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# Application of numerical methods in modern astrophysical analysis

PhD thesis statements

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## Scientific background

In our time, scientific measurements are becoming increasingly data intensive, and computers have become indispensable for their management. These computers become the everyday devices in our lives, and serious calculations can also be done by personal computers. With this, the numerical methods become readily available. Complex problems can almost only be modeled numerically, so these problems can be handled with finite precision.

One of the most important methods is the so-called inversion, where a modelling method that produces (e.g., light curve) a model from parameter inputs is inverted using an algorithm, where the parameters and their errors and correlations are inferred from the fit of the model to the data. However, several factors, such as invertibility, convergence, run time, and plausibility of the model have to be taken into account. I applied the inversion method to two complex problems within astronomy.

One application is the lightcurve modelling of supernova explosions, where from modelling the lightcurves, the physical parameters of the exploding star, such as ejected mass, radius and energies can be determined. To this day, there is considerable research into the physics of supernova explosions, because it allows us to study extreme physical phenomena. There are countless classes and sub-classes of explosions, which is due to the final physical state of stars. One such subclass is the low luminous (LL, Pastorello et al. 2004; Spiro et al. 2014) supernova, which, because of its low luminosity, has not been well studied and therefore is still largely unknown.

The other application of the inversion is to model the measured fluxes of exomoons (moons of planets orbiting other stars), and thus determining their chances of detection. Although many exoplanets are known, none of exomoon are known (only canidates). The main reason is that the planet it orbits and the star of the system are much brighter than the moon itself, so the exomoon gets lost in the background. For this reason, it makes sense to look for an exomoon with a strong intrinsic luminosity. This is possible with tidal heating, similar in the case of Io.

## Research topics

For the inversion I have written a Markov Chain Monte Carlo (MCMC, Metropolis et al. 1953; Hastings 1970; Gilks et al. 1996) environment that is well suited for sampling physical parameter spaces for inverse problems.

During the supernova modelling 2 Hungarian (SZTE Baja Observatory and Piszkéstető Konkoly Observatory) instruments and my own measurements used to produce the low luminosity (LL) PSN J17292918+7542390 supernova BVRI light curves in an IRAF (Image Reduction and Analysis Facility[1]) environment. During the process, the g'r'i'z' filters used at Baja, were converted to BVRI based on Jordi et al. ( 2006) formulas. The BVRI light curves were converted to bolometric light

[1] http://iraf.noao.edu/

curves using trapezoidal-mode and diluted blackbody radiation (Eastman et al. 1996; Hamuy et al. 2001 fitting with proprietary code. From the fit, the radius and temperature of the SN photosphere as a function of time can be determined. The single spectra of the of PSN J17292918+7542390 (Tomasella et al. 2015) were fitted with the Syn++ (Fisher et al. 1997, Thomas et al. 2011) algorithm, which fits the P Cygni profiles with the Sobolev approximation (Sobolev 1960).

I have improved the accuracy and the runtime of the semi-analytical supernova (SN) light curve modelling code (Arnett ( 1980, 1982), Arnett & Fu 1989; Popov 1993) developed by my colleagues (Nagy et al. 2014, Nagy & Vinkó 2016) using more advanced numerical algorithms, preparing it for MCMC sampling. This allowed me to successfully determine the physical parameters of PSN J17292918+7542390 supernovae and their uncertainties.

During the exomoon flux modelling, the fixed Q and viscoelastic models (Reynolds et al. 1987, Segatz et al. 1988, Fischer & Spohn 1990, Moore 2003, Meyer & Wisdom 2007, Henning et al. 2009, Dobos & Turner 2015) has been improved with non-homogeneous tidal heating, taking into account the internal convection. The time scale of energy transport during convection is much smaller than for conduction, and the ratio of the two is described by the so-called Nusselt number. The Nusselt number depends largely on the internal structure of the moon, but its value is in the order of magnitude of 60 (Pavel et al. 2014). As a consequence, heat from tidal heating comes out on so-called hot spots, as in the case of Io. I integrated this model into the MCMC code. This was used to investigate if it is possible, and if so, which physical parameters are important during the detection of exomoons with current technology.

## Theses

1. With inverting and optimizing well-proven models, I have developed and fine-tuned the MCMC algorithm that can fit synthetic SN and tidally heated models to a to a given lightcurve and to map the parameter space, thus determining the parameter values and their uncertainties. **(T1, T2)**

2. I have made my own measurements of PSN J17292918+7542390 (SN-NGC6412 for short) with the 50 cm diameter optical telescope of the Baja Observatory. I have made BVRI, griz and bolometric lightcurves of the SN. In the process, I determined the photospheric radii and temperatures of the expanding remnant as a function of time. The temperature is similar for all SN, but the photospheric radius is much smaller (~3) for the LL subclass compared to the conventional II-P. From the SN spectra, I have determined the photospheric velocity in the early phase, which is very low 7000 km $s^{-1}$, I also determined the early chemical composition of the SN atmosphere. The atmosphere has strong He II lines, but also weak H, He I and N III lines. **(T1)**

