UNIVERSITY OF SZEGED

PH.D. THESIS

Image Quality Improvement in Computed and Binary Tomography

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"... most people don't get those experiences because they never ask. I've never found anybody who didn't want to help me when I've asked them for help... You gotta act... If you're afraid of failing, you won't get very far."

- Steve Jobs

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List of Abbreviations

2D	two dimensional
3D	three dimensional
AM	additive manufacturing
ANN	Artificial Neural Network
ART	Algebraic Reconstruction Technique
ATCM	Automatic Tube Current Modulation
ATVS	Automatic Tube Voltage Selection
BCE	binary cross-entropy
BT	Binary Tomography
CAD	Computer-aided design
CGLS	Conjugate Gradient Least Squares
CNN	Convolutional Neural Network
CT	Computed Tomography
DNS	direct numerical simulation
DOF	degree of freedom
DT	Discrete Tomography
FBP	Filtered Backprojection
HD	Hausdorff distance
LCSS	level crossing sampling scheme
ME	Misclassification error
MSE	Mean Squared Error
NDT	Non-destructive testing
PFOM	Pratt's Figure of Merit
RAE	Relative foreground area error
RME	Relative Mean Error
SA	Simulated Annealing
SART	Simultaneous Algebraic Reconstruction Technique
SIRT	Simultaneous Iterative Reconstruction Technique
SNR	Signal-to-noise ratio
STD	standard deviation

List of Symbols

$\mathbf{A}$	matrix that describes the projection geometry
$\mathbf{x}$	the vector of all n^2 unknown image pixels
$\mathbf{b}$	the vector of all m measured projection values
m	total number of projection lines used
n	height and width of the image to be reconstructed
N	stands for n^2
$\mathcal{N}$	$\{1, 2, \dots, N\}$
$\mathcal{M}$	$\{1, 2, \dots, m\}$
$\text{RME}(\mathbf{x}^*, \mathbf{x}^S)$	Relative Mean Error between $\mathbf{x}^*$ and $\mathbf{x}^S$, where $\mathbf{x}^*$ is the blueprint and $\mathbf{x}^S$ is the reconstructed image from angle set S
$\text{SNR}(\mathbf{x})$	Signal-to-noise ratio of $\mathbf{x}$
L	number of levels in Gaussian pyramid
G_l	downscaled version of the original image using Gaussian pyramid on level l
D_l	downscaled version of the original detector intensities on level l
S	set of angles
$\text{PC}(\mathbf{x})$	perimetric complexity of $\mathbf{x}$
p_j	probability of pixel x_j taking value 1
$\mathcal{H}(x_j)$	uncertainty of pixel x_j
$\mathcal{U}(\mathbf{x})$	global uncertainty of $\mathbf{x}$
r	number of projections
E_k	error function after acquiring the k th projection
$\mathcal{P}_{\mathbf{A}}(i)$	i -th perturbed reconstