Summary of dissertation

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STUDY OF STRATEGY USE IN MENTAL COUNTING IN THE AGE OF 10-12

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INTRODUCTION

As a practicing educators we can observe in many cases that even graduating students often make major errors in estimating the outcome of a basic operation when working without a calculator. The deteriorating Hungarian results of the international PISA mathematical measurements (cf. OECD, 2019) also draw attention to the need to develop numeracy skills. One application of mathematics in practical life and school is fast, accurate mental arithmetic. Therefore, it is important for children to learn to use mental arithmetic strategies so that they can acquire knowledge that can be applied to other areas of life. The application and creation of different strategies is also useful because in the meantime the students' metacognitive, problem-solving and problem-creating skills develop (Csíkos, 2017).

In the six studies presented in the dissertation, we aimed to create a paper-and-pencil-based measuring device. With the help of the Multiplication Strategies Test, the research area that has not been studied among Hungarian students so far can be mapped, the use of multiplication strategies in mind during calculation. The test was developed to examine the development of the numeracy skills of 10-12 year old students. The test can be used to filter out students' counting errors in the head during multiplication, and to further develop students' counting skills based on error patterns (Vígh-Kiss, 2016a, 2016b, 2017a, 2017c). In dissertation we describe the process of test development, the statistical analysis of the test. We present the results of the measurement system consisting of six tests and the development experiment, the strategies used by the examined students, rational errors.

THEORETICAL BACKGROUND OF RESEARCH

A significant proportion of students are unable to apply the concepts they have learned in solving complex tasks (Csapó, 2006). This is also the case with elementary numeracy skills, if they are underdeveloped, solving textual problems is difficult (Vígh-Kiss, 2015a). Five factors play a key role in the development of mathematical profiency (Kilpatrick, Swafford, & Findell, 2001): (1) conceptual understanding, (2) procedural ease, (3) strategic competencies, (4) adaptive thinking and (5) result orientation. In order to develop applicable knowledge, we first need to develop the skills that make up the competence (Vidákovich, 2013).

The concept, models, and examination of strategy use

Students make many mistakes in longer written calculations (e.g., multiplication, division). This is because they forget, misunderstand computational algorithms. The same phenomenon can be observed during head counts (Vígh-Kiss, Csíkos & Steklács, 2013, 2019). There are individual differences between students' arithmetic abilities (Dowker, 2005). It would be expedient to eliminate and reduce this difference (Vígh-Kiss, 2016b). The teaching of mental arithmetic also plays a role in the development of students' memory (Szendrei, 2005). Teaching different strategies is also important during talent development (Vígh-Kiss, 2014a, 2014b, 2014c).

Studies related to strategic research have been present in international educational research for almost 40 years. A strategy is an action or sequence of actions used to achieve some higher goal (Lemaire, & Reder, 1999). The use of strategy can be observed in all areas of life: human cognition, scientific reasoning (Kuhn, Schäuble, & Garcia-Milla, 1992), decision making

(Payne, Bettman, & Johnson, 1988), telling time (Siegle, & McGilly, 1989), currency exchange (Lemaire, & Lecacheur, 2001); appears in many areas of education and learning: mathematics (Csíkos, 2013, 2016, 2017; Csíkos, Szitányi, & Kelemen 2012), reading (Molitorisz, 2012; Steklács, 2013), language learning, word learning, serial memory (Siegler, & Jenkins, 1989), spelling (Rittle-Johnson, Siegler, & Alibali, 2001), sheet music reading (Buzás, 2016). Strategy use and metacognition are closely related concepts.

The flexibility or adaptability of a strategy can be defined in many ways. Some researchers (e.g., Thompson, 1999; Blöte, Van der Burg, & Klein, 2001) consider combinations of strategies and task types to be flexible or inflexible depending on how well they fit together. However, according to Verschaffel, Torbeyns, Luwel, Van Dooren, and De Smedt (2007), the effectiveness of choosing between different strategies is also an important issue. The strategy to suit the characteristics of the task may vary from individual to individual and from task to task. Siegler and Shipley (1995) ASCM (Adaptive Strategy Choice Model) shows how we select the most appropriate strategy to solve a given problem and how strategy choice develops. In children, the application of certain strategies during development becomes automatic: they solve certain problems faster and with less attention.

The strategy appeared decades ago in the reform curricula of several countries, including the United Kingdom (DfEE, 1999), the Netherlands (Treffers, De Moor and Feijs, 1990) and the United States (NCTM, 1989), and Australia (Australian Education Council, 1991). improving its flexibility. Thus, teachers place more emphasis on teaching different solution strategies. Adaptive strategy use can be developed. However, there is still few research (e.g., Barrrody, 1999; Carr, Alexander & Folds-Bennett, 1994; Geary, 2003; Verschaffel, Greer, & De Corte, 2007; Jacob, & Mulligan, 2014) reports on the effects of these developmental experiments.

In a study of multiplications in the head, Cooper, Heirdsfield, Mulligan, and Irons (1999) observed five strategies in lower-grade children. There is no Hungarian equivalent of the names, as no research has been conducted in Hungary before. We give the Hungarian names we suggest (Vígh-Kiss, Csíkos, & Steklács, 2013, 2019; Vígh-Kiss, 2014d, 2020a):

• Counting (CO) or counting strategy (számlálás stratégia)

• Basic fact (BF) strategy (tényeken alapuló stratégia)

• RL separated (RLS) (helyiérték szerinti jobbról balra stratégia)

• LR separated (LRS) (helyiérték szerinti balról jobbra stratégia)

• Wholistic (WH) strategy (holisztikus stratégia).

Hope and Sherrill (1987) conducted a paper-and-pencil-based study among 11th- and 12th-grade students (N = 286) and recorded oral interviews. In his studies on the use of multiplication strategies, four solution methods were identified, including 12 strategies.

