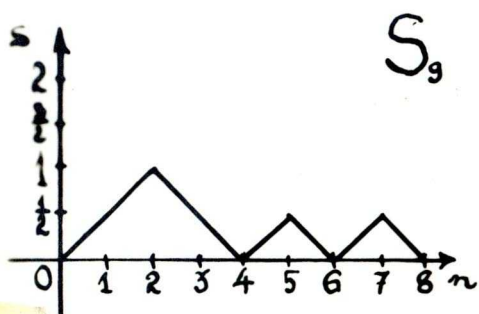
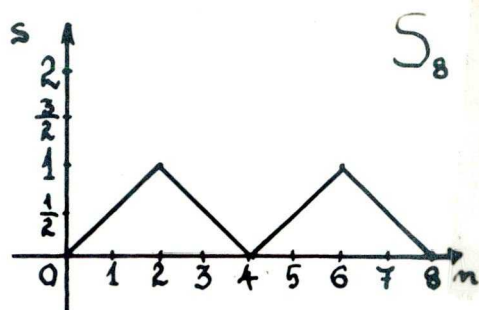
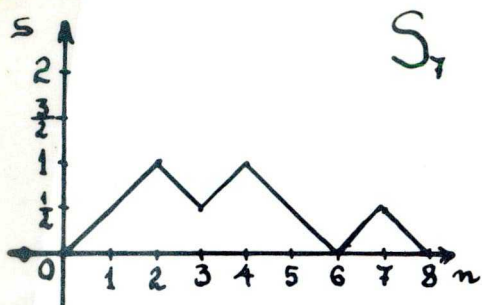
$$\begin{aligned}
 \varphi_{31}^0 &= |\alpha \beta \beta \alpha \beta \beta \alpha \alpha| \\
 \varphi_{32}^0 &= |\alpha \beta \beta \beta \alpha \alpha \alpha \beta| \\
 \varphi_{33}^0 &= |\alpha \beta \beta \beta \alpha \alpha \beta \alpha| \\
 \varphi_{34}^0 &= |\alpha \beta \beta \beta \alpha \beta \alpha \alpha| \\
 \varphi_{35}^0 &= |\alpha \beta \beta \beta \beta \alpha \alpha \alpha| \\
 \varphi_{36}^0 &= |\beta \alpha \alpha \alpha \alpha \beta \beta \beta| \\
 \varphi_{37}^0 &= |\beta \alpha \alpha \alpha \beta \alpha \beta \beta| \\
 \varphi_{38}^0 &= |\beta \alpha \alpha \alpha \beta \beta \alpha \beta| \\
 \varphi_{39}^0 &= |\beta \alpha \alpha \alpha \beta \beta \beta \alpha| \\
 \varphi_{40}^0 &= |\beta \alpha \alpha \beta \alpha \alpha \beta \beta| \\
 \varphi_{41}^0 &= |\beta \alpha \alpha \beta \alpha \beta \alpha \beta| \\
 \varphi_{42}^0 &= |\beta \alpha \alpha \beta \alpha \beta \beta \alpha| \\
 \varphi_{43}^0 &= |\beta \alpha \alpha \beta \beta \alpha \alpha \beta| \\
 \varphi_{44}^0 &= |\beta \alpha \alpha \beta \beta \alpha \beta \alpha| \\
 \varphi_{45}^0 &= |\beta \alpha \alpha \beta \beta \beta \alpha \alpha| \\
 \varphi_{46}^0 &= |\beta \alpha \beta \alpha \alpha \alpha \beta \beta| \\
 \varphi_{47}^0 &= |\beta \alpha \beta \alpha \alpha \beta \alpha \beta| \\
 \varphi_{48}^0 &= |\beta \alpha \beta \alpha \alpha \beta \beta \alpha| \\
 \varphi_{49}^0 &= |\beta \alpha \beta \alpha \beta \alpha \alpha \beta| \\
 \varphi_{50}^0 &= |\beta \alpha \beta \alpha \beta \alpha \beta \alpha| \\
 \varphi_{51}^0 &= |\beta \alpha \beta \alpha \beta \beta \alpha \alpha| \\
 \varphi_{52}^0 &= |\beta \alpha \beta \beta \alpha \alpha \alpha \beta| \\
 \varphi_{53}^0 &= |\beta \alpha \beta \beta \alpha \alpha \beta \alpha| \\
 \varphi_{54}^0 &= |\beta \alpha \beta \beta \alpha \beta \alpha \alpha| \\
 \varphi_{55}^0 &= |\beta \alpha \beta \beta \beta \alpha \alpha \alpha| \\
 \varphi_{56}^0 &= |\beta \beta \alpha \alpha \alpha \alpha \beta \beta| \\
 \varphi_{57}^0 &= |\beta \beta \alpha \alpha \alpha \beta \alpha \beta| \\
 \varphi_{58}^0 &= |\beta \beta \alpha \alpha \alpha \beta \beta \alpha| \\
 \varphi_{59}^0 &= |\beta \beta \alpha \alpha \beta \alpha \alpha \beta| \\
 \varphi_{60}^0 &= |\beta \beta \alpha \alpha \beta \alpha \beta \alpha|
 \end{aligned}$$

$$\begin{aligned}\varphi_{61}^0 &= |\beta\beta\alpha\alpha\beta\beta\alpha\alpha| \\ \varphi_{62}^0 &= |\beta\beta\alpha\beta\alpha\alpha\beta| \\ \varphi_{63}^0 &= |\beta\beta\alpha\beta\alpha\beta\alpha| \\ \varphi_{64}^0 &= |\beta\beta\alpha\beta\alpha\alpha\alpha| \\ \varphi_{65}^0 &= |\beta\beta\alpha\beta\beta\alpha\alpha\alpha|\end{aligned}$$

$$\begin{aligned}\varphi_{66}^0 &= |\beta\beta\beta\alpha\alpha\alpha\alpha\beta| \\ \varphi_{67}^0 &= |\beta\beta\beta\alpha\alpha\beta\alpha| \\ \varphi_{68}^0 &= |\beta\beta\beta\alpha\alpha\beta\alpha\alpha| \\ \varphi_{69}^0 &= |\beta\beta\beta\alpha\beta\alpha\alpha\alpha| \\ \varphi_{70}^0 &= |\beta\beta\beta\beta\alpha\alpha\alpha\alpha|\end{aligned}$$

Szinguletteket az elágazási diagram szerint
14 féleképpen szerkeszthetünk :





A megfelelő sajátfüggvények / 5.10 / alapján a következők:

$$\begin{aligned}
 \varphi_{S_1} = & \frac{\sqrt{5}}{60} \{ 12 (\varphi_1^\circ + \varphi_{70}^\circ) + \\
 & + 2 (\varphi_{10}^\circ + \varphi_{11}^\circ + \varphi_{12}^\circ + \varphi_{13}^\circ + \varphi_{14}^\circ + \varphi_{15}^\circ + \varphi_{20}^\circ + \varphi_{21}^\circ + \varphi_{22}^\circ + \varphi_{23}^\circ + \varphi_{27}^\circ + \varphi_{25}^\circ + \\
 & + \varphi_{26}^\circ + \varphi_{27}^\circ + \varphi_{28}^\circ + \varphi_{29}^\circ + \varphi_{30}^\circ + \varphi_{31}^\circ + \varphi_{40}^\circ + \varphi_{41}^\circ + \varphi_{42}^\circ + \varphi_{43}^\circ + \varphi_{44}^\circ + \varphi_{45}^\circ + \\
 & + \varphi_{46}^\circ + \varphi_{47}^\circ + \varphi_{48}^\circ + \varphi_{49}^\circ + \varphi_{50}^\circ + \varphi_{51}^\circ + \varphi_{56}^\circ + \varphi_{57}^\circ + \varphi_{58}^\circ + \varphi_{59}^\circ + \varphi_{60}^\circ + \varphi_{61}^\circ) - \\
 & - 3 (\varphi_2^\circ + \varphi_3^\circ + \varphi_4^\circ + \varphi_5^\circ + \varphi_6^\circ + \varphi_7^\circ + \varphi_8^\circ + \varphi_9^\circ + \\
 & + \varphi_{16}^\circ + \varphi_{17}^\circ + \varphi_{18}^\circ + \varphi_{19}^\circ + \varphi_{36}^\circ + \varphi_{37}^\circ + \varphi_{38}^\circ + \varphi_{39}^\circ + \\
 & + \varphi_{32}^\circ + \varphi_{39}^\circ + \varphi_{34}^\circ + \varphi_{35}^\circ + \varphi_{52}^\circ + \varphi_{53}^\circ + \varphi_{54}^\circ + \varphi_{55}^\circ + \\
 & + \varphi_{62}^\circ + \varphi_{63}^\circ + \varphi_{64}^\circ + \varphi_{65}^\circ + \varphi_{66}^\circ + \varphi_{67}^\circ + \varphi_{68}^\circ + \varphi_{69}^\circ) \} \\
 \\
 \varphi_{S_2} = & \frac{\sqrt{3}}{36} \{ 9 (\varphi_2^\circ + \varphi_{69}^\circ) + \\
 & + 2 (\varphi_{13}^\circ + \varphi_{14}^\circ + \varphi_{15}^\circ + \varphi_{23}^\circ + \varphi_{24}^\circ + \varphi_{25}^\circ + \varphi_{26}^\circ + \varphi_{27}^\circ + \varphi_{28}^\circ + \varphi_{43}^\circ + \\
 & + \varphi_{44}^\circ + \varphi_{45}^\circ + \varphi_{46}^\circ + \varphi_{47}^\circ + \varphi_{48}^\circ + \varphi_{56}^\circ + \varphi_{57}^\circ + \varphi_{58}^\circ) + \\
 & + 1 (\varphi_7^\circ + \varphi_8^\circ + \varphi_9^\circ + \varphi_{17}^\circ + \varphi_{18}^\circ + \varphi_{19}^\circ + \varphi_{32}^\circ + \varphi_{33}^\circ + \varphi_{34}^\circ + \varphi_{37}^\circ + \\
 & + \varphi_{38}^\circ + \varphi_{39}^\circ + \varphi_{52}^\circ + \varphi_{53}^\circ + \varphi_{54}^\circ + \varphi_{62}^\circ + \varphi_{63}^\circ + \varphi_{64}^\circ) - \\
 & - 3 (\varphi_3^\circ + \varphi_4^\circ + \varphi_5^\circ + \varphi_6^\circ + \varphi_{16}^\circ + \varphi_{35}^\circ + \varphi_{36}^\circ + \varphi_{55}^\circ + \varphi_{65}^\circ + \varphi_{66}^\circ + \varphi_{67}^\circ + \varphi_{68}^\circ) - \\
 & - 2 (\varphi_{10}^\circ + \varphi_{11}^\circ + \varphi_{12}^\circ + \varphi_{20}^\circ + \varphi_{21}^\circ + \varphi_{22}^\circ + \varphi_{29}^\circ + \varphi_{30}^\circ + \varphi_{31}^\circ + \varphi_{40}^\circ + \\
 & + \varphi_{41}^\circ + \varphi_{42}^\circ + \varphi_{49}^\circ + \varphi_{50}^\circ + \varphi_{51}^\circ + \varphi_{59}^\circ + \varphi_{60}^\circ + \varphi_{61}^\circ) \} \\
 \\
 \varphi_{S_3} = & \frac{\sqrt{6}}{36} \{ 6 (\varphi_3^\circ + \varphi_{68}^\circ) + \\
 & + 2 (\varphi_{15}^\circ + \varphi_{25}^\circ + \varphi_{26}^\circ + \varphi_{45}^\circ + \varphi_{46}^\circ + \varphi_{56}^\circ) + \\
 & + 1 (\varphi_8^\circ + \varphi_9^\circ + \varphi_{11}^\circ + \varphi_{12}^\circ + \varphi_{18}^\circ + \varphi_{19}^\circ + \varphi_{21}^\circ + \varphi_{22}^\circ + \varphi_{29}^\circ + \varphi_{30}^\circ + \varphi_{32}^\circ + \varphi_{33}^\circ + \\
 & + \varphi_{38}^\circ + \varphi_{39}^\circ + \varphi_{41}^\circ + \varphi_{42}^\circ + \varphi_{49}^\circ + \varphi_{50}^\circ + \varphi_{52}^\circ + \varphi_{53}^\circ + \varphi_{59}^\circ + \varphi_{60}^\circ + \varphi_{62}^\circ + \varphi_{63}^\circ) - \\
 & - 3 (\varphi_4^\circ + \varphi_5^\circ + \varphi_{66}^\circ + \varphi_{67}^\circ) - \\
 & - 2 (\varphi_7^\circ + \varphi_{10}^\circ + \varphi_{17}^\circ + \varphi_{20}^\circ + \varphi_{31}^\circ + \varphi_{34}^\circ + \varphi_{37}^\circ + \varphi_{40}^\circ + \varphi_{51}^\circ + \varphi_{54}^\circ + \varphi_{61}^\circ + \varphi_{64}^\circ) - \\
 & - 1 (\varphi_{13}^\circ + \varphi_{14}^\circ + \varphi_{23}^\circ + \varphi_{24}^\circ + \varphi_{27}^\circ + \varphi_{28}^\circ + \varphi_{43}^\circ + \varphi_{44}^\circ + \varphi_{47}^\circ + \varphi_{48}^\circ + \varphi_{57}^\circ + \varphi_{58}^\circ) \}
 \end{aligned}$$