3. Based on my results, the SN is a typical representative of the low luminosity (LL) II-P SN class

with low synthesized $^{56}$Ni mass. I compared it with 6 other II-P SNs in the literature, with the closest similarity being to SN 2005cs. Main parameters in order for the parent (progenitor) star radius, ejected mass, kinetic energy and expansion rate: $R_0 = 91^{+119}_{-70} \cdot 10^{11}$ cm, $M_{\rm ej} = 9.89^{+2.10}_{-1.00}\ M_\odot$, $E_{\rm kin} = 0.65^{+0.19}_{-0.18}$ foe, $v_{\rm exp} = 3332^{+216}_{-347}$ km s$^{-1}$, The initial mass of $^{56}$Ni is $1.55^{+0.75}_{-0.70} \cdot 10^{-3} M_\odot$. **(T1)**

4. I have added a formalism to models describing the tidal heating, that takes the Nusselt number into account to deal with the conventional energy spread, which depends on the complex internal structure of the exomoon. This leads to one or more hot spots at the surface, with temperatures reaching thousands of Kelvins, while the rest of the surface will be balanced by the irradiance energy received from the star. The hot spot has a significant effect on the detection, with bolometric energy output being given a factor of 2.5 to 4, and significantly shifts the maximum of the energy distribution, which can make detection easier or more difficult depending on the size of the hot spot. I integrated and optimized the method for my MCMC modeler, which allows the mapping of parameters from synthetic observations. **(T2)**

5. My results suggest that the detection of exomoons is possible under lucky but realistic conditions. Super-Earths orbiting Jupiter with a 1-day orbital period and $e = 0.1$ eccentricity yield a signal-to-per-noise ratio of $\approx$100 ppm, which can be detected by the ARIEL and the James Webb space telescopes in the near future. Other exomoons with similar orbital elements far from the star ($>$1 AU) are brighter than the planets they orbit. The exomoon's own luminosity reaches 21 AB and 24 bolometric magnitude for super-Earths orbiting Jupiter. An Earth-sized moon yields 23 AB and 26 bolometric magnitude, while a moon the size of Ganymede gives 25 AB and 28 bolometric magnitudes. This suggests the possibility of discovering systems well separated from their stars and starless, so-called rogue planets via their moons. The modelling showed that the flux is not monotonically dependent on the eccentricity. The dependence of flux on eccentricity has a maximum at $e = 0.21$. This is due to the dynamical constraint that the moon must avoid colliding with the planet, or fragmentation due to the Roche limit. **(T2)**

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## Publications

**Publications associated with the thesis**

Refereed papers:

- **Jäger Z. Jr.**, Vinkó J., Bíró I. B., Hegedüs T., Borkovits T., Jäger Z. Sr., Nagy A. P., Molnár L., Kriskovics L.: A low-luminosity core-collapse supernova very similar to SN 2005cs, 2020, Monthly Notices of the Royal Astronomical Society, 496, 3725J **(T1)**
- **Jäger Z.**, Szabó Gy. M.: Enhanced thermal radiation from a tidally heated exomoon with a single hotspot, 2021, Monthly Notices of the Royal Astronomical Society, 508, 5524J **(T2)**

**Other matters related to the topic of the thesis**

Conference talks:

- **Jäger Z.** Inferring physical parameters of supernova explosions via light curve modeling, Connections in astronomy, astrophysics, space and planetary sciences, International Conference – Cluj Academic Days 2017, 29-30 May 2017, Cluj-Napoca/Kolozsvár, Romania
- **Jäger Z.** Inferring physical parameters of supernova explosions via light curve modeling, Conference about successes of stellar astronomy – Bezovec 2017, 2-4 June 2017, Hlohovec, Szlovakia
- **Jäger Z.** Inferring physical parameters of supernova explosions via light curve modeling, International meeting on variable stars research – KOLOS 2017, 7-9 December 2017, Stakcin, Szlovakia
- **Jäger Z.** Inferring physical parameters of supernova explosions via light curve modeling, Outlook in astronomy, astrophysics, space and planetary sciences, International Conference – Cluj Academic Days 2018, 17-19 May 2018, Cluj-Napoca/Kolozsvár, Romania
- **Jäger Z.** A numerical optimization code for supernova light curves, International meeting on variable stars research – KOLOS 2018, 6-8 December 2018, Stakcin, Szlovakia
- **Jäger Z.** A numerical optimization code for supernova light curves, Joint Conference of the Sub-Regional European Astronomical Committee and the Bulgarian Astronomical Society, 4-8 June 2019, Sofia, Bulgaria
- **Jäger Z.** A numerical optimization code for supernova light curves, International meeting on variable stars research – KOLOS 2019, 5-7 December 2019, Stakcin, Szlovakia

Seminar talk:

- **Jäger Z.**, Szabó Gy. M., Enhanced thermal radiation from a tidally heated exomoon with a single hot spot, Invited lecturer at Deutsches Zentrum für Luft- und Raumfahrt (German Aerospace Center), Institut für Planetenforschung (Institute of Planetary Research), Abteilung von Extrasolar Planeten und Atmospheren (Department of Extrasolar Planets and Atmospheres), 12 November 2021, Berlin, Germany

Conference publication:

- **Jäger Z.**, Nagy A. P., Bíró I. B., Vinkó J., A new supernova lightcurve modelling program, 2017, Romanian Astronomical Journal, 27, 203J