Mistakes made in mental calculation during multiplication

Examining mathematical errors can help you learn about mental representations related to mathematics. In dissertation, we analyze the strategies used during multiplication in the head, the mistakes made during the calculation, and we examine the students' erroneous thinking.

Ben-Zeev (1998) believes that we find fundamental mental processes behind innumerable different errors. The proper development of the new rule may be hampered by some partial knowledge due to shortcomings in the monitoring mechanism; analogous thinking can result in a rational error; syntactic induction is also a rich source of error; as well as over-generalization (Ben-Zeev, 1998). Student errors should be seen as an opportunity for learning (cf. Althoff, 1998; Szendrei, 2005). Debugging, examining the origins of student errors, and discussing them with students can contribute to a deeper understanding of mathematics.

Using computer simulations, Lemaire and Siegler (1995) observed that, with a few exceptions, children's mental arithmetic errors observed in the early stages of learning are caused by erroneous retrieval of knowledge about addition. Error patterns can be easily identified because errors are consistently dropped by learners because of their relatively inflexible estimation strategies (VanLehn, 1990). In the case of beginners, several researchers, e.g. Siegler (1988) observed that existing knowledge sometimes has a blocking effect on learning. Since the multiplication can be related to the addition, the addition here can result in two kinds of type errors (1) related-operation errors, e.g. $8 \cdot 3 = 11$, the sum of the two numbers is the result, (2) addition-like odd-even errors, e.g. $5 \cdot 3 = 12$, $4 \cdot 6 = 20$, $8 \cdot 3 = 25$. In learning, children build an increasingly complex knowledge network about operations, applying calculation strategies more and more accurately and flexibly. Students are also likely to use a number of uncharted strategies (LeFevre, Smith-Chant, Hischock, Daley, & Morris, 2013). Their research has not been completed since.

RESEARCH QUESTIONS

Based on the literature and more than 25 years of teaching experience, we formulated the following questions:

1. How and by what means is it expedient to measure the flexibility of strategy use in relation to multiplication in the head in a pedagogically relevant way?

2. What strategy do students aged 10-12 use in solving each multiplication problem in the mind count?

3. What background variables can be associated with the strategy used for multiplication in the head?

4. In head multiplication, what are the most commonly used strategies in each grade?

5. How does the number of multiplication strategies used by students change in each grade?

6. What characterizes the use of strategy by children who perform better on the Mathematics Knowledge Level Test? Is their use of strategy adaptive in all cases? How many strategies are used?

7. What characterizes people with learning difficulties, respectively the use of strategies by children with special educational needs?

8. To what extent is the level of mathematical knowledge of children related to the use of adaptive strategies?

9. What mathematical problems can help develop adaptive strategy use?

EMPIRICAL INVESTIGATIONS

Samples of research

The schools participating in the study were randomly selected and contacted in person. Between 2013 and 2019, we conducted research among 10–18-year-old students. Five of the six studies we graduated among 4-6. grade students. The first study was conducted in 2013 among fourth graders (N = 13). The second study in 2014 was 8-12. The third study was conducted in 2015 among seventh graders (N = 61), and the fourth study was conducted in 2016 among sixth graders (N = 154).

In 2016, during the fifth study (N = 270), we performed a one-month development. Sixth-year students of two eight-year grammar schools in Budapest took part in the experiment. Development took place in one grammar school, the control group in the other grammar school class. Content-embedded development took place within class frameworks, in math classes, 20 times, in the second half of the math class (Vígh-Kiss, 2016c).

Most information was collected from 4th, 5th, and 6th grade students in 2019 in the central study (N = 850).

Research methods and tools

We conducted first study with the help of Tobii T120 eyetracker, recorded an oral interview, and made video recordings of the students' eye movements. The results of TobiiStudio 2.2.7. evaluated using software. In the further investigations carried out between 2013 and 2019, we mainly used self-developed paper-and-pencil-based measuring instruments. The Multiplication Strategies Test contains 40 items, two-digit number by a two-digit number in 28 items. In first studies, we first coded the strategies used by students based on the strategies described by Cooper, Heirdsfield, Mulligan, and Irons (1999). We refined this system, classifying the strategies used by the students into 35 different strategies.

Many factors have an effect on school performance and the creation of knowledge, such as attitude towards school, attitude towards subjects, social conditions, mother's and father's educational level (B. Németh, 2002, 2003; Csapó, 2002a, 2002b). Therefore, we also included own Mathematics Knowledge Level Test and Background Questionnaire for studies. During the development embedded in the one-month content, we recorded pre-measurement, post-measurement, and then three months later, during the delayed post-measurement, we interviewed the students.

We have adhered to the ethical standards applicable to educational research throughout the planning, implementation and analysis of results. The students and parents were informed about the measurements by the surveyors. The anonymity of the students was ensured during the data processing. The schools participating in the study were informed about the measurement results.

RESULTS OF EMPIRICAL TESTS

The results of the measurement system consisting of six studies are summarized along hypotheses. The hypotheses were largely confirmed.

Hypotheses about the reliability of measuring instruments:

H1a: The Multiplication Strategies Test reliably measures the strategy use of students in each grade in solving multiplication tasks in the head. In the first study, the measured reliability was acceptable (Cronbach-$\alpha = 0.68$). In the five paper-and-pencil-based studies, students who also described the partial results during the head count were excluded from the study to ensure validity. In the second study, the reliability of the measuring device was 0.88, in six groups it was between 0.53 and 0.78, therefore further test development was needed. In the other four studies, this hypothesis was confirmed, with a reliability of around 0.90.

H1b: The Math Proficiency Testing Strategies Test reliably measures students' math proficiency. This hypothesis was confirmed, with a reliability of at least 0.87 from the third study.