$$\begin{aligned} \varphi_{S_4} = & \frac{\sqrt{2}}{12} \{ 3(\varphi_4^\circ + \varphi_{67}^\circ) + \\ & + 1(\varphi_3^\circ + \varphi_{12}^\circ + \varphi_{14}^\circ + \varphi_{27}^\circ + \varphi_{29}^\circ + \varphi_{32}^\circ + \varphi_{19}^\circ + \varphi_{22}^\circ + \varphi_{24}^\circ + \varphi_{39}^\circ + \\ & + \varphi_{42}^\circ + \varphi_{44}^\circ + \varphi_{47}^\circ + \varphi_{49}^\circ + \varphi_{52}^\circ + \varphi_{57}^\circ + \varphi_{59}^\circ + \varphi_{62}^\circ) - \\ & - 3(\varphi_5^\circ + \varphi_{66}^\circ) - \\ & - 1(\varphi_8^\circ + \varphi_{11}^\circ + \varphi_{13}^\circ + \varphi_{18}^\circ + \varphi_{21}^\circ + \varphi_{23}^\circ + \varphi_{28}^\circ + \varphi_{30}^\circ + \varphi_{33}^\circ + \varphi_{38}^\circ + \\ & + \varphi_{41}^\circ + \varphi_{43}^\circ + \varphi_{48}^\circ + \varphi_{50}^\circ + \varphi_{53}^\circ + \varphi_{58}^\circ + \varphi_{60}^\circ + \varphi_{63}^\circ) \} \end{aligned}$$

$$\begin{aligned} \varphi_{S_5} = & \frac{\sqrt{6}}{36} \{ 6(\varphi_6^\circ + \varphi_{65}^\circ) + \\ & + 2(\varphi_{13}^\circ + \varphi_{14}^\circ + \varphi_{15}^\circ + \varphi_{56}^\circ + \varphi_{57}^\circ + \varphi_{58}^\circ) + \\ & + 1(\varphi_6^\circ + \varphi_{17}^\circ + \varphi_{18}^\circ + \varphi_{20}^\circ + \varphi_{21}^\circ + \varphi_{22}^\circ + \varphi_{29}^\circ + \varphi_{30}^\circ + \varphi_{31}^\circ + \varphi_{32}^\circ + \varphi_{33}^\circ + \varphi_{34}^\circ + \\ & + \varphi_{37}^\circ + \varphi_{38}^\circ + \varphi_{39}^\circ + \varphi_{40}^\circ + \varphi_{41}^\circ + \varphi_{42}^\circ + \varphi_{43}^\circ + \varphi_{50}^\circ + \varphi_{51}^\circ + \varphi_{52}^\circ + \varphi_{53}^\circ + \varphi_{54}^\circ) - \\ & - 3(\varphi_{16}^\circ + \varphi_{35}^\circ + \varphi_{55}^\circ + \varphi_{66}^\circ) - \\ & - 2(\varphi_{10}^\circ + \varphi_{11}^\circ + \varphi_{12}^\circ + \varphi_7^\circ + \varphi_8^\circ + \varphi_9^\circ + \varphi_{59}^\circ + \varphi_{60}^\circ + \varphi_{61}^\circ + \varphi_{62}^\circ + \varphi_{63}^\circ + \varphi_{64}^\circ) - \\ & - 1(\varphi_{23}^\circ + \varphi_{24}^\circ + \varphi_{25}^\circ + \varphi_{26}^\circ + \varphi_{27}^\circ + \varphi_{28}^\circ + \varphi_{43}^\circ + \varphi_{44}^\circ + \varphi_{45}^\circ + \varphi_{46}^\circ + \varphi_{47}^\circ + \varphi_{48}^\circ) \} \end{aligned}$$

$$\begin{aligned} \varphi_{S_6} = & \frac{\sqrt{3}}{36} \{ 8(\varphi_7^\circ + \varphi_{64}^\circ) + \\ & + 4(\varphi_{15}^\circ + \varphi_{56}^\circ) \\ & + 2(\varphi_{11}^\circ + \varphi_{12}^\circ + \varphi_{18}^\circ + \varphi_{19}^\circ + \varphi_{25}^\circ + \varphi_{31}^\circ + \varphi_{32}^\circ + \varphi_{33}^\circ + \\ & + \varphi_{38}^\circ + \varphi_{39}^\circ + \varphi_{45}^\circ + \varphi_{51}^\circ + \varphi_{59}^\circ + \varphi_{60}^\circ) + \\ & + 1(\varphi_{23}^\circ + \varphi_{24}^\circ + \varphi_{27}^\circ + \varphi_{28}^\circ + \varphi_{43}^\circ + \varphi_{44}^\circ + \varphi_{47}^\circ + \varphi_{48}^\circ) - \\ & - 4(\varphi_8^\circ + \varphi_9^\circ + \varphi_{10}^\circ + \varphi_{17}^\circ + \varphi_{34}^\circ + \varphi_{37}^\circ + \varphi_{54}^\circ + \varphi_{61}^\circ) - \\ & - 2(\varphi_{13}^\circ + \varphi_{14}^\circ + \varphi_{25}^\circ + \varphi_{26}^\circ + \varphi_{45}^\circ + \varphi_{46}^\circ + \varphi_{52}^\circ + \varphi_{53}^\circ + \varphi_{57}^\circ + \varphi_{58}^\circ) - \\ & - 1(\varphi_{21}^\circ + \varphi_{22}^\circ + \varphi_{29}^\circ + \varphi_{30}^\circ + \varphi_{41}^\circ + \varphi_{42}^\circ + \varphi_{49}^\circ + \varphi_{50}^\circ) \} \end{aligned}$$

$$\begin{aligned} \varphi_{S_7} = & \frac{1}{12} \{ 4(\varphi_8^\circ + \varphi_{63}^\circ) + \\ & + 2(\varphi_{12}^\circ + \varphi_{14}^\circ + \varphi_{19}^\circ + \varphi_{32}^\circ + \varphi_{39}^\circ + \varphi_{52}^\circ + \varphi_{57}^\circ + \varphi_{59}^\circ) + \end{aligned}$$

$$\begin{aligned}
 & + 1(\varphi_{21}^{\circ} + \varphi_{23}^{\circ} + \varphi_{28}^{\circ} + \varphi_{30}^{\circ} + \varphi_{41}^{\circ} + \varphi_{43}^{\circ} + \varphi_{48}^{\circ} + \varphi_{50}^{\circ}) - \\
 & \quad - 4(\varphi_3^{\circ} + \varphi_{62}^{\circ}) - \\
 & - 2(\varphi_{11}^{\circ} + \varphi_{13}^{\circ} + \varphi_{18}^{\circ} + \varphi_{33}^{\circ} + \varphi_{38}^{\circ} + \varphi_{53}^{\circ} + \varphi_{58}^{\circ} + \varphi_{60}^{\circ}) - \\
 & - 1(\varphi_{22}^{\circ} + \varphi_{24}^{\circ} + \varphi_{27}^{\circ} + \varphi_{29}^{\circ} + \varphi_{42}^{\circ} + \varphi_{44}^{\circ} + \varphi_{47}^{\circ} + \varphi_{49}^{\circ}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{S_8} = \frac{1}{12} \{ & 4(\varphi_{10}^{\circ} + \varphi_{15}^{\circ} + \varphi_{56}^{\circ} + \varphi_{61}^{\circ}) \\
 & + 1(\varphi_{21}^{\circ} + \varphi_{22}^{\circ} + \varphi_{23}^{\circ} + \varphi_{24}^{\circ} + \varphi_{27}^{\circ} + \varphi_{28}^{\circ} + \varphi_{29}^{\circ} + \varphi_{30}^{\circ} + \\
 & \quad + \varphi_{41}^{\circ} + \varphi_{42}^{\circ} + \varphi_{43}^{\circ} + \varphi_{44}^{\circ} + \varphi_{47}^{\circ} + \varphi_{48}^{\circ} + \varphi_{49}^{\circ} + \varphi_{50}^{\circ}) - \\
 & - 2(\varphi_{11}^{\circ} + \varphi_{12}^{\circ} + \varphi_{13}^{\circ} + \varphi_{14}^{\circ} + \varphi_{20}^{\circ} + \varphi_{25}^{\circ} + \varphi_{26}^{\circ} + \varphi_{31}^{\circ} + \\
 & \quad + \varphi_{40}^{\circ} + \varphi_{45}^{\circ} + \varphi_{46}^{\circ} + \varphi_{51}^{\circ} + \varphi_{57}^{\circ} + \varphi_{58}^{\circ} + \varphi_{59}^{\circ} + \varphi_{60}^{\circ}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{S_9} = \frac{\sqrt{3}}{12} \{ & 2(\varphi_{11}^{\circ} + \varphi_{14}^{\circ} + \varphi_{57}^{\circ} + \varphi_{60}^{\circ}) + \\
 & + 1(\varphi_{22}^{\circ} + \varphi_{23}^{\circ} + \varphi_{28}^{\circ} + \varphi_{29}^{\circ} + \varphi_{42}^{\circ} + \varphi_{43}^{\circ} + \varphi_{48}^{\circ} + \varphi_{49}^{\circ}) - \\
 & \quad - 2(\varphi_{12}^{\circ} + \varphi_{13}^{\circ} + \varphi_{58}^{\circ} + \varphi_{59}^{\circ}) - \\
 & - 1(\varphi_{21}^{\circ} + \varphi_{24}^{\circ} + \varphi_{27}^{\circ} + \varphi_{30}^{\circ} + \varphi_{41}^{\circ} + \varphi_{44}^{\circ} + \varphi_{47}^{\circ} + \varphi_{50}^{\circ}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{S_{10}} = \frac{\sqrt{2}}{12} \{ & 3(\varphi_{16}^{\circ} + \varphi_{55}^{\circ}) + \\
 & + 1(\varphi_{23}^{\circ} + \varphi_{24}^{\circ} + \varphi_{25}^{\circ} + \varphi_{29}^{\circ} + \varphi_{30}^{\circ} + \varphi_{31}^{\circ} + \varphi_{32}^{\circ} + \varphi_{33}^{\circ} + \varphi_{34}^{\circ} + \varphi_{37}^{\circ} + \\
 & \quad + \varphi_{38}^{\circ} + \varphi_{39}^{\circ} + \varphi_{40}^{\circ} + \varphi_{41}^{\circ} + \varphi_{42}^{\circ} + \varphi_{46}^{\circ} + \varphi_{47}^{\circ} + \varphi_{48}^{\circ}) - \\
 & \quad - 3(\varphi_{35}^{\circ} + \varphi_{36}^{\circ}) \\
 & - 1(\varphi_{17}^{\circ} + \varphi_{18}^{\circ} + \varphi_{19}^{\circ} + \varphi_{20}^{\circ} + \varphi_{21}^{\circ} + \varphi_{22}^{\circ} + \varphi_{26}^{\circ} + \varphi_{27}^{\circ} + \varphi_{28}^{\circ} + \varphi_{43}^{\circ} + \\
 & \quad + \varphi_{44}^{\circ} + \varphi_{45}^{\circ} + \varphi_{49}^{\circ} + \varphi_{50}^{\circ} + \varphi_{51}^{\circ} + \varphi_{52}^{\circ} + \varphi_{53}^{\circ} + \varphi_{54}^{\circ}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{S_{11}} = \frac{1}{12} \{ & 4(\varphi_{17}^{\circ} + \varphi_{54}^{\circ} + \varphi_{55}^{\circ}) + \\
 & + 2(\varphi_{32}^{\circ} + \varphi_{33}^{\circ} + \varphi_{40}^{\circ} + \varphi_{45}^{\circ} + \varphi_{46}^{\circ} + \varphi_{51}^{\circ}) + \\
 & + 1(\varphi_{21}^{\circ} + \varphi_{22}^{\circ} + \varphi_{23}^{\circ} + \varphi_{24}^{\circ} + \varphi_{27}^{\circ} + \varphi_{28}^{\circ} + \varphi_{29}^{\circ} + \varphi_{30}^{\circ}) -
 \end{aligned}$$

$$\begin{aligned}
 & -4(\varphi_{34}^{\circ} + \varphi_{35}^{\circ} + \varphi_{37}^{\circ}) - \\
 & -2(\varphi_{20}^{\circ} + \varphi_{25}^{\circ} + \varphi_{26}^{\circ} + \varphi_{31}^{\circ} + \varphi_{52}^{\circ} + \varphi_{53}^{\circ}) - \\
 & -1(\varphi_{41}^{\circ} + \varphi_{42}^{\circ} + \varphi_{43}^{\circ} + \varphi_{44}^{\circ} + \varphi_{47}^{\circ} + \varphi_{48}^{\circ} + \varphi_{49}^{\circ} + \varphi_{50}^{\circ}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{S_{12}} = \frac{\sqrt{3}}{6} \{ & 2(\varphi_{18}^{\circ} + \varphi_{32}^{\circ} + \varphi_{39}^{\circ} + \varphi_{53}^{\circ}) + \\
 & + 1(\varphi_{22}^{\circ} + \varphi_{24}^{\circ} + \varphi_{28}^{\circ} + \varphi_{30}^{\circ} + \varphi_{41}^{\circ} + \varphi_{43}^{\circ} + \varphi_{47}^{\circ} + \varphi_{49}^{\circ}) - \\
 & - 2(\varphi_{19}^{\circ} + \varphi_{33}^{\circ} + \varphi_{38}^{\circ} + \varphi_{52}^{\circ}) - \\
 & - 1(\varphi_{21}^{\circ} + \varphi_{23}^{\circ} + \varphi_{27}^{\circ} + \varphi_{29}^{\circ} + \varphi_{42}^{\circ} + \varphi_{44}^{\circ} + \varphi_{48}^{\circ} + \varphi_{50}^{\circ}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{S_{13}} = \frac{\sqrt{3}}{24} \{ & 4(\varphi_{20}^{\circ} + \varphi_{25}^{\circ} + \varphi_{46}^{\circ}) + \\
 & + 2(\varphi_{27}^{\circ} + \varphi_{28}^{\circ} + \varphi_{29}^{\circ} + \varphi_{30}^{\circ} + \varphi_{41}^{\circ} + \varphi_{42}^{\circ} + \varphi_{43}^{\circ} + \varphi_{44}^{\circ}) + \\
 & + 1(\varphi_{51}^{\circ}) - \\
 & - 4(\varphi_{26}^{\circ} + \varphi_{40}^{\circ} + \varphi_{45}^{\circ}) - \\
 & - 2(\varphi_{21}^{\circ} + \varphi_{22}^{\circ} + \varphi_{23}^{\circ} + \varphi_{24}^{\circ} + \varphi_{47}^{\circ} + \varphi_{48}^{\circ} + \varphi_{49}^{\circ} + \varphi_{50}^{\circ}) - \\
 & - 1(\varphi_{31}^{\circ}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{S_{14}} = \frac{1}{4} \{ & +1(\varphi_{21}^{\circ} + \varphi_{24}^{\circ} + \varphi_{28}^{\circ} + \varphi_{29}^{\circ} + \varphi_{42}^{\circ} + \varphi_{43}^{\circ} + \varphi_{47}^{\circ} + \varphi_{50}^{\circ}) - \\
 & -1(\varphi_{12}^{\circ} + \varphi_{23}^{\circ} + \varphi_{27}^{\circ} + \varphi_{30}^{\circ} + \varphi_{41}^{\circ} + \varphi_{44}^{\circ} + \varphi_{48}^{\circ} + \varphi_{49}^{\circ}) \}
 \end{aligned}$$