Hypotheses about strategy use:

H2a: In multiplication problems that can be solved by mental arithmetic, 10–18-year-olds use at least five different strategies (cf. Hope & Sherrill, 1987; Heirdsfield, Cooper, Mulligan & Irons, 1999). This hypothesis was confirmed by all studies.

Students examined in the central measurement used 21 strategies that led to correct results. Strategies observed: "Written multiplication in the head", Multiply each detail by digits (P and P0, Hope & Sherrill), Additive distribution (Hope and Sherrill), Multiply by two digits starts with right (right to left separated strategy, Heirdsfield et al.), starts with multiples by tens (left to right separated strategy, Heirdsfield et al.), Divides one factor into adders, Divides both factors into adders, fractional distribution, (Hope & Sherrill), Counting (Counting strategy, Heirdsfield et al.), Subtractive distribution (Hope & Sherrill); wholistic strategy (Heirdsfield et al.), Quadratic distribution (Hope and Sherrill), General factoring, general factoring, Hope and Sherrill, Halving-doubling: halving one factor and doubling the other d-double, Hope & Sherrill), Halving-doubling, by subtraction, Aliquot, Hope & Sherrill, one-factor transformation, Applying a "known rule", Applying "Algebraic identity", sum square, "Algebraic transformation", Retrieval of a numerical equivalent, Hope and Sherrill, Basic fact strategy Heirdsfield et al., Exponential factoring (Hope & Sherrill).

H2e: Rational errors can be observed among the students studied (cf. Ben-Zeev, 1998). This hypothesis was confirmed by all studies. In the first study, e.g. students made a rational error in 23% of the items (Vígh-Kiss, Csíkos & Steklács, 2019). Among the strategies that lead to an incorrect result, we can find one that may have been created by the erroneous analogy of the stepwise strategy found among the addition strategies. Here, the student multiplied the tens by the tens and the ones by the ones and then added the two products, which is a rational error (cf. Ben-Zeev, 1998). Some students multiplied the multiplier by the decimal place of the second multiplier, and then multiplied the multiplier or the multiplied by the multiplicative of the digits in the multiplier by the actual values of the digits in the multiplier. Other studies have expanded this experience.

In the other five studies, we observed more strategies leading to erroneous results. In the central study, 14.2% of the items were calculated by the students as follows: "Tens with tens and ones with ones" were multiplied by the addition pattern, and then these two installments were added together, which is a rational error (cf. Ben-Zeev, 1998). This affects 37.3 percent of students. For twenty items, one in five students counted on this strategy. For 8% of the items, students used other strategies that led to incorrect results; other errors were made by 49.3 percent of students. In 5.3% of cases, their calculation was not justified. We saw a reference to written counting in 4.6% of the items. For each item, at least 12.6% of the students left the task unanswered, the highest rate was for item 38 (22.4%), and in previous studies the calculation of 77 · 99 was the most difficult. Overall, the relative frequency of items left blank is 17.5%. Students calculated 50.1% of the items with a strategy leading to a correct result, but made a mistake in a third of the cases.

H4: In solving multiplication problems, students most often use the following strategies: counting, fact-based, place-based (from left to right, and from right to left), and wholistic strategy (cf. Hope & Sherrill, 1987). This hypothesis was confirmed by all studies. In first study, e.g. students used the evidence-based strategy for 27.9% of the items, the left-to-right strategy for 24.04%, and the right-to-left strategy for the local value for 6.73%. Hungarian students with lower abilities also use the "imagining in my head" strategy observed by Hope and Sherrill (1987), which was reported by 3% of students. 5% of students used a wholistic strategy. In addition, the use of individual strategies was also observed. students also made many calculation errors when applying strategies that led to the right result.

In the central, sixth study, on average, 50.1% of students solved the 40 items with a strategy leading to a correct result, but 30.1% made mistakes in the calculation. Of the strategies that lead to the correct result, the strategy "For two-digit numbers, I multiply one number first by the number of the other two-digit number in the local value of tens, and then by the number in place of each one" the relative frequency of use ranges from 5.9% to 49.4%, with more than 30% in 23 cases. Other strategies are used in much smaller proportions and cases.

Hypotheses on the impact of development:

In fifth study, these hypotheses were confirmed.

H9a: Students who participate in the explicit teaching of multiplication strategies perform better on the Multiplication Strategies post-test than their non-development peers (cf. Mulligan & Mitchelmore, 2009). The mean solution of 34.11% points (experimental group) and 34.86% points (control group) of the pre-test of Multiplication Strategies indicates that the test seemed too difficult for the sample, the standard deviations are also relatively large (28.84 and 16, 82). The resolution of the post-tests was 50.33 and 45.45 in the control group, and the standard deviations were relatively high (28.88 and 33.23). The control group had a higher relative standard deviation: 73.11 on the multiplication strategies test compared to the 57.38 relative standard deviation of the experimental group. The control group was selected so that it did not deviate significantly from the mean of the experimental group. Due to the development, there was a significant difference between the performance of the two groups. The development affected all students, but it had a better effect on students with better results, and the difference in student performance remained.

H9b: Students who participated in the development achieve better results in the Mathematics Knowledge Level post-test than their peers who did not participate in the development (cf. Csíkos, 2007). In the pre-test, the experimental group scored an average of 52.01% points and the control group 54.03% points. In the experimental group the average in the post-test was 72.65% points (standard deviation 32.38), in the control group 51.89% points (standard deviation 39.71).

H9c: The effect of development can also be detected during the delayed post-test (Cf. Csíkos, 2007).

Hypothesis of explicitly teaching multiplication strategies has been confirmed. The formula described by Keppel (1991) is used to calculate the experimental effect (quoted by Csíkos, 2007). Based on the differences found for the post-tests, the experimental effect for the multiplication strategies test is 10.73, i.e. 10.73% of the performance differences observed at the end of the experiment. In the case of the mathematics test, the calculated $\omega^2 = 12.03$, which values also indicate a medium effect size. The experiment explains 12.03% of the student performance difference measured at the end of development. Developmental experiment showed a moderate impact, and based on interviews conducted three months later, we believe that even a one-month, content-embedded development has a positive effect on the effectiveness of multiplication in the mind by showing the advantages and disadvantages of strategies and using metacognitive strategies (Vígh-Kiss, & Buzás, 2017).

Relationships of tests with background variables:

During measurements, we found a number of significant correlations.

H2b: The level of development of children differs in the flexibility of strategy use, there is a significant difference between the strategy use of individual children - mathematically gifted children, majority students with SEN. (cf. Hope, & Sherrill, 1987; Heirdsfield, Cooper, Mulligan, & Irons, 1999) In first study, students with poorer (average) academic performance in mathematics used a counting strategy; while those with a grade of 4, 5 have a wholistic strategy, 4.8% of items. Effectiveness of the applied strategies: counting 42.9%, facts 86.2%, right-to-left strategy according to place value 42.9%, left-to-right strategy according to place value 44%, wholistic strategy 60%.

In second study, students with weaker mathematics (sufficient, moderate) more often resorted to the enumeration strategy in upper grades, with 5% of students surveyed reporting this. For many of the students with excellent results in mathematics, multiplying even a two-digit number by a two-digit number appeared to be a known fact, 10% of the students reported using this strategy, they were all eighth-graders. In 47.5% of cases, students used a left-to-right strategy for multiplying two-digit numbers by place value. The right-to-left local value strategy was used by 2.5% of students. Hungarian students with lower abilities also use the “imagining in my head” strategy observed by Hope and Sherrill (1987), which was reported by 3% of students. 5% of students used a wholistic strategy. In addition, the use of individual strategies was also observed.

We found significant differences between the performance of individual schools, grades and classes, the hypotheses were confirmed:

H2c: There is a significant difference in strategy use between different schools.

H2d: There is a significant difference in strategy use between different classes.

H3: The multiplication strategy used in the solution and its adaptability can be related to the following background variables (cf. B. Németh, 2002, 2003; Csapó, 2002a, 2002b):

a) the mother's education,

b) the gender of the student,

c) the learning outcome of the learner,

d) learning difficulties and disorders.

In the sixth study, the hypotheses H3a, H3c were confirmed, the other two were not. The relative standard deviations calculated for the sample of boys and girls in the sixth study (N = 850): RSD boys = 72.60%, RSD girls = 69.62%, the median value for boys was 14 points, for girls 15.5 points, the sample median 15 points. The girls performed better on average in the multiplication test, but this is not significant.

H5a: In solving multiplication problems, 4th grade students perform worse than students in higher grades. This hypothesis was confirmed.

The difference between the average scores of each grade is significant ($p < 0.001$). The average scores for each grade on the 40-point test were: 13.40; 18.84 and 19.89 (standard deviations: 10.79; 11.94 and 13.25). In the central measurement, we found a difference between the performance of the individual grades, the fourth-graders scored significantly lower than those of the higher grades ($p < 0.001$).

H5b: In solving multiplication problems, students in lower grades achieve worse results than students in higher grades. This hypothesis has not been confirmed.

Based on the central study, the performance of 5th and 6th graders does not differ significantly. On the two test variants, the performance of 4th graders was 35.58% and 31.45%, respectively; the performance of 5th graders was 47.48% and 46.65%, respectively; while the performance of 6th graders was 46.9% and 52.83%, respectively.

H5c: The number of strategies used by students in higher grades shows a decreasing trend (cf. Siegler & Lin, 2010) model of “overlapping waves”).

The number of multiplication strategies for 4th grade students is less than for 5th and 6th grade students. Although a spontaneous development seems to take place, it does not end at the end of the lower grades. For this reason, based on the “overlapping waves” model described by Siegler and Lin (2010), we can conclude that the teaching of computational strategies should be continued in the upper grades, similar to the teaching of reading strategies (cf. Józsa & Józsa, 2014).

H6: The use of strategies by children who perform better on the Maths Test of Knowledge is twofold: on the one hand, they are more flexible and use different strategies than students who perform worse on the Maths Test of Knowledge and on the other hand their use of strategies is not always adaptive (Cf. de Smedt, St. Verchaffel, 2010).

H7: The use of strategies by children with special educational needs is characterized by a high degree of inflexibility. At the same time, with one or two known strategies - due to their diligence, striving for precision - they often do better than their fellow students who perform better on the Mathematics Knowledge Level Test. Due to limited data, we could neither confirm nor reject this hypothesis.

H8a: Performance on the Mathematical Knowledge Level Test shows a moderate correlation with the result on the Multiplication Strategies Test.

H8b: According to the result of the Mathematics Knowledge Level Test, the difference is significant (Cf. Hermann, 2018)

(1) the schools participating in the measurement,

(2) the classes involved in the measurement,

(3) between boys and girls.

We found significant differences between the performance of schools and classes (as expected based on the results of the national competency measurement), this was established with the help of homogeneity analysis and analysis of variance.

H8c: Performance on the Mathematical Test of Knowledge shows a medium correlation (Cf. Hermann, 2018)

(1) the educational level of the parents,

(2) satisfaction with school performance,

(3) the child's continuing education plans,

(4) the child’s semester math grade.

These hypotheses were confirmed.

SUMMARY

In the course of research, we obtained answers to research questions, established the characteristics of students' strategy use, error patterns, and showed correlations between the effectiveness of the multiplication strategy and the background variables. Most of hypotheses were confirmed.