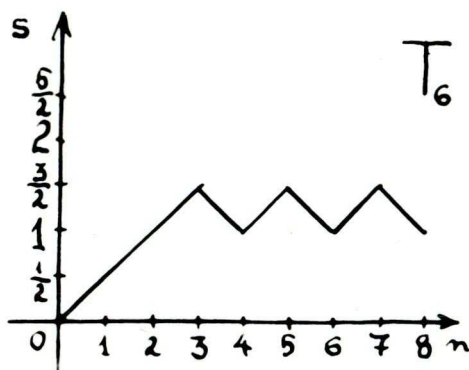
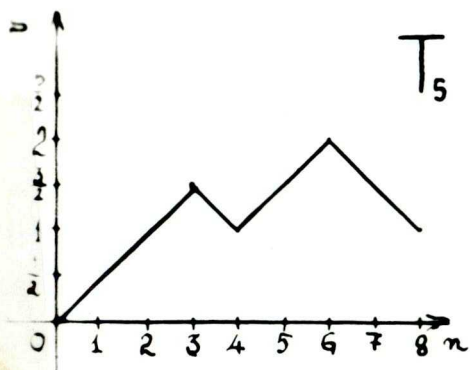
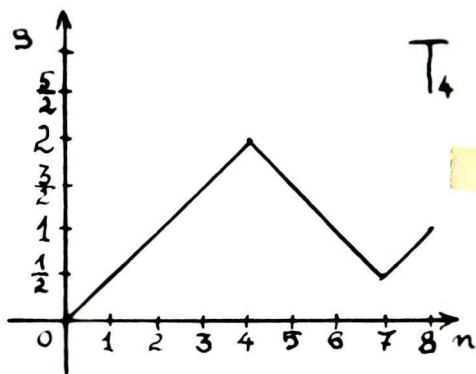
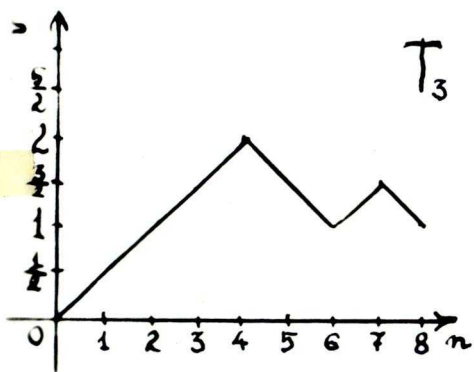
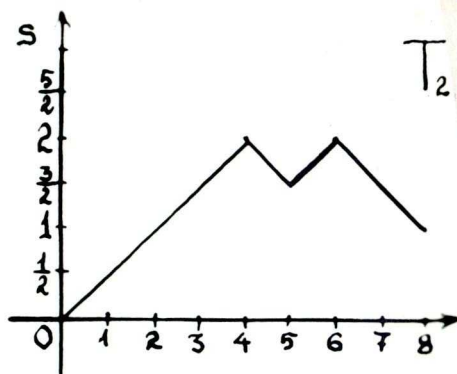
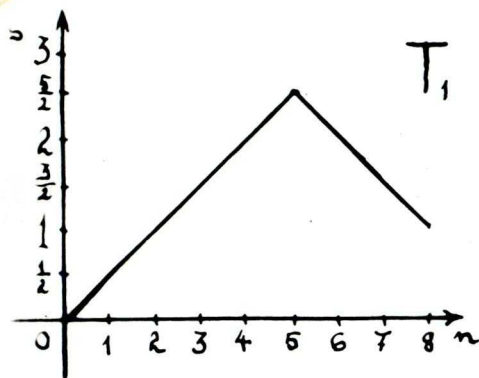
B./ Triplettek

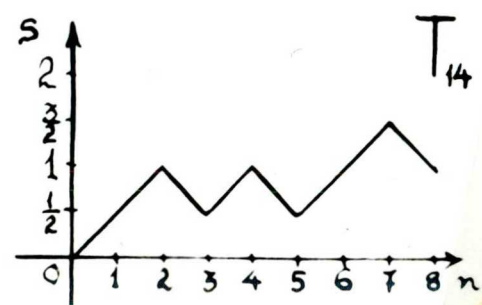
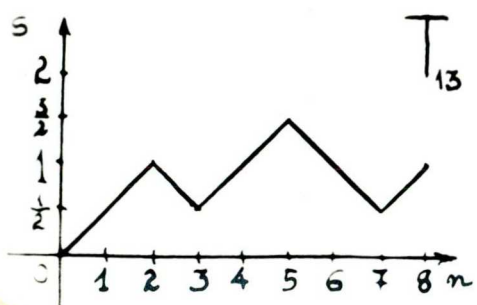
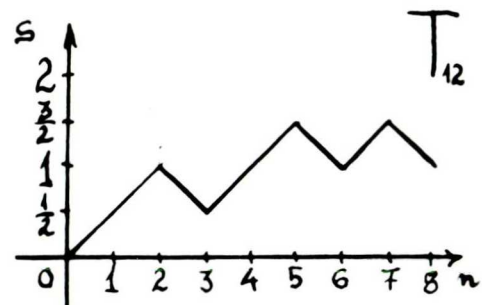
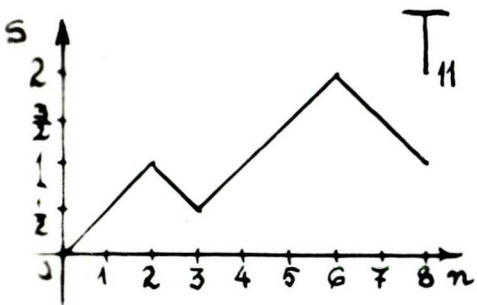
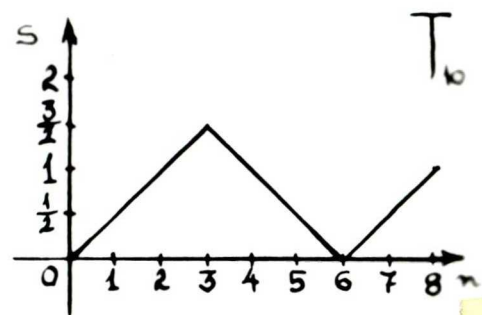
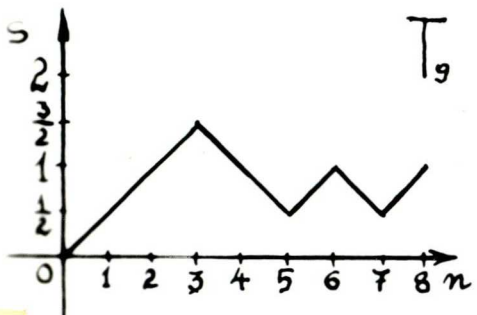
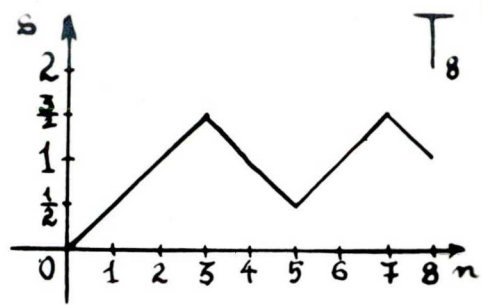
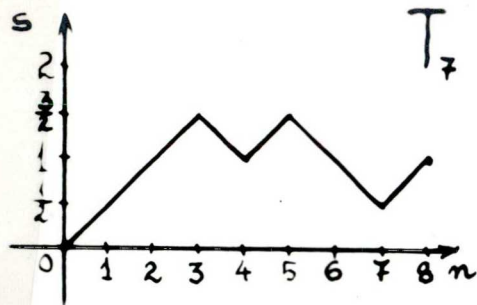
Az 1 eredő spinhez tartozó SLATER- determinánsokat a következőképpen jelöljük :

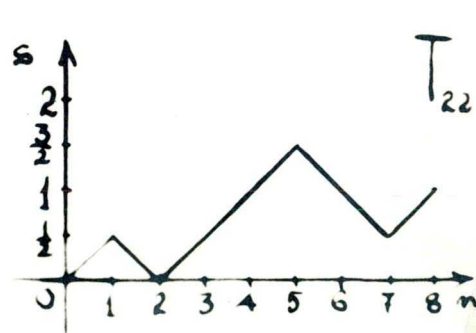
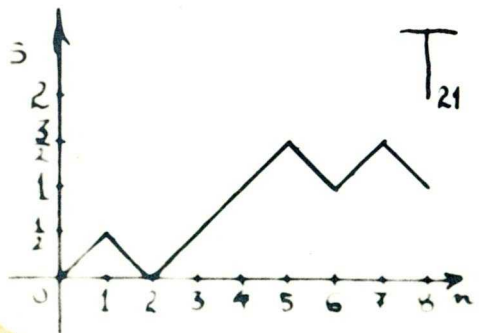
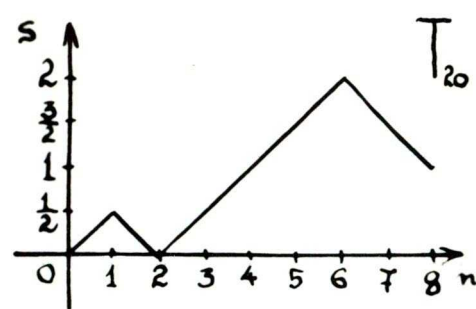
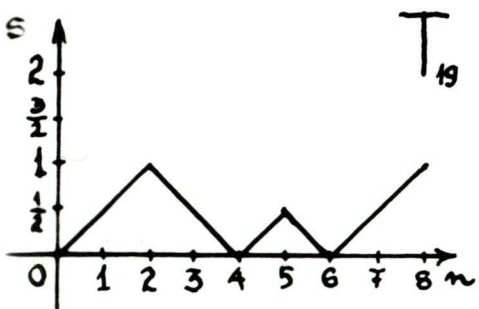
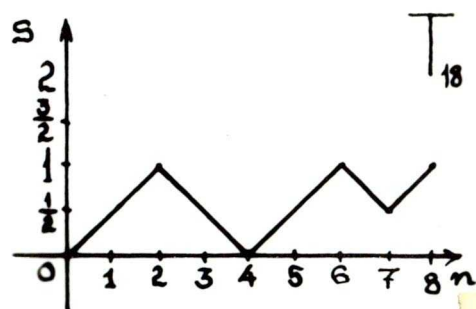
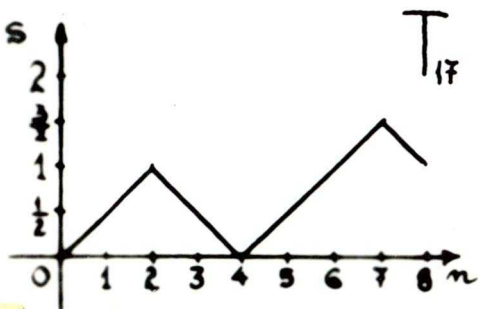
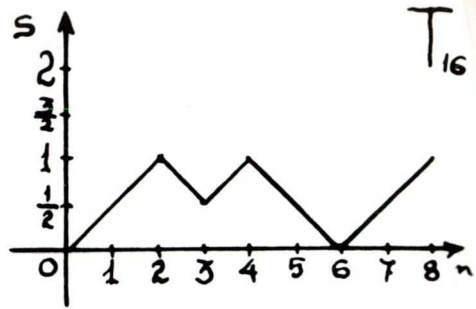
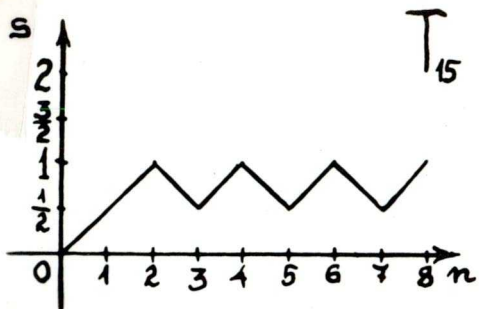
$$\begin{aligned}\varphi_1' &= | \alpha \alpha \alpha \alpha \alpha \beta \beta \beta | \\ \varphi_2' &= | \alpha \alpha \alpha \alpha \beta \alpha \beta \beta | \\ \varphi_3' &= | \alpha \alpha \alpha \alpha \beta \beta \alpha \beta | \\ \varphi_4' &= | \alpha \alpha \alpha \alpha \beta \beta \beta \alpha | \\ \varphi_5' &= | \alpha \alpha \alpha \beta \alpha \alpha \beta \beta | \\ \varphi_6' &= | \alpha \alpha \alpha \beta \alpha \beta \alpha \beta | \\ \varphi_7' &= | \alpha \alpha \alpha \beta \alpha \beta \beta \alpha | \\ \varphi_8' &= | \alpha \alpha \alpha \beta \beta \alpha \alpha \beta | \\ \varphi_9' &= | \alpha \alpha \alpha \beta \beta \alpha \beta \alpha | \\ \varphi_{10}' &= | \alpha \alpha \alpha \beta \beta \beta \alpha \alpha | \\ \varphi_{11}' &= | \alpha \alpha \beta \alpha \alpha \alpha \beta \beta | \\ \varphi_{12}' &= | \alpha \alpha \beta \alpha \alpha \beta \alpha \beta | \\ \varphi_{13}' &= | \alpha \alpha \beta \alpha \alpha \beta \beta \alpha | \\ \varphi_{14}' &= | \alpha \alpha \beta \alpha \beta \alpha \alpha \beta | \\ \varphi_{15}' &= | \alpha \alpha \beta \alpha \beta \alpha \beta \alpha | \\ \varphi_{16}' &= | \alpha \alpha \beta \alpha \beta \beta \alpha \alpha | \\ \varphi_{17}' &= | \alpha \alpha \beta \beta \alpha \alpha \alpha \beta | \\ \varphi_{18}' &= | \alpha \alpha \beta \beta \alpha \beta \alpha \alpha | \\ \varphi_{19}' &= | \alpha \alpha \beta \beta \beta \alpha \alpha \alpha | \\ \varphi_{20}' &= | \alpha \beta \alpha \alpha \alpha \alpha \beta \beta | \\ \varphi_{21}' &= | \alpha \beta \alpha \alpha \alpha \beta \alpha \beta | \\ \varphi_{22}' &= | \alpha \beta \alpha \alpha \alpha \beta \beta \alpha | \\ \varphi_{23}' &= | \alpha \beta \alpha \alpha \beta \alpha \alpha \beta | \\ \varphi_{24}' &= | \alpha \beta \alpha \alpha \beta \alpha \beta \alpha | \\ \varphi_{25}' &= | \alpha \beta \alpha \alpha \beta \beta \alpha \alpha | \\ \varphi_{26}' &= | \alpha \beta \alpha \alpha \beta \beta \alpha \alpha | \\ \varphi_{27}' &= | \alpha \beta \alpha \beta \alpha \alpha \alpha \beta | \\ \varphi_{28}' &= | \alpha \beta \alpha \beta \alpha \alpha \beta \alpha | \end{aligned}$$