POSSIBILITIES OF USING THE RESULTS

The present research deals with the examination of one of the basic skills, numeracy, including multiplication strategies in the mind. The work we do can be considered basic research, as such studies have not yet been conducted in Hungary. In research, we developed a well-functioning measurement tool to examine the multiplication strategies of 10–12-year-old students. One of the main conclusions of strategy research is that students' use of strategy is characterized by diversity, the strategy used depends on the characteristics of the task and the individual (cf. Siegler, 2007). We analyzed the strategies and error patterns used by the students. Among the Hungarian research methods, the ocular camera examination used during the first measurement is a rarity. Fifth study included a one-month, content-embedded development conducted in math classes. The effect of the development was also detectable during the delayed post-measurement. It is advisable to teach students several counting strategies. The teaching of metacognitive strategies resulted in a transfer effect, significantly increasing the level of mathematical knowledge of the students involved in the development. The central measurement showed that the development of strategies does not end in the fourth grade (cf. Siegler & Lin, 2010), so it is advisable to continue teaching strategies in the upper grade, similar to reading strategies.

The 7-12. no general conclusions can be drawn from smaller sample measurements for graders. However, findings made during the examination of the 4-6. grade are presumably general for Hungarian students. Based on results, the applied tests and background questionnaires are suitable for conducting measurements related to the multiplication strategies used in the mind counting among the examined age group. The tests we use can help teachers and students to assess their condition, which can also serve as a starting point for development based on metacognition, and can help expand the strategy repertoire. Measurement tools can be used in research where researchers look for answers to the relationship between the effectiveness of strategy use and the relationship between mathematical knowledge level and background factors in a model of mathematical knowledge level, multiplication strategies, and math learning beliefs.

In the course of research, the aim was to formulate conclusions and recommendations that can be used by practicing teachers. With research, we would like to draw the attention of prospective and practicing teachers to the development of students' numeracy skills, counting in their minds, and multiplication strategies. We have tried to shed light on the fact that the development of metacognitive strategies can result in a number of transfer effects, and it is worthwhile to continue teaching numeracy strategies in upper secondary school and high school.

ADDITIONAL RESEARCH TASKS

With the help of the pattern and systematization of the errors discovered during the examination of the calculation strategies, we can develop teaching methods that can be used to reduce the differences between the students' ability level and strategy use; students will be more effective in solving arithmetic problems, thus in solving textual problems, from all subjects, and during both domestic and international measurements and in everyday life. With the help of appropriate and sufficient research, the expected ability level of calculations and multiplications performed in the head can be standardized. At the same time, new teaching aids may be created. More conscious and frequent teaching of numeracy strategies and metacognitive strategies can become a tool for the differentiated development of students, and can help the development of a new classroom culture that is better suited to the challenges of time. The development of children's adaptive counting and multiplication skills can also be utilized in practical life, and presumably the approach of adaptability can result in a significant transfer effect in other areas of human thinking as well. We consider it expedient to carry out more development and to include the teaching of metacognitive strategies at several points in the training of teachers. Another research task is technology-based testing using the eDia system: the creation of a digital valid and reliable measurement tool that can increase the objectivity of data collection and evaluation and help us get answers to questions sooner (Vígh-Kiss, 2017c).

LITERATURE

Australian Education Council (1991). *A National Statement on Mathematics for Australian schools*. Curriculum Corporation.

Baroody, A. J. (1999). The roles of estimation and the communtativity principle in the development of third-graders mental multiplication. *Journal of Experimental Child Psychology, 74*, 157-193.

Ben-Zeev, T. (1998). Rational errors and the mathematical mind. *Review of General Psychology, 2,* 366-383.

Blöte, A. W., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instruction effects. *Journal of Educational Psychology, 93, Sep 2001*, 627-638. http://dx.doi.org/10.1037/0022-0663.93.3.627 Utolsó letöltés: 2021.05.08.

B. Németh Mária (2002): Az iskolai és a hasznosítható tudás: természettudományos ismeretek alkalmazása. In Csapó Benő (szerk.), Az iskolai tudás (pp. 123-148). Osiris Kiadó.

B. Németh Mária (2003): A természettudományos műveltség mérése. *Magyar Pedagógia*, *103*(4), 499-526.

Buzás Zsuzsa (2016). Kottaolvasási stratégiák vizsgálata zeneiskolás diákok körében. In Devosa Iván és Steklács János (szerk.), *II. Magyar Szemmozgáskutatás Konferencia - II. Hungarian Conference on Eye Movements: A konferencia programja és absztraktjai. - Program and abstracts of the conference.* Konferencia helye, ideje: Kecskemét, Magyarország, 2016.06.10 Kecskeméti Főiskola Tanítóképző Főiskolai Kar, 8. https://libres.uncg.edu/ir/uncg/f/Carrier_uncg_0154D_10324.pdf Utolsó letöltés: 2021.05.08.

Carr, M., Alexander, J., & Folds-Bennett, T. (1994). Metacognition and mathematics strategy use. *Applied Cognitive Psychology, 8,* 583-595. https://doi.org/10.1002/acp.2350080605 Utolsó letöltés: 2021.05.08.

Cooper, T., Heirdsfield, A., Mulligan, J., & Irons, C. (1999). *Children's multiplication and division strategies*. In Zaslavsky, O. (Ed.), Proceedings of the 23rd Annual Conference of the International Group for the Psychology of Mathematics Education, 3, (pp. 89-96). University of Haifa, Israel, Program Committee.

Csapó Benő (2002a, szerk): *Az iskolai tudás.* 2.kiadás. Osiris Kiadó.

Csapó Benő (2002b): A tudáskoncepció változása. *Új Pedagógiai Szemle*. **52.,** 2.sz.38-45.

Csapó Benő (2006). A formális és nem-formális tanulás során szerzett tudás integrálása. Az előzetes tudás felmérése és elismerése. *Iskolakultúra, 16*(2), 3-16. http://www.edu.u-szeged.hu/~csapo/publ/Csapo_ElozetesTudas.pdf Utolsó letöltés: 2021.05.08.

Csíkos Csaba (2007). *Metakogníció*. A tudásra vonatkozó tudás pedagógiája. Tanítás és tanulás sorozat. Műszaki Könyvkiadó.

Csíkos Csaba (2013). A fejben számolás stratégiáinak vizsgálata háromjegyű számok összeadásával negyedik osztályos tanulók körében. In Molnár Gyöngyvér és Korom Erzsébet (szerk.), *Az iskolai sikerességet befolyásoló kognitív és affektív tényezők értékelése* (pp. 31-45). Nemzetek Tudása Tankönyvkiadó.

Csíkos, C. (2016). Strategies and performance in elementary students' three-digit mental addition. *Educational Studies in Mathematics, 91*, 123-139.

Csíkos Csaba (2017). *A gondolkodás stratégiai összetevőinek fejlesztése iskoláskorban*. Akadémiai doktori értekezés. Szeged.

Csíkos, C., Szitányi, J., & Kelemen, R. (2012). The effects of using drawings in developing young children's mathematical problem solving: A design experiment with third-grade Hungarian students. *Educational Studies in Mathematics, 81*, 47-65.

Department for Education and Employment (1999). *The National Numeracy Strategy Framework for Teaching Mathematics from Reception to Year 6*. London.

De Smedt,B., Torbeyns,J., Stassens,N., Ghesquiére, P., & Verschaffel, L.(2010). Frequency, efficiency and flexibility of indirect addition in two learning environments. *Learning and Instruction, 20*, 205-215.

Dowker, A. (2005). Early Identification and Intervention for Students with Mathematics Difficulties. *Journal of Learning Disabilities, 38*, 324. http://dx.doi.org/10.1177/00222194050380040801 Utolsó letöltés: 2021. 05. 09.

Geary, D. C. (2003). Arithmetical development: Commentary on Chapters 9 through 15 and future directions. In Baroody, A. J., & Dowker, A. (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 453–464). Lawrence Erlbaum Associates.

Hermann Zoltán (2018): A nemek szerinti tesztpontszám-különbségek nemzetközi összehasonlításban. In Munkaerőpiaci tükör 2017, pp. 103-109. Budapest: MTA Közgazdaság- és Regionális Tudományi Kutatóközpont Közgazdaság-tudományi Intézet. https://kti.krtk.hu/wp-content/uploads/2018/12/mt_2017_hun_103-109.pdf Utolsó letöltés: 202105.08.

Hope, J. A. & Sherrill, J. M. (1987). Characteristics of unskilled and skilled Mental Calculation. *Journal for Reserches in Mathematics Education., 18* (2), 98-111.

Jacob, L. & Mulligan, J. (2014). Using arrays to build towards multiplicative thinking in early years. *Australian Primary Mathematics Classroom, 19(1),* 35-40.

Józsa Gabriella és Józsa Krisztián (2014). A szövegértés, az olvasási motiváció és a stratégiahasználat összefüggése. *Magyar Pedagógia, 114* (2), 67-89.

Kilpatrick, J., Swafford, J. & Findell, B. (2001). *Adding it Up: helping children learn mathematics*. National Research Council.

Kuhn, D., Schauble, L., & Garcia-Mila, M. (1992). Cross-domain development of scientific reasoning. *Cognition and Instruction, 9*(4), 285–327. https://doi.org/10.1207/s1532690xci0904_1 Utolsó letöltés: 2021.05.08.

LeFevre, J., Smith, B. L., Hiscock, K., Daley, K. E., & Morris, J. (2013). Young adults' strategy choices in simple arithmetic: Implications for the development of mathematical representations. In A. J. Baroody, & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Recent research and theory* (pp.203-228). Erlbaum.

Lemaire, P., & Lecacheur, M. (2001). Older and Younger Adults' Strategy Use and Execution in Currency Conversion Tasks: Insights From French Franc to Euro and Euro to French Franc Conversions. *Journal of experimental psychology. Applied. 7*, 195-206. 10.1037/1076-898X.7.3.195.

Lemaire.P., & Reder, L. (1999). What affects strategy selection in arithmetic? The example of parity and five effectson product verification. *Memory & Cognition, 27* (2), 364-382.

Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General, 124*(1), 83–97. https://doi.org/10.1037/0096-3445.124.1.83

Molitorisz Anikó (2012). Tankönyvolvasási stratégiák vizsgálata 12–18 éves tanulók körében. *Anyanyelvpedagógia, 5,* 1. sz. http://www.anyp.hu/cikkek.php?id=375 Utolsó letöltés: 2021.05.08.

Mulligan, J.T. & Mitchelmore, M. C. (2009). Awareness of pattern and structure in early mathematical development. Mathematics Education Research Journal, 21(2), 33-49. Mathematics Education Research Journal. 21. 33-49. 10.1007/BF03217544.

National Council of Teachers of Mathematics (1989). *Principles and standards for school mathematic* Reston.101: National Council of Teachers of Mathematics

OECD (2019). *PISA 2018 Results.* COMBINED EXECUTIVE SUMMARIES VOLUME I, II & III. https://www.oecd.org/pisa/Combined_Executive_Summaries_PISA_2018.pdf Utolsó letöltés: 2021.05.08.

Payne, J.W., Bettman, J.R., & Johnson, E.J. (1988). Adaptive Strategy Selection in Decision Making. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 14,* 534-552.

Rittle-Johnson, B., Siegler, R., & Alibali, M. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93,* 346-362. 10.1037//0022-0663.93.2.346. Utolsó letöltés: 2021.05.08.

Siegler, R. S. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General, 117*(3), 258–275. https://doi.org/10.1037/0096-3445.117.3.258 Utolsó letöltés: 2021.05.08.