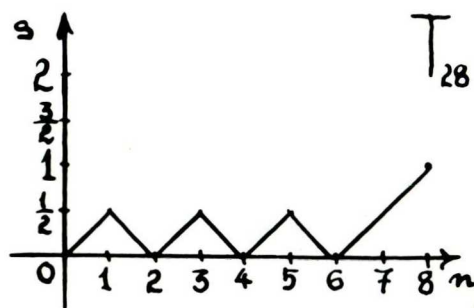
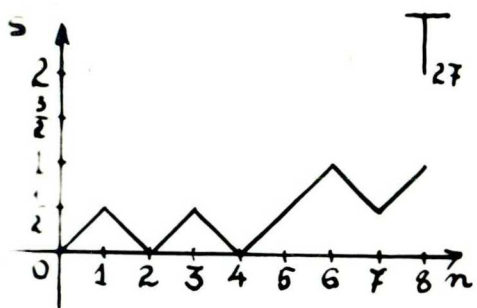
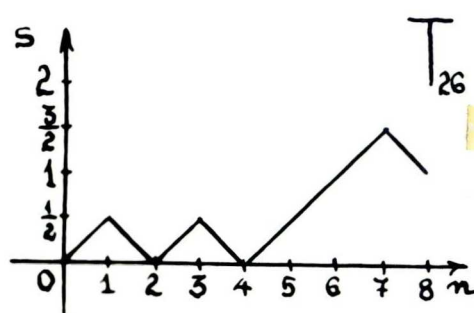
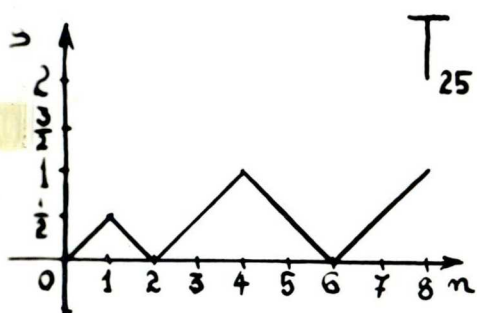
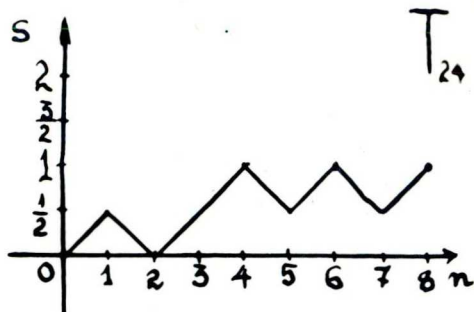
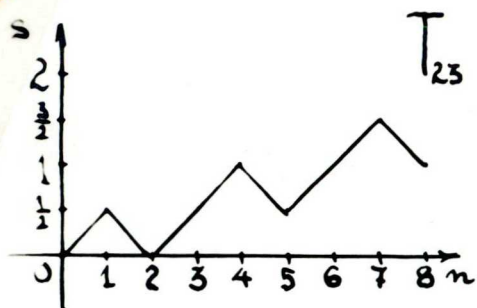
$$\begin{aligned}\varphi_{29}' &= | \alpha \beta \alpha \beta \alpha \beta \alpha \alpha | \\ \varphi_{30}' &= | \alpha \beta \alpha \beta \beta \alpha \alpha \alpha | \\ \varphi_{31}' &= | \alpha \beta \beta \alpha \alpha \alpha \alpha \beta | \\ \varphi_{32}' &= | \alpha \beta \beta \alpha \alpha \alpha \beta \alpha | \\ \varphi_{33}' &= | \alpha \beta \beta \alpha \alpha \beta \alpha \alpha | \\ \varphi_{34}' &= | \alpha \beta \beta \alpha \beta \alpha \alpha \alpha | \\ \varphi_{35}' &= | \alpha \beta \beta \beta \alpha \alpha \alpha \alpha | \\ \varphi_{36}' &= | \beta \alpha \alpha \alpha \alpha \alpha \beta \beta | \\ \varphi_{37}' &= | \beta \alpha \alpha \alpha \alpha \beta \alpha \beta | \\ \varphi_{38}' &= | \beta \alpha \alpha \alpha \alpha \beta \beta \alpha | \\ \varphi_{39}' &= | \beta \alpha \alpha \alpha \beta \alpha \alpha \beta | \\ \varphi_{40}' &= | \beta \alpha \alpha \alpha \beta \alpha \beta \alpha | \\ \varphi_{41}' &= | \beta \alpha \alpha \alpha \beta \beta \alpha \alpha | \\ \varphi_{42}' &= | \beta \alpha \alpha \beta \alpha \alpha \alpha \beta | \\ \varphi_{43}' &= | \beta \alpha \alpha \beta \alpha \alpha \beta \alpha | \\ \varphi_{44}' &= | \beta \alpha \alpha \beta \alpha \beta \alpha \alpha | \\ \varphi_{45}' &= | \beta \alpha \alpha \beta \beta \alpha \alpha \alpha | \\ \varphi_{46}' &= | \beta \alpha \beta \alpha \alpha \alpha \alpha \beta | \\ \varphi_{47}' &= | \beta \alpha \beta \alpha \alpha \alpha \beta \alpha | \\ \varphi_{48}' &= | \beta \alpha \beta \alpha \alpha \beta \alpha \alpha | \\ \varphi_{49}' &= | \beta \alpha \beta \alpha \beta \alpha \alpha \alpha | \\ \varphi_{50}' &= | \beta \alpha \beta \beta \alpha \alpha \alpha \alpha | \\ \varphi_{51}' &= | \beta \beta \alpha \alpha \alpha \alpha \alpha \beta | \\ \varphi_{52}' &= | \beta \beta \alpha \alpha \alpha \alpha \beta \alpha | \\ \varphi_{53}' &= | \beta \beta \alpha \alpha \alpha \beta \alpha \alpha | \\ \varphi_{54}' &= | \beta \beta \alpha \alpha \beta \alpha \alpha \alpha | \\ \varphi_{55}' &= | \beta \beta \alpha \beta \alpha \alpha \alpha \alpha | \\ \varphi_{56}' &= | \beta \beta \beta \alpha \alpha \alpha \alpha \alpha | \end{aligned}$$

Tripletteket az elágazási diagram szerint 28 féleképpen szerkeszthetünk :









A megfelelő sajátfüggvények a következők :

$$\varphi_{T_1} = \frac{\sqrt{2}}{20} \{ 10 (\varphi'_1) + \\ + 1 (\varphi'_8 + \varphi'_9 + \varphi'_{10} + \varphi'_{14} + \varphi'_{15} + \varphi'_{16} + \varphi'_{17} + \varphi'_{18} + \varphi'_{19} + \varphi'_{24} +$$

$$\begin{aligned}
 & + \varphi_{25}' + \varphi_{26}' + \varphi_{27}' + \varphi_{28}' + \varphi_{29}' + \varphi_{31}' + \varphi_{32}' + \varphi_{33}' + \varphi_{39}' + \varphi_{40}' \\
 & + \varphi_{41}' + \varphi_{42}' + \varphi_{43}' + \varphi_{44}' + \varphi_{46}' + \varphi_{47}' + \varphi_{48}' + \varphi_{51}' + \varphi_{52}' + \varphi_{53}') - \\
 & - 2(\varphi_2' + \varphi_3' + \varphi_4' + \varphi_5' + \varphi_6' + \varphi_7' + \varphi_{11}' + \varphi_{12}' + \\
 & + \varphi_{13}' + \varphi_{21}' + \varphi_{22}' + \varphi_{23}' + \varphi_{36}' + \varphi_{37}' + \varphi_{38}') - \\
 & - 1(\varphi_{20}' + \varphi_{30}' + \varphi_{34}' + \varphi_{35}' + \varphi_{45}' + \varphi_{49}' + \varphi_{50}' + \varphi_{54}' + \varphi_{55}' + \varphi_{56}') \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{T_2} = \frac{\sqrt{3}}{120} \{ & 48(\varphi_2') + \\
 & + 6(\varphi_{17}' + \varphi_{18}' + \varphi_{27}' + \varphi_{28}' + \varphi_{31}' + \varphi_{32}' + \varphi_{42}' + \varphi_{43}' + \\
 & + \varphi_{46}' + \varphi_{47}' + \varphi_{51}' + \varphi_{52}' + \varphi_{10}' + \varphi_{16}' + \varphi_{26}' + \varphi_{41}') + \\
 & + 4(\varphi_{20}' + \varphi_{30}' + \varphi_{34}' + \varphi_{45}' + \varphi_{49}' + \varphi_{54}') + \\
 & + 3(\varphi_6' + \varphi_7' + \varphi_{12}' + \varphi_{13}' + \varphi_{22}' + \varphi_{23}' + \varphi_{37}' + \varphi_{38}') - \\
 & - 12(\varphi_3' + \varphi_4' + \varphi_5' + \varphi_{11}' + \varphi_{21}' + \varphi_{36}') - \\
 & - 9(\varphi_8' + \varphi_9' + \varphi_{14}' + \varphi_{15}' + \varphi_{24}' + \varphi_{25}' + \varphi_{39}' + \varphi_{40}') - \\
 & - 6(\varphi_{35}' + \varphi_{50}' + \varphi_{55}' + \varphi_{56}') - \\
 & - 4(\varphi_{19}' + \varphi_{29}' + \varphi_{33}' + \varphi_{44}' + \varphi_{48}' + \varphi_{53}') \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{T_3} = \frac{\sqrt{5}}{120} \{ & 36(\varphi_3') + \\
 & + 6(\varphi_{17}' + \varphi_{27}' + \varphi_{31}' + \varphi_{42}' + \varphi_{46}' + \varphi_{51}') + \\
 & + 4(\varphi_{19}' + \varphi_{20}' + \varphi_{29}' + \varphi_{30}' + \varphi_{33}' + \varphi_{34}' + \varphi_{44}' + \varphi_{45}' + \varphi_{48}' + \varphi_{49}' + \varphi_{53}' + \varphi_{54}') + \\
 & + 3(\varphi_7' + \varphi_9' + \varphi_{13}' + \varphi_{15}' + \varphi_{23}' + \varphi_{25}' + \varphi_{38}' + \varphi_{40}') - \\
 & - 12(\varphi_4') - \\
 & - 9(\varphi_6' + \varphi_8' + \varphi_{12}' + \varphi_{14}' + \varphi_{22}' + \varphi_{24}' + \varphi_{37}' + \varphi_{39}') - \\
 & - 6(\varphi_{10}' + \varphi_{16}' + \varphi_{26}' + \varphi_{35}' + \varphi_{41}' + \varphi_{50}' + \varphi_{55}' + \varphi_{56}') - \\
 & - 2(\varphi_{18}' + \varphi_{28}' + \varphi_{32}' + \varphi_{43}' + \varphi_{47}' + \varphi_{52}') \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{T_4} = \frac{\sqrt{10}}{60} \{ & 12(\varphi_4') + \\
 & 2(\varphi_{18}' + \varphi_{19}' + \varphi_{20}' + \varphi_{28}' + \varphi_{29}' + \varphi_{30}' + \varphi_{32}' + \varphi_{33}' + \varphi_{34}' + \varphi_{43}' +
 \end{aligned}$$

$$+ \varphi'_{44} + \varphi'_{45} + \varphi'_{47} + \varphi'_{48} + \varphi'_{49} + \varphi'_{52} + \varphi'_{53} + \varphi'_{54}) - \\ - 3(\varphi'_{7} + \varphi'_{9} + \varphi'_{10} + \varphi'_{13} + \varphi'_{15} + \varphi'_{16} + \varphi'_{23} + \varphi'_{25} + \\ + \varphi'_{26} + \varphi'_{35} + \varphi'_{38} + \varphi'_{40} + \varphi'_{41} + \varphi'_{50} + \varphi'_{55} + \varphi'_{56}) \}$$

$$\varphi_{T_5} = \frac{\sqrt{5}}{120} \{ 36(\varphi'_5) + \\ + 6(\varphi'_{10} + \varphi'_{31} + \varphi'_{32} + \varphi'_{46} + \varphi'_{47} + \varphi'_{51} + \varphi'_{52}) + \\ + 4(\varphi'_{19} + \varphi'_{20} + \varphi'_{29} + \varphi'_{30} + \varphi'_{44} + \varphi'_{45}) + \\ + 3(\varphi'_{12} + \varphi'_{13} + \varphi'_{14} + \varphi'_{15} + \varphi'_{22} + \varphi'_{23} + \varphi'_{24} + \varphi'_{25} + \varphi'_{37} + \varphi'_{38} + \varphi'_{39} + \varphi'_{40}) + \\ + 2(\varphi'_{35} + \varphi'_{50} + \varphi'_{55}) - \\ - 12(\varphi'_{11} + \varphi'_{21} + \varphi'_{36}) - \\ - 9(\varphi'_{6} + \varphi'_{7} + \varphi'_{8} + \varphi'_{9}) - \\ - 6(\varphi'_{17} + \varphi'_{18} + \varphi'_{27} + \varphi'_{28} + \varphi'_{42} + \varphi'_{43} + \varphi'_{56}) - \\ - 4(\varphi'_{33} + \varphi'_{34} + \varphi'_{48} + \varphi'_{49} + \varphi'_{53} + \varphi'_{54}) - \\ - 2(\varphi'_{16} + \varphi'_{26} + \varphi'_{41}) \}$$

$$\varphi_{T_6} = \frac{\sqrt{3}}{72} \{ 27(\varphi'_6) + \\ + 6(\varphi'_{31} + \varphi'_{46} + \varphi'_{51}) + \\ + 4(\varphi'_{20} + \varphi'_{30} + \varphi'_{33} + \varphi'_{45} + \varphi'_{48} + \varphi'_{53}) + \\ + 3(\varphi'_{9} + \varphi'_{13} + \varphi'_{14} + \varphi'_{23} + \varphi'_{24} + \varphi'_{38} + \varphi'_{39}) + \\ + 2(\varphi'_{16} + \varphi'_{18} + \varphi'_{26} + \varphi'_{28} + \varphi'_{35} + \varphi'_{41} + \varphi'_{43} + \varphi'_{50} + \varphi'_{55}) - \\ - 9(\varphi'_{7} + \varphi'_{8} + \varphi'_{12} + \varphi'_{22} + \varphi'_{37}) - \\ - 6(\varphi'_{10} + \varphi'_{17} + \varphi'_{27} + \varphi'_{42} + \varphi'_{56}) - \\ - 4(\varphi'_{19} + \varphi'_{29} + \varphi'_{34} + \varphi'_{44} + \varphi'_{49} + \varphi'_{54}) - \\ - 2(\varphi'_{32} + \varphi'_{47} + \varphi'_{52}) - \\ - 1(\varphi'_{15} + \varphi'_{25} + \varphi'_{40}) \}$$

$$\begin{aligned} \varphi_{T_7} = & \frac{\sqrt{6}}{36} \{ 9(\varphi'_7) + \\ & + 2(\varphi'_{20} + \varphi'_{30} + \varphi'_{32} + \varphi'_{33} + \varphi'_{45} + \varphi'_{47} + \varphi'_{48} + \varphi'_{52} + \varphi'_{53}) + \\ & + 1(\varphi'_{15} + \varphi'_{16} + \varphi'_{15} + \varphi'_{26} + \varphi'_{35} + \varphi'_{40} + \varphi'_{41} + \varphi'_{50} + \varphi'_{55}) - \\ & - 3(\varphi'_9 + \varphi'_{10} + \varphi'_{13} + \varphi'_{23} + \varphi'_{38} + \varphi'_{56}) - \\ & - 2(\varphi'_{18} + \varphi'_{19} + \varphi'_{28} + \varphi'_{29} + \varphi'_{43} + \varphi'_{44} + \varphi'_{34} + \varphi'_{49} + \varphi'_{54}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_8} = & \frac{\sqrt{6}}{36} \{ 9(\varphi'_8) + \\ & + 2(\varphi'_{20} + \varphi'_{30} + \varphi'_{45}) + \\ & + 1(\varphi'_{15} + \varphi'_{16} + \varphi'_{18} + \varphi'_{19} + \varphi'_{25} + \varphi'_{26} + \varphi'_{28} + \varphi'_{29} + \varphi'_{34} + \\ & + \varphi'_{35} + \varphi'_{40} + \varphi'_{41} + \varphi'_{43} + \varphi'_{44} + \varphi'_{49} + \varphi'_{50} + \varphi'_{54} + \varphi'_{55}) - \\ & - 3(\varphi'_9 + \varphi'_{10} + \varphi'_{14} + \varphi'_{20} + \varphi'_{24} + \varphi'_{30} + \varphi'_{39} + \varphi'_{45} + \varphi'_{56}) - \\ & - 1(\varphi'_{32} + \varphi'_{33} + \varphi'_{47} + \varphi'_{48} + \varphi'_{52} + \varphi'_{53}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_9} = & \frac{\sqrt{3}}{18} \{ 6(\varphi'_9) + \\ & + 2(\varphi'_{32} + \varphi'_{47} + \varphi'_{52}) + \\ & + 1(\varphi'_{16} + \varphi'_{19} + \varphi'_{26} + \varphi'_{29} + \varphi'_{34} + \varphi'_{35} + \varphi'_{41} + \varphi'_{44} + \varphi'_{49} + \varphi'_{50} + \varphi'_{54} + \varphi'_{55}) - \\ & - 3(\varphi'_{10} + \varphi'_{56}) - \\ & - 2(\varphi'_{15} + \varphi'_{18} + \varphi'_{25} + \varphi'_{28} + \varphi'_{40} + \varphi'_{43}) - \\ & - 1(\varphi'_{20} + \varphi'_{30} + \varphi'_{33} + \varphi'_{45} + \varphi'_{48} + \varphi'_{53}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_{10}} = & \frac{1}{6} \{ 3(\varphi'_{10}) + \\ & + 1(\varphi'_{33} + \varphi'_{34} + \varphi'_{35} + \varphi'_{18} + \varphi'_{49} + \varphi'_{50} + \varphi'_{55} + \varphi'_{53} + \varphi'_{54}) - \\ & - 3(\varphi'_{56}) - \\ & - 1(\varphi'_{16} + \varphi'_{19} + \varphi'_{20} + \varphi'_{29} + \varphi'_{30} + \varphi'_{26} + \varphi'_{41} + \varphi'_{44} + \varphi'_{45}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_{11}} = & \frac{\sqrt{10}}{120} \{ 24(\varphi'_{11}) + \\ & + 6(\varphi'_{51} + \varphi'_{52}) + \end{aligned}$$

$$\begin{aligned}
 & + 4(\varphi_{16}' + \varphi_{19}' + \varphi_{20}') + \\
 & + 3(\varphi_{22}' + \varphi_{23}' + \varphi_{24}' + \varphi_{25}' + \varphi_{27}' + \varphi_{28}' + \varphi_{37}' + \varphi_{38}' + \varphi_{39}' + \varphi_{40}' + \varphi_{42}' + \varphi_{43}') + \\
 & + 2(\varphi_{33}' + \varphi_{34}' + \varphi_{35}' + \varphi_{48}' + \varphi_{49}' + \varphi_{50}') - \\
 & - 12(\varphi_{21}' + \varphi_{36}') - \\
 & - 6(\varphi_{12}' + \varphi_{13}' + \varphi_{14}' + \varphi_{15}' + \varphi_{17}' + \varphi_{18}') - \\
 & - 4(\varphi_{53}' + \varphi_{54}' + \varphi_{55}') - \\
 & - 3(\varphi_{31}' + \varphi_{32}' + \varphi_{46}' + \varphi_{47}') - \\
 & - 2(\varphi_{26}' + \varphi_{29}' + \varphi_{30}' + \varphi_{41}' + \varphi_{44}' + \varphi_{45}') \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{T_{12}} = \frac{\sqrt{6}}{72} \{ & 18(\varphi_{12}') + \\
 & + 6(\varphi_{51}') + \\
 & + 4(\varphi_{20}' + \varphi_{53}') + \\
 & + 3(\varphi_{23}' + \varphi_{24}' + \varphi_{27}' + \varphi_{38}' + \varphi_{39}' + \varphi_{42}') + \\
 & + 2(\varphi_{15}' + \varphi_{18}' + \varphi_{26}' + \varphi_{29}' + \varphi_{34}' + \varphi_{35}' + \varphi_{41}' + \varphi_{44}' + \varphi_{49}' + \varphi_{50}') \\
 & + 1(\varphi_{32}' + \varphi_{47}') - \\
 & - 9(\varphi_{22}' + \varphi_{37}') - \\
 & - 6(\varphi_{13}' + \varphi_{14}' + \varphi_{17}') - \\
 & - 4(\varphi_{16}' + \varphi_{19}' + \varphi_{54}' + \varphi_{55}') \\
 & - 3(\varphi_{31}' + \varphi_{46}') - \\
 & - 2(\varphi_{30}' + \varphi_{33}' + \varphi_{45}' + \varphi_{48}' + \varphi_{52}') - \\
 & - 1(\varphi_{25}' + \varphi_{28}' + \varphi_{40}' + \varphi_{43}') \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{T_{13}} = \frac{\sqrt{3}}{3} \{ & 6(\varphi_{13}') + \\
 & + 2(\varphi_{20}' + \varphi_{52}' + \varphi_{53}') + \\
 & + 1(\varphi_{15}' + \varphi_{26}' + \varphi_{28}' + \varphi_{29}' + \varphi_{34}' + \varphi_{35}' + \varphi_{40}' + \varphi_{41}' + \varphi_{43}' + \varphi_{44}' + \varphi_{49}' + \varphi_{50}') - \\
 & - 3(\varphi_{23}' + \varphi_{38}') -
 \end{aligned}$$

$$-2(\varphi'_{15} + \varphi'_{16} + \varphi'_{18} + \varphi'_{19} + \varphi'_{54} + \varphi'_{55}) - \\ -1(\varphi'_{30} + \varphi'_{32} + \varphi'_{33} + \varphi'_{45} + \varphi'_{47} + \varphi'_{48})\}$$

$$\varphi_{T_{14}} = \frac{\sqrt{3}}{36} \{ 12(\varphi'_{14}) + \\ + 6(\varphi'_{51}) + \\ + 3(\varphi'_{27} + \varphi'_{42}) + \\ + 2(\varphi'_{18} + \varphi'_{19} + \varphi'_{25} + \varphi'_{26} + \varphi'_{35} + \varphi'_{40} + \varphi'_{41} + \varphi'_{50} + \varphi'_{54}) + \\ + 1(\varphi'_{30} + \varphi'_{32} + \varphi'_{33} + \varphi'_{43} + \varphi'_{47} + \varphi'_{48}) - \\ - 6(\varphi'_{17} + \varphi'_{24} + \varphi'_{39}) - \\ - 4(\varphi'_{15} + \varphi'_{16} + \varphi'_{55}) - \\ - 3(\varphi'_{31} + \varphi'_{46}) - \\ - 2(\varphi'_{20} + \varphi'_{52} + \varphi'_{53}) - \\ - 1(\varphi'_{28} + \varphi'_{29} + \varphi'_{34} + \varphi'_{43} + \varphi'_{44} + \varphi'_{49}) \}$$

$$\varphi_{T_{15}} = \frac{\sqrt{6}}{36} \{ 8(\varphi'_{15}) + \\ + 4(\varphi'_{52}) + \\ + 2(\varphi'_{19} + \varphi'_{26} + \varphi'_{28} + \varphi'_{32} + \varphi'_{35} + \varphi'_{41} + \varphi'_{48} + \varphi'_{50} + \varphi'_{54}) + \\ + 1(\varphi'_{30} + \varphi'_{33} + \varphi'_{45}) - \\ - 4(\varphi'_{16} + \varphi'_{18} + \varphi'_{25} + \varphi'_{40} + \varphi'_{55}) - \\ - 2(\varphi'_{20} + \varphi'_{43} + \varphi'_{47} + \varphi'_{53}) - \\ - 1(\varphi'_{29} + \varphi'_{34} + \varphi'_{44} + \varphi'_{48} + \varphi'_{49}) \}$$

$$\varphi_{T_{16}} = \frac{\sqrt{2}}{12} \{ 4(\varphi'_{16}) + \\ + 2(\varphi'_{35} + \varphi'_{50} + \varphi'_{53} + \varphi'_{54}) + \\ + 1(\varphi'_{29} + \varphi'_{30} + \varphi'_{44} + \varphi'_{45}) - \\ - 4(\varphi'_{55}) -$$