Siegler, R., & Jenkins, E (1989). How children discover new strategies. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc. https://psycnet.apa.org/record/1989-98401-000 Utolsó letöltés: 2021.05.09.

Siegler, R.S., & Lin, X. (2010). Self-explanations promote children's learning. In H. Salatas Waters, & W. Schmeider (Eds.), *Metacognition, strategy use, and instruction* (pp. 85-112). New York, London: The Guilford Press.

Siegler, R. S., & McGilly, K. (1989). Strategy choices in children's time-telling. In I. Levin & D. Zakay (Eds.), *Advances in psychology, 59. Time and human cognition: A life-span perspective* (p. 185–218). North-Holland. https://doi.org/10.1016/S0166-4115(08)61042-0 Utolsó letöltés: 2021.05.08.

Siegler, R. S., & Shipley, C. (1995). Variation, Selection and Cognitive Change. In T. Simon, & G. Halford (Eds.), Developing cognitive competence: New approaches to process modelling (pp. 31-76). Lawrence Erlbaum Associates, Inc.

Siegler, R.S. (2007). Cognitive variability. *Developmental Science, 10*, 104-109.

Snyder, A. (2019): Why Teacsers shoud celebrate Failure in the Classroom https://medium.com/inspired-ideas-prek-12/success-failure-the-key-to-learning-cc337cc286c Utolsó letöltés: 2021.05.08.

Steklács János (2013). *Olvasási stratégiák tanítása, tanulása és az olvasásra vonatkozó meggyőződés*. Nemzedékek Tudása Tankönyvkiadó.

Szendrei Julianna (2005). *Gondolod, hogy egyre megy?* Typotex Kiadó.

Thompson, I. (1999). Getting your head around mental calculation. In I. Thompson (Ed.), Issues in teaching numeracy in primary schools (pp. 145–156). Open University Press.

Treffers, A., De Moor, E., & Feijs, E. (1990). *Proeve van een national programma voor het reken/wiksundeonderwijs op de basisschool. Deel 1. Overzicht einddoelen*. [Towards a national curriculum for mathematics education in the elementary school. part 1. Overview of the goals.] Zwijsen, Tilburg.

VanLehn, K. (1990). *Mind bugs. The orogin of procedural misconceptions*. MIT Press.

Verschaffel, L., Greer, B. és De Corte, E. (2007): Whole number concepts and operations.In F. Lester (Ed.), *Second Handbook of research on mathematics teaching and learning* (pp. 557-628). Information Age Publishing.

Verschaffel, L., Torbeyns, J., Luwel, K., Van Dooren, W. & De Smedt, B. (2007). A stratégiahasználat rugalmassága az alsótagozatos matematikában: elemzés és fejlesztés. *Iskolakultúra, 17*, 11-12.sz., 92-102.

Vidákovich Tibor (2013). Kompetenciákon alapuló korszerű oktatás a középiskolákban - a Magyar Nemzeti Tanács, a szabadkai Vajdasági Módszertani Központ és a magyarkanizsai Regionális Szakmai Pedagógus-továbbképző Központ közös nemzetközi konferenciája. 2013.06.06., Szabadka, Magyar Tannyelvű Tanítóképző Kar. https://www.youtube.com/watch?v=PTP_6HBJYnQ Utolsó letöltés: 2021.05.08.

PUBLICATIONS ON THE TOPIC OF THE DISSERTATION

Vígh-Kiss, E.R. (2013a). Adaptive Strategy Use in Mathematics Education. In J.T. Karlovitz (Ed.), Questions and perspectives in education, (pp. 342-352). International Research Institute.

Vígh-Kiss Erika (2013b). Adaptív stratégiahasználat a matematikaoktatásban. In Józsa Krisztián és Fejes József Balázs (szerk.), PÉK 2013. XI. Pedagógiai Értékelési Konferencia: CEA 2013. 11th Conference on Educational Assessment (p.118). Szeged: SZTE BTK Neveléstudományi Doktori Iskola.

Vígh-Kiss, E. (2014a). Advocacy and support for mathematically talented adolescents (aged 14-18): Matematikai tehetség gondozása 10-18 éves korban. In M. Takács, G. Czékus, & L. Major (Eds.), *3rd International Methodologocal Conference: The influence of teaching methodology ont he quality teacher and preschool teacher training. Abstracts*, p. 43. Újvidéki Egyetem Magyar Tannyelvű Tanítóképző Kar.

Vígh-Kiss, E.R. (2014b). The Development of mathematical Gifted Students. In J. T. Karlovitz (Ed.), *2nd IRI International educational conference: abstracts*, p. 64. International Research Institute.

Vígh-Kiss, E.R. (2014c). The Development of Talented Students. In J.T. Karlovitz (Ed.), *PRACTICE AND THEORY IN SYSTEMS OF EDUCATION* (pp. 350-356). Elérés forrás: http://eduscience.fw.hu/3007VighKissErika.pdf Utolsó letöltés: 2021.05.08.

Vígh-Kiss, E.R. (2014d). 14-18 éves tanulók adaptív készséghasználatának vizsgálata. In Korom Erzsébet és Pásztor Attila (szerk.), *PÉK 2014: XII. Pedagógiai Értékelési Konferencia: program, előadás-összefoglalók,* p. 177. SZTE BTK Neveléstudományi Doktori Iskola.

Vígh-Kiss, E.R. (2014e). 14-18 year old students' use of adaptive skills. In A. Engin (Ed.), *METAcognition: 6th Biennal Meating of the EARLI Special Interest Gruop 16*, 2014.09. 03-2014.09.06, p. 154. Bogaziçi University.

Vígh-Kiss Erika Rozália (2015a). Tudás és gazda(g)ság: A gazdasági versenyképesség és az oktatás kapcsolata. *Új Kép: Pedagógusok és Szülők folyóirata*, 7-17.