$$\begin{aligned} & -2(\varphi'_{19} + \varphi'_{20} + \varphi'_{26} + \varphi'_{41}) \\ & -1(\varphi'_{33} + \varphi'_{34} + \varphi'_{48} + \varphi'_{49}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_{17}} = & \frac{1}{12} \{ 6(\varphi'_{17} + \varphi'_{51}) + \\ & + 1(\varphi'_{28} + \varphi'_{29} + \varphi'_{30} + \varphi'_{32} + \varphi'_{33} + \varphi'_{34} + \varphi'_{43} + \varphi'_{44} + \varphi'_{45} + \varphi'_{47} + \varphi'_{48} + \varphi'_{49}) - \\ & - 2(\varphi'_{18} + \varphi'_{19} + \varphi'_{20} + \varphi'_{52} + \varphi'_{53} + \varphi'_{54}) - \\ & - 3(\varphi'_{27} + \varphi'_{31} + \varphi'_{42} + \varphi'_{46}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_{18}} = & \frac{\sqrt{2}}{12} \{ 4(\varphi'_{18} + \varphi'_{52}) + \\ & + 1(\varphi'_{29} + \varphi'_{30} + \varphi'_{33} + \varphi'_{34} + \varphi'_{44} + \varphi'_{45} + \varphi'_{48} + \varphi'_{49}) - \\ & - 2(\varphi'_{19} + \varphi'_{20} + \varphi'_{28} + \varphi'_{32} + \varphi'_{43} + \varphi'_{47} + \varphi'_{53} + \varphi'_{54}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_{19}} = & \frac{\sqrt{6}}{12} \{ 2(\varphi'_{19} + \varphi'_{53}) + \\ & + 1(\varphi'_{29} + \varphi'_{33} + \varphi'_{44} + \varphi'_{48}) - \\ & - 2(\varphi'_{20} + \varphi'_{54}) - \\ & - 1(\varphi'_{30} + \varphi'_{34} + \varphi'_{45} + \varphi'_{49}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_{20}} = & \frac{\sqrt{5}}{60} \{ 12(\varphi'_{21}) + \\ & + 3(\varphi'_{37} + \varphi'_{38} + \varphi'_{39} + \varphi'_{40} + \varphi'_{42} + \varphi'_{43} + \varphi'_{46} + \varphi'_{47}) + \\ & + 2(\varphi'_{26} + \varphi'_{29} + \varphi'_{30} + \varphi'_{33} + \varphi'_{34} + \varphi'_{35}) - \\ & - 12(\varphi'_{36}) - \\ & - 3(\varphi'_{22} + \varphi'_{23} + \varphi'_{24} + \varphi'_{25} + \varphi'_{27} + \varphi'_{28} + \varphi'_{31} + \varphi'_{32}) - \\ & - 2(\varphi'_{41} + \varphi'_{44} + \varphi'_{45} + \varphi'_{48} + \varphi'_{49} + \varphi'_{50}) \} \end{aligned}$$

$$\begin{aligned} \varphi_{T_{21}} = & \frac{\sqrt{2}}{24} \{ 9(\varphi'_{22}) + \\ & + 3(\varphi'_{38} + \varphi'_{39} + \varphi'_{42} + \varphi'_{46}) + \end{aligned}$$

$$\begin{aligned}
 & + 2(\varphi'_{30} + \varphi'_{34} + \varphi'_{35} + \varphi'_{41} + \varphi'_{44} + \varphi'_{48}) + \\
 & + 1(\varphi'_{25} + \varphi'_{28} + \varphi'_{32}) - \\
 & - 9(\varphi'_{37}) - \\
 & - 3(\varphi'_{23} + \varphi'_{24} + \varphi'_{27} + \varphi'_{31}) - \\
 & - 2(\varphi'_{26} + \varphi'_{29} + \varphi'_{33} + \varphi'_{45} + \varphi'_{49} + \varphi'_{50}) - \\
 & - 1(\varphi'_{40} + \varphi'_{43} + \varphi'_{47}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{T_{22}} = \frac{1}{6} \{ & 3(\varphi'_{23}) + \\
 & + 1(\varphi'_{29} + \varphi'_{32} + \varphi'_{35} + \varphi'_{40} + \varphi'_{41} + \varphi'_{43} + \varphi'_{44} + \varphi'_{47} + \varphi'_{48}) - \\
 & - 3(\varphi'_{38}) - \\
 & - 1(\varphi'_{25} + \varphi'_{26} + \varphi'_{28} + \varphi'_{29} + \varphi'_{32} + \varphi'_{33} + \varphi'_{45} + \varphi'_{49} + \varphi'_{50}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{T_{23}} = \frac{1}{12} \{ & 6(\varphi'_{24}) + \\
 & + 3(\varphi'_{42} + \varphi'_{46}) + \\
 & + 2(\varphi'_{35} + \varphi'_{40} + \varphi'_{41}) + \\
 & + 1(\varphi'_{28} + \varphi'_{29} + \varphi'_{32} + \varphi'_{33} + \varphi'_{45} + \varphi'_{49}) - \\
 & - 6(\varphi'_{39}) - \\
 & - 3(\varphi'_{27} + \varphi'_{31}) - \\
 & - 2(\varphi'_{25} + \varphi'_{26} + \varphi'_{50}) - \\
 & - 1(\varphi'_{30} + \varphi'_{34} + \varphi'_{43} + \varphi'_{44} + \varphi'_{47} + \varphi'_{48}) \}
 \end{aligned}$$

$$\begin{aligned}
 \varphi_{T_{24}} = \frac{\sqrt{2}}{12} \{ & 4(\varphi'_{25}) \\
 & + 2(\varphi'_{35} + \varphi'_{41} + \varphi'_{43} + \varphi'_{47}) + \\
 & + 1(\varphi'_{29} + \varphi'_{33} + \varphi'_{45} + \varphi'_{49}) - \\
 & - 4(\varphi'_{40}) - \\
 & - 2(\varphi'_{26} + \varphi'_{28} + \varphi'_{32} + \varphi'_{50}) - \\
 & - 1(\varphi'_{30} + \varphi'_{34} + \varphi'_{44} + \varphi'_{48}) \}
 \end{aligned}$$

$$\varphi_{T_{25}} = \frac{\sqrt{6}}{12} \{ 2(\varphi'_{26} + \varphi'_{35}) + \\ + 1(\varphi'_{44} + \varphi'_{45} + \varphi'_{48} + \varphi'_{49}) - \\ - 2(\varphi'_{44} + \varphi'_{50}) - \\ - 1(\varphi'_{29} + \varphi'_{30} + \varphi'_{33} + \varphi'_{34}) \}$$

$$\varphi_{T_{26}} = \frac{\sqrt{3}}{12} \{ 3(\varphi'_{27} + \varphi'_{46}) + \\ + 1(\varphi'_{32} + \varphi'_{33} + \varphi'_{34} + \varphi'_{43} + \varphi'_{44} + \varphi'_{45}) - \\ - 3(\varphi'_{34} + \varphi'_{42}) - \\ - 1(\varphi'_{28} + \varphi'_{29} + \varphi'_{30} + \varphi'_{47} + \varphi'_{48} + \varphi'_{49}) \}$$

$$\varphi_{T_{27}} = \frac{\sqrt{6}}{12} \{ 2(\varphi'_{28} + \varphi'_{47}) + \\ + 1(\varphi'_{33} + \varphi'_{34} + \varphi'_{44} + \varphi'_{45}) - \\ - 2(\varphi'_{32} + \varphi'_{43}) - \\ - 1(\varphi'_{29} + \varphi'_{30} + \varphi'_{48} + \varphi'_{49}) \}$$

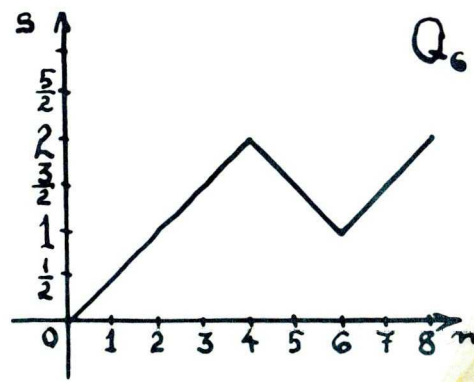
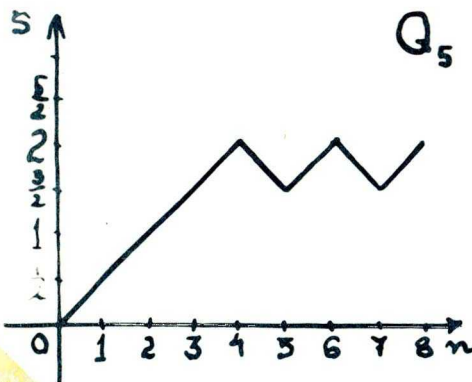
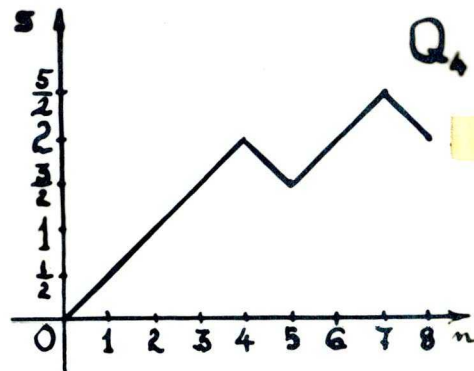
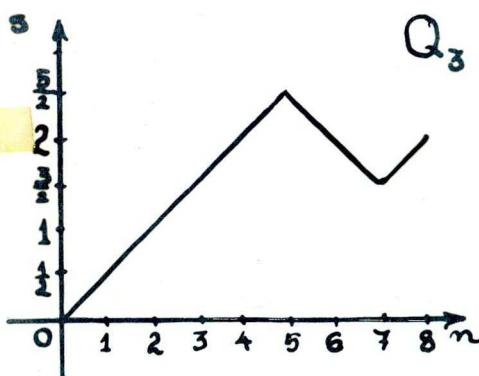
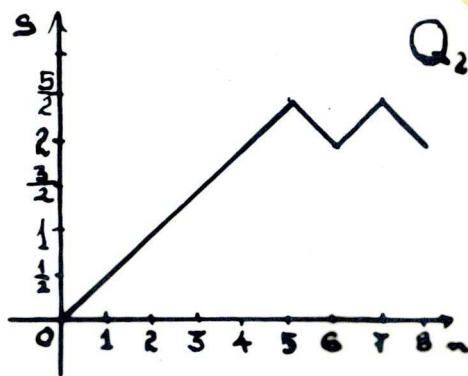
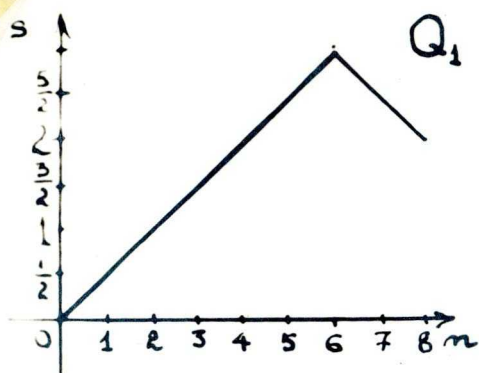
$$\varphi_{T_{28}} = \frac{\sqrt{2}}{4} \{ 1(\varphi'_{29} + \varphi'_{34} + \varphi'_{45} + \varphi'_{48}) - \\ - 1(\varphi'_{30} + \varphi'_{33} + \varphi'_{43} + \varphi'_{49}) \}$$

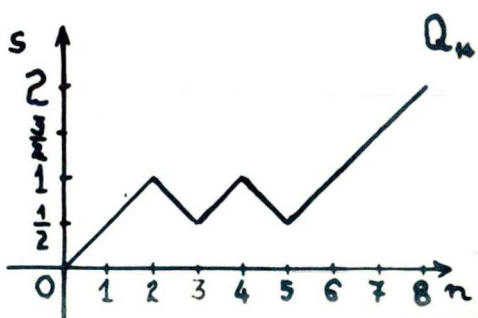
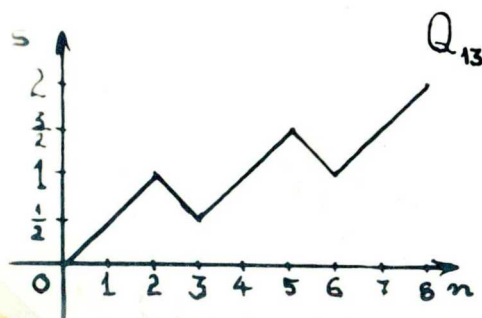
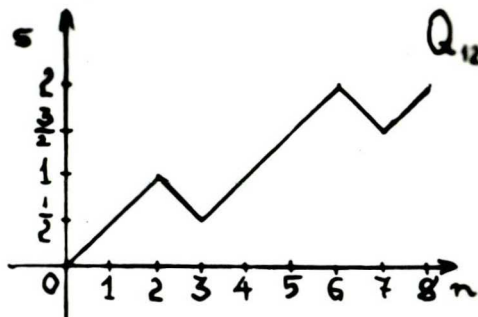
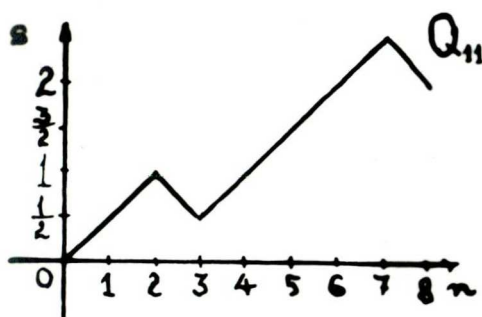
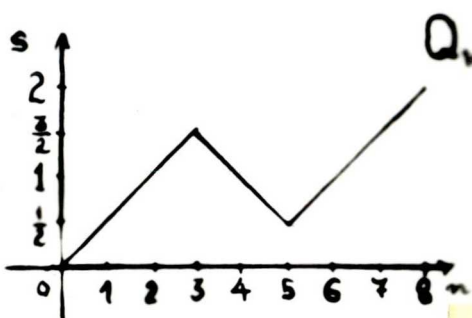
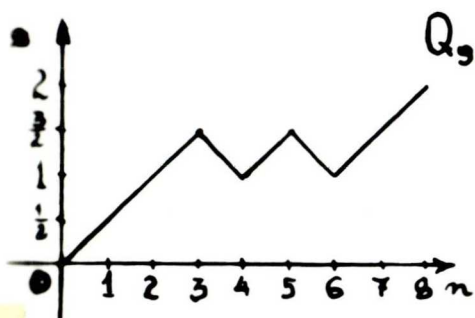
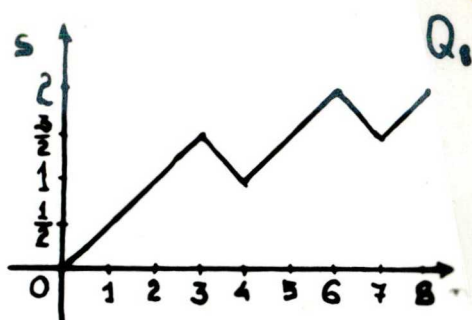
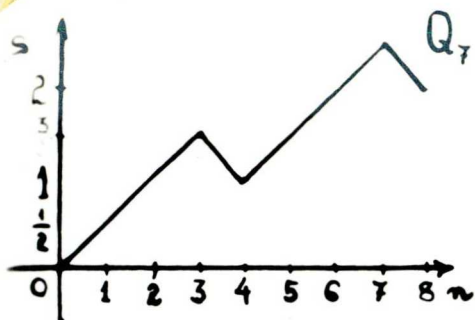
C./ Quintettek