Vígh-Kiss Erika Rozália (2015b). A metakogníció körében végzett legújabb kutatások eredményei: Beszámoló az EARLI Metacognition szakcsoportjának SIG, Special Interest Group) 2014-es konferenciájáról. *Iskolakultúra: Pedagógusok szakmai tudományos folyóirata 25*(12), 121-133.
Elérés forrás: http://www.iskolakultura.hu/index.php/iskolakultura/article/view/21741

Vígh-Kiss Erika Rozália (2015c). A szorzási stratégiák vizsgálata negyedik osztályos tanulók körében a szemmozgás-elemzés módszerével. In Zs. Buzás, I. Devosa, J. Steklács, & Á. Maródi (szerk.), *Nemzetközi Szemmozgáskutatás Konferencia: International Conference on Eye Movements 2015*, 5. Kecskeméti Főiskola Tanítóképző Főiskolai Kar.

Vígh-Kiss Erika Rozália (2015d). 10-12 éves tanulók szorzási stratégiáinak vizsgálata. In Tóth Péter, Holik Ildikó és Tordai Zita (szerk.), *Pedagógusok, tanulók, iskolák- az értékformálás, az értékközvetítés és az értékteremtés világa: tartalmi összefoglalók: XV. Országos Neveléstudományi Konferencia,* 320. Óbudai Egyetem.

Vígh-Kiss Erika Rozália (2016a). Development of 11-12 years–old children's multiplication strategies. In Cs. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of the 40th conference of the International Group for the Psychology of Mathematics Education: PME 40*, 260. Szeged: International Group for the Psychology of Mathematics Education.

Vígh-Kiss Erika Rozália (2016b). A segítés értelmezése a matematika tanítása során. In Buda András és Kiss Endre (szerk.), *Interdiszciplináris pedagógia és az oktatási rendszer újraformálása*: A IX. Kiss Árpád Emlékkonferencia előadásainak szerkesztett változata (pp. 374-382). Kiss Árpád Archívum Könyvtára, DE Neveléstudományok Intézete.

Vígh-Kiss Erika Rozália (2016c). 11-12 éves tanulók szorzási stratégiáinak fejlesztése. In Zsolnai Anikó és Kasik László (szerk.), *A tanulás és nevelés interdiszciplináris megközelítése: XVI. Országos Neveléstudományi Konferencia, Szeged, 2016. november 17-19.: program és absztraktköte*t, 356. MTA Pedagógiai Bizottság, SZTE BTK Neveléstudományi Intézet.

Vígh-Kiss Erika Rozália (2017a). A fejben számolás stratégiáinak vizsgálata hatodik évfolyamos tanulók körében. In Talata István (szerk.), *Matematikát, Fizikát és Informatikát Oktatók 41. Országos Konferenciája: MAFIÓK 2017*, 42. Szent István Egyetem Ybl Miklós Építéstudományi Kar.

Vígh-Kiss Erika Rozália (2017b). A fejben számolás stratégiáinak vizsgálata hatodik évfolyamos tanulók körében.

In Talata István (szerk.), *Matematikát, Fizikát és Informatikát Oktatók 41. Országos Konferenciája: MAFIÓK 2017* (pp. 261-266). Szent István Egyetem Ybl Miklós Építéstudományi Kar.

Vígh-Kiss Erika Rozália (2017c). Az IKT eszközök használatának lehetőségei a matematikaoktatásban. In [s. n.] (szerk.), *Innováció, kutatás, pedagógusok. HUCER 2017: Absztraktkötet*, 241. Magyar Nevelés- és Oktatáskutatók Egyesülete, Hungarian Educational Research Association (HERA).

Vígh-Kiss Erika Rozália (2017d). Oktatás – Egészség – Tudásgazda(g)ság. In I. Devosa, Á. Maródi, Zs. Buzás, & J. Steklács (Eds.), *International HEART 2017 Conference: Program and abstracts of the conference*, pp. 29-30. Pallasz Athéné Egyetem.

Vígh-Kiss Erika Rozália (2017e). Hatodik évfolyamos tanulók szorzási stratégiáinak vizsgálata. In Kerülő Judit, Jenei Teréz és Gyarmati Imre (szerk.), *XVII. Országos Neveléstudományi Konferencia*, 528. MTA Pedagógiai Tudományos Bizottság, Nyíregyházi Egyetem.

Vígh-Kiss Erika és Buzás Zsuzsanna (2017). Metakognitív stratégiák a tanulási folyamatban. In Janurik Márta és Szabó Norbert (szerk.), *III. Digitális Zenepedagógiai és Szakmódszertani Konferencia.*, 15. JATEPress Kiadó.

Vígh-Kiss Erika Rozália (2019). OKTATÁS – TUDÁS – GAZDA(G)SÁG.In Devosa Iván (szerk.), *Aszklépiosz tanulmányok 2019* (pp. 5-21). Neumann János Egyetem, Pedagógusképző Kar.

Vígh-Kiss Erika, Csíkos Csaba és Steklács János (2013). Negyedikes tanulók adaptív készséghasználatának vizsgálata a szemmozgás-elemzés módszerével. In Bárdos Jenő, Kis-Tóth Lajos és Racskó Réka (szerk.), *XIII: Országos Neveléstudományi Konferencia: Változó életformák – Régi és új tanulási környezetek: Absztraktkötet*. Konferencia helyszíne: Eger, Magyarország, 2013.11.06-2016.11.09. Líceum Kiadó, 2013.p71.

Vígh-Kiss Erika, Csíkos Csaba és Steklács János (2019). Negyedik osztályos tanulók szorzási stratégiáinak vizsgálata szemkamerás módszerrel. In Steklács János (szerk.), *Szemkamerás vizsgálatok a pedagógiai kutatásban: Tanulmánykötet* (pp. 119-131). Kaposvári Egyetem Pedagógiai Kar.