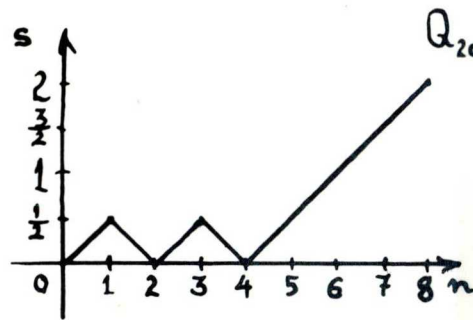
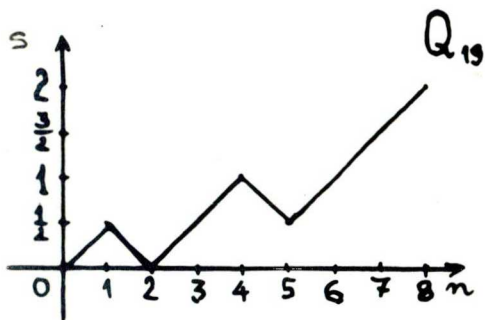
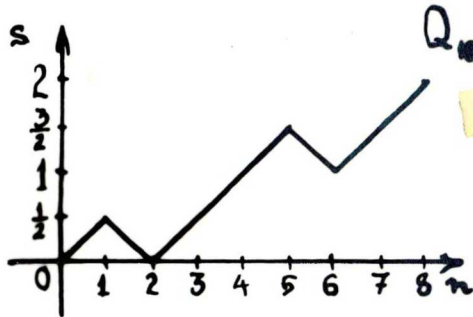
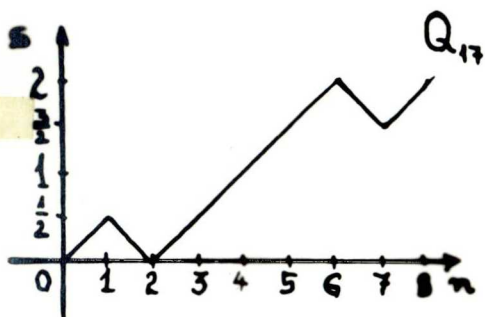
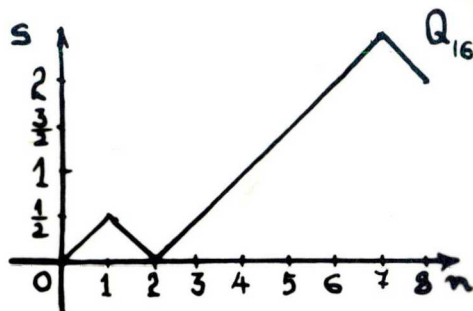
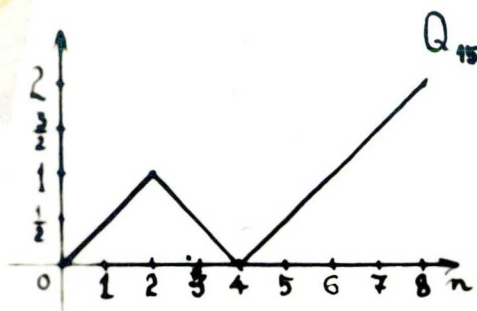
A 3 eredő spinhez tartozó SLATER-determinánsokat a következőképpen jelöljük :

$$\begin{aligned}
 \varphi_1^2 &= | \alpha \alpha \alpha \alpha \alpha \alpha \beta \beta | \\
 \varphi_2^2 &= | \alpha \alpha \alpha \alpha \alpha \beta \alpha \beta | \\
 \varphi_3^2 &= | \alpha \alpha \alpha \alpha \alpha \beta \beta \alpha | \\
 \varphi_4^2 &= | \alpha \alpha \alpha \alpha \beta \alpha \alpha \beta | \\
 \varphi_5^2 &= | \alpha \alpha \alpha \alpha \beta \alpha \beta \alpha | \\
 \varphi_6^2 &= | \alpha \alpha \alpha \alpha \beta \beta \alpha \alpha | \\
 \varphi_7^2 &= | \alpha \alpha \alpha \beta \alpha \alpha \alpha \beta | \\
 \varphi_8^2 &= | \alpha \alpha \alpha \beta \alpha \alpha \beta \alpha | \\
 \varphi_9^2 &= | \alpha \alpha \alpha \beta \alpha \beta \alpha \alpha | \\
 \varphi_{10}^2 &= | \alpha \alpha \alpha \beta \beta \alpha \alpha \alpha | \\
 \varphi_{11}^2 &= | \alpha \alpha \beta \alpha \alpha \alpha \alpha \beta | \\
 \varphi_{12}^2 &= | \alpha \alpha \beta \alpha \alpha \alpha \beta \alpha | \\
 \varphi_{13}^2 &= | \alpha \alpha \beta \alpha \alpha \beta \alpha \alpha | \\
 \varphi_{14}^2 &= | \alpha \alpha \beta \alpha \beta \alpha \alpha \alpha | \\
 \varphi_{15}^2 &= | \alpha \alpha \beta \beta \alpha \alpha \alpha \alpha | \\
 \varphi_{16}^2 &= | \alpha \beta \alpha \alpha \alpha \alpha \alpha \beta | \\
 \varphi_{17}^2 &= | \alpha \beta \alpha \alpha \alpha \alpha \beta \alpha | \\
 \varphi_{18}^2 &= | \alpha \beta \alpha \alpha \alpha \beta \alpha \alpha | \\
 \varphi_{19}^2 &= | \alpha \beta \alpha \alpha \beta \alpha \alpha \alpha | \\
 \varphi_{20}^2 &= | \alpha \beta \alpha \beta \alpha \alpha \alpha \alpha | \\
 \varphi_{21}^2 &= | \alpha \beta \beta \alpha \alpha \alpha \alpha \alpha | \\
 \varphi_{22}^2 &= | \beta \alpha \alpha \alpha \alpha \alpha \alpha \beta | \\
 \varphi_{23}^2 &= | \beta \alpha \alpha \alpha \alpha \alpha \beta \alpha | \\
 \varphi_{24}^2 &= | \beta \alpha \alpha \alpha \alpha \beta \alpha \alpha | \\
 \varphi_{25}^2 &= | \beta \alpha \alpha \alpha \beta \alpha \alpha \alpha | \\
 \varphi_{26}^2 &= | \beta \alpha \alpha \beta \alpha \alpha \alpha \alpha | \\
 \varphi_{27}^2 &= | \beta \alpha \beta \alpha \alpha \alpha \alpha \alpha | \\
 \varphi_{28}^2 &= | \beta \beta \alpha \alpha \alpha \alpha \alpha \alpha |
 \end{aligned}$$

Quintetteket az elágazási diagram szerint 20 főle-
képpen szerkeszthetünk :







A megfelelő sajátfüggvények a következők :

$$\begin{aligned} \varphi_{Q_1} = & \frac{\sqrt{35}}{210} \{ 30(\varphi_1^2) + \\ & + 2(\varphi_6^2 + \varphi_9^2 + \varphi_{10}^2 + \varphi_{13}^2 + \varphi_{14}^2 + \varphi_{15}^2 + \varphi_{18}^2 + \varphi_{19}^2 + \varphi_{20}^2 + \varphi_{21}^2 + \\ & + \varphi_{24}^2 + \varphi_{25}^2 + \varphi_{26}^2 + \varphi_{27}^2 + \varphi_{28}^2) - \\ & - 5(\varphi_2^2 + \varphi_3^2 + \varphi_4^2 + \varphi_5^2 + \varphi_7^2 + \varphi_8^2 + \varphi_{11}^2 + \varphi_{12}^2 + \varphi_{16}^2 + \varphi_{17}^2 + \varphi_{22}^2 + \varphi_{23}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_2} = & \frac{1}{30} \{ 25(\varphi_2^2) + \\ & + 2(\varphi_{10}^2 + \varphi_{14}^2 + \varphi_{15}^2 + \varphi_{19}^2 + \varphi_{20}^2 + \varphi_{21}^2 + \varphi_{25}^2 + \varphi_{26}^2 + \varphi_{27}^2 + \varphi_{28}^2) + \\ & + 1(\varphi_5^2 + \varphi_8^2 + \varphi_{12}^2 + \varphi_{17}^2 + \varphi_{23}^2) - \\ & - 5(\varphi_3^2 + \varphi_4^2 + \varphi_7^2 + \varphi_{11}^2 + \varphi_{16}^2 + \varphi_{22}^2) - \\ & - 4(\varphi_6^2 + \varphi_9^2 + \varphi_{13}^2 + \varphi_{18}^2 + \varphi_{24}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_3} = & \frac{\sqrt{6}}{30} \{ 10(\varphi_3^2) + \\ & + 1(\varphi_{10}^2 + \varphi_{14}^2 + \varphi_{15}^2 + \varphi_{19}^2 + \varphi_{25}^2 + \varphi_{20}^2 + \varphi_{21}^2 + \varphi_{26}^2 + \varphi_{27}^2 + \varphi_{28}^2) - \\ & - 2(\varphi_5^2 + \varphi_6^2 + \varphi_8^2 + \varphi_9^2 + \varphi_{12}^2 + \varphi_{13}^2 + \varphi_{17}^2 + \varphi_{18}^2 + \varphi_{23}^2 + \varphi_{24}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_4} = & \frac{\sqrt{6}}{60} \{ 20(\varphi_4^2) + \\ & + 2(\varphi_{15}^2 + \varphi_{20}^2 + \varphi_{21}^2 + \varphi_{26}^2 + \varphi_{27}^2 + \varphi_{28}^2) + \\ & + 1(\varphi_8^2 + \varphi_9^2 + \varphi_{12}^2 + \varphi_{13}^2 + \varphi_{17}^2 + \varphi_{18}^2 + \varphi_{23}^2 + \varphi_{24}^2) - \\ & - 5(\varphi_7^2 + \varphi_{11}^2 + \varphi_{16}^2 + \varphi_{22}^2) - \\ & - 4(\varphi_5^2 + \varphi_6^2) - \\ & - 3(\varphi_{10}^2 + \varphi_{14}^2 + \varphi_{19}^2 + \varphi_{25}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_5} = & \frac{1}{20} \{ 16(\varphi_5^2) + \\ & + 2(\varphi_{15}^2 + \varphi_{20}^2 + \varphi_{21}^2 + \varphi_{26}^2 + \varphi_{27}^2 + \varphi_{28}^2) + \\ & + 1(\varphi_9^2 + \varphi_{13}^2 + \varphi_{18}^2 + \varphi_{24}^2) - \\ & - 4(\varphi_6^2 + \varphi_8^2 + \varphi_{12}^2 + \varphi_{17}^2 + \varphi_{23}^2) - \\ & - 3(\varphi_{10}^2 + \varphi_{14}^2 + \varphi_{19}^2 + \varphi_{25}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_6} = & \frac{\sqrt{15}}{60} \{ 12(\varphi_6^2) + \\ & + 2(\varphi_{15}^2 + \varphi_{20}^2 + \varphi_{21}^2 + \varphi_{26}^2 + \varphi_{27}^2 + \varphi_{28}^2) - \\ & - 3(\varphi_9^2 + \varphi_{10}^2 + \varphi_{13}^2 + \varphi_{14}^2 + \varphi_{18}^2 + \varphi_{19}^2 + \varphi_{24}^2 + \varphi_{25}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_7} = & \frac{\sqrt{10}}{60} \{ 15(\varphi_7^2) + \\ & + 2(\varphi_{21}^2 + \varphi_{27}^2 + \varphi_{28}^2) + \\ & + 1(\varphi_{12}^2 + \varphi_{13}^2 + \varphi_{14}^2 + \varphi_{17}^2 + \varphi_{18}^2 + \varphi_{19}^2 + \varphi_{23}^2 + \varphi_{24}^2 + \varphi_{25}^2) - \\ & - 5(\varphi_{11}^2 + \varphi_{16}^2 + \varphi_{22}^2) - \\ & - 3(\varphi_8^2 + \varphi_9^2 + \varphi_{10}^2) \\ & - 2(\varphi_{15}^2 + \varphi_{20}^2 + \varphi_{26}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_8} = & \frac{\sqrt{15}}{60} \{ 12(\varphi_8^2) + \\ & + 2(\varphi_{21}^2 + \varphi_{27}^2 + \varphi_{28}^2) + \\ & + 1(\varphi_{13}^2 + \varphi_{14}^2 + \varphi_{18}^2 + \varphi_{19}^2 + \varphi_{24}^2 + \varphi_{25}^2) - \\ & - 4(\varphi_{12}^2 + \varphi_{17}^2 + \varphi_{23}^2) - \\ & - 3(\varphi_9^2 + \varphi_{10}^2) - \\ & - 2(\varphi_{15}^2 + \varphi_{20}^2 + \varphi_{26}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_9} = & \frac{1}{12} \{ 9(\varphi_9^2) + \\ & + 2(\varphi_{21}^2 + \varphi_{27}^2 + \varphi_{28}^2) + \\ & + 1(\varphi_{14}^2 + \varphi_{19}^2 + \varphi_{25}^2) - \\ & - 3(\varphi_{10}^2 + \varphi_{13}^2 + \varphi_{18}^2 + \varphi_{24}^2) - \\ & - 2(\varphi_{15}^2 + \varphi_{20}^2 + \varphi_{26}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_{10}} = & \frac{\sqrt{2}}{6} \{ 3(\varphi_{10}^2) + \\ & + 1(\varphi_{21}^2 + \varphi_{27}^2 + \varphi_{28}^2) - \\ & - 1(\varphi_{14}^2 + \varphi_{15}^2 + \varphi_{19}^2 + \varphi_{20}^2 + \varphi_{25}^2 + \varphi_{26}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_{11}} = & \frac{\sqrt{5}}{30} \{ 10(\varphi_{11}^2) + \\ & + 2(\varphi_{28}^2) + \end{aligned}$$

$$+ 1(\varphi_{17}^2 + \varphi_{18}^2 + \varphi_{19}^2 + \varphi_{20}^2 + \varphi_{23}^2 + \varphi_{24}^2 + \varphi_{25}^2 + \varphi_{26}^2) - \\ - 5(\varphi_{16}^2 + \varphi_{22}^2) - \\ - 2(\varphi_{12}^2 + \varphi_{13}^2 + \varphi_{14}^2 + \varphi_{15}^2) - \\ - 1(\varphi_{21}^2 + \varphi_{27}^2)\}$$

$$\varphi_{Q_{12}} = \frac{\sqrt{30}}{60} \{ 8(\varphi_{12}^2) + \\ + 2(\varphi_{28}^2) + \\ + 1(\varphi_{18}^2 + \varphi_{19}^2 + \varphi_{20}^2 + \varphi_{24}^2 + \varphi_{25}^2 + \varphi_{26}^2) - \\ - 4(\varphi_{17}^2 + \varphi_{23}^2) - \\ - 2(\varphi_{13}^2 + \varphi_{14}^2 + \varphi_{15}^2) - \\ - 1(\varphi_{21}^2 + \varphi_{27}^2)\}$$

$$\varphi_{Q_{13}} = \frac{\sqrt{2}}{12} \{ 6(\varphi_{13}^2) + \\ + 2(\varphi_{28}^2) + \\ + 1(\varphi_{19}^2 + \varphi_{20}^2 + \varphi_{25}^2 + \varphi_{26}^2) - \\ - 3(\varphi_{18}^2 + \varphi_{24}^2) - \\ - 2(\varphi_{14}^2 + \varphi_{15}^2) - \\ - 1(\varphi_{21}^2 + \varphi_{27}^2)\}$$

$$\varphi_{Q_{14}} = \frac{1}{6} \{ 4(\varphi_{14}^2) + \\ + 2(\varphi_{28}^2) + \\ + 1(\varphi_{20}^2 + \varphi_{26}^2) - \\ - 2(\varphi_{15}^2 + \varphi_{19}^2 + \varphi_{25}^2) - \\ - 1(\varphi_{21}^2 + \varphi_{27}^2)\}$$

$$\varphi_{Q_{15}} = \frac{\sqrt{3}}{6} \{ 2(\varphi_{15}^2 + \varphi_{28}^2) - \\ - 1(\varphi_{20}^2 + \varphi_{21}^2 + \varphi_{26}^2 + \varphi_{27}^2)\}$$

$$\begin{aligned} \varphi_{Q_{16}} = & \frac{\sqrt{15}}{30} \{ 5(\varphi_{16}^2) + \\ & + 1(\varphi_{23}^2 + \varphi_{24}^2 + \varphi_{25}^2 + \varphi_{26}^2 + \varphi_{27}^2) - \\ & - 5(\varphi_{22}^2) - \\ & - 1(\varphi_{17}^2 + \varphi_{18}^2 + \varphi_{19}^2 + \varphi_{20}^2 + \varphi_{21}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_{17}} = & \frac{\sqrt{10}}{20} \{ 4(\varphi_{17}^2) + \\ & + 1(\varphi_{24}^2 + \varphi_{25}^2 + \varphi_{26}^2 + \varphi_{27}^2) - \\ & - 4(\varphi_{23}^2) - \\ & - 1(\varphi_{18}^2 + \varphi_{19}^2 + \varphi_{20}^2 + \varphi_{21}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_{18}} = & \frac{\sqrt{6}}{12} \{ 3(\varphi_{18}^2) + \\ & + 1(\varphi_{25}^2 + \varphi_{26}^2 + \varphi_{27}^2) - \\ & - 3(\varphi_{24}^2) - \\ & - 1(\varphi_{19}^2 + \varphi_{20}^2 + \varphi_{21}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_{19}} = & \frac{\sqrt{3}}{6} \{ 2(\varphi_{19}^2) + \\ & + 1(\varphi_{26}^2 + \varphi_{27}^2) - 2(\varphi_{25}^2) - \\ & - 1(\varphi_{20}^2 + \varphi_{21}^2) \} \end{aligned}$$

$$\begin{aligned} \varphi_{Q_{20}} = & \frac{1}{2} \{ 1(\varphi_{20}^2 + \varphi_{27}^2) - \\ & - 1(\varphi_{21}^2 + \varphi_{26}^2) \} \end{aligned}$$

D./ Szeptettek

A 4 eredő spinhez tartozó SLATER-determinánsokat a következőképpen jelöljük :

$$\varphi_1^3 = |\alpha\alpha\alpha\alpha\alpha\alpha\alpha\beta|$$

$$\varphi_2^3 = |\alpha\alpha\alpha\alpha\alpha\alpha\beta\alpha|$$

$$\varphi_3^3 = |\alpha\alpha\alpha\alpha\alpha\beta\alpha\alpha|$$

$$\varphi_4^3 = |\alpha\alpha\alpha\alpha\beta\alpha\alpha\alpha|$$

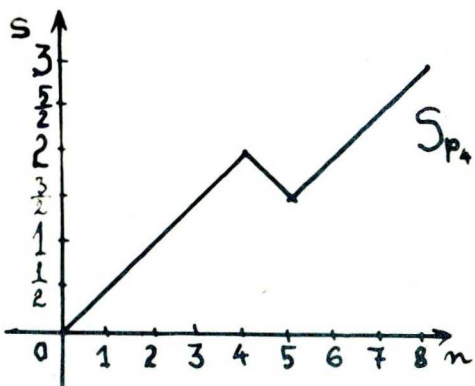
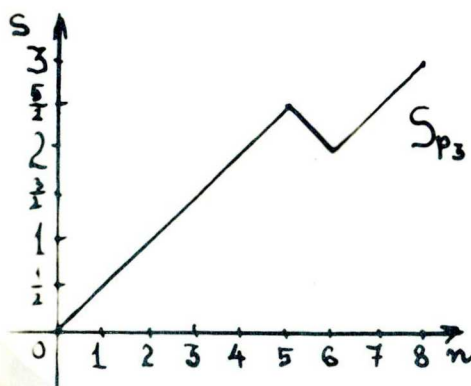
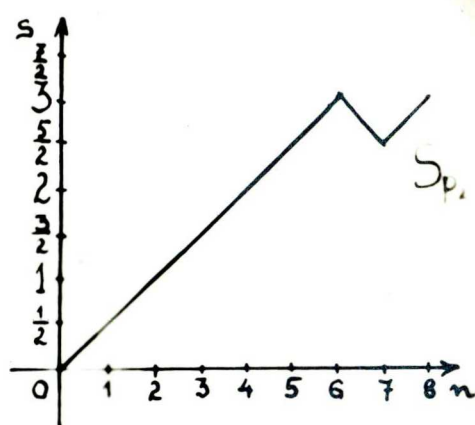
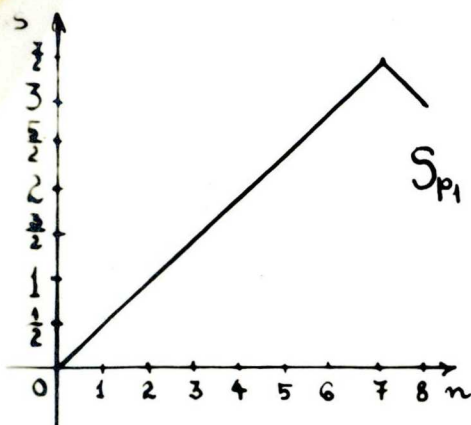
$$\varphi_5^3 = |\alpha\alpha\alpha\beta\alpha\alpha\alpha\alpha|$$

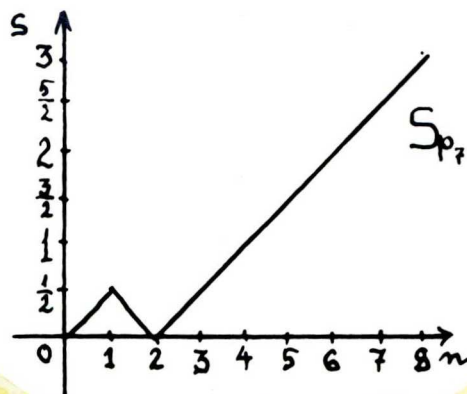
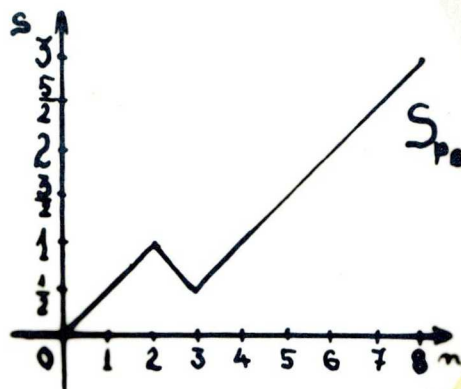
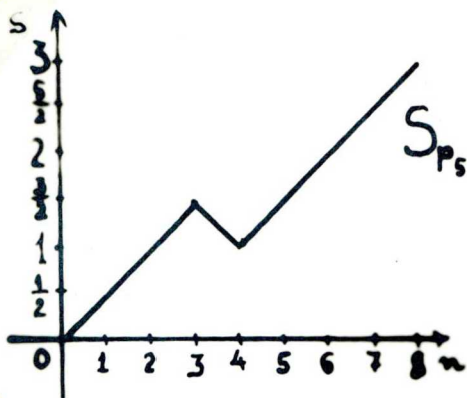
$$\varphi_6^3 = |\alpha\alpha\beta\alpha\alpha\alpha\alpha\alpha|$$

$$\varphi_7^3 = |\alpha\beta\alpha\alpha\alpha\alpha\alpha\alpha|$$

$$\varphi_8^3 = |\beta\alpha\alpha\alpha\alpha\alpha\alpha\alpha|$$

Szeptetteket az előgazási diagram szerint hétfelé-
leképpen szerkeszthetünk :





A megfelelő sajátfüggvények a következők :

$$\varphi_{S_{p_1}} = \frac{\sqrt{14}}{28} \{ 7(\varphi_1^3) - \\ - 1(\varphi_2^3 + \varphi_3^3 + \varphi_4^3 + \varphi_5^3 + \varphi_6^3 + \varphi_7^3 + \varphi_8^3) \}$$

$$\varphi_{S_{p_2}} = \frac{\sqrt{42}}{42} \{ 6(\varphi_2^3) - \\ - 1(\varphi_3^3 + \varphi_4^3 + \varphi_5^3 + \varphi_6^3 + \varphi_7^3 + \varphi_8^3) \}$$

$$\varphi_{Sp3} = \frac{\sqrt{30}}{30} \{ 5 (\varphi_3^3) - \\ - 1 (\varphi_4^3 + \varphi_5^3 + \varphi_6^3 + \varphi_7^3 + \varphi_8^3) \}$$

$$\varphi_{Sp4} = \frac{\sqrt{5}}{10} \{ 4 (\varphi_4^3) - \\ - 1 (\varphi_5^3 + \varphi_6^3 + \varphi_7^3 + \varphi_8^3) \}$$

$$\varphi_{Sp5} = \frac{\sqrt{3}}{6} \{ 3 (\varphi_5^3) - \\ - 1 (\varphi_6^3 + \varphi_7^3 + \varphi_8^3) \}$$

$$\varphi_{Sp6} = \frac{\sqrt{6}}{6} \{ 2 (\varphi_6^3) - \\ - 1 (\varphi_7^3 + \varphi_8^3) \}$$

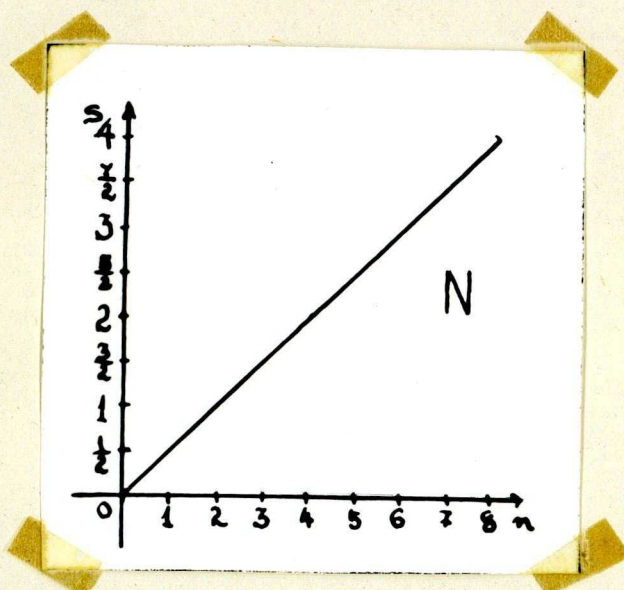
$$\varphi_{Sp7} = \frac{\sqrt{2}}{2} \{ \varphi_7^3 - \varphi_8^3 \}$$

B./ Nonett

Az 5 eredő spinhez tartozó SLATER-determináns a következőképpen jelöljük :

$$\varphi_1^4 = |\alpha\alpha\alpha\alpha\alpha\alpha\alpha|$$

Nonettet az elágazási diagram szerint egyféleképpen szerkeszthetünk :



A megfelelő sajátfüggvény a következő :

$$\varphi_N = \varphi_1^4$$

Irodalom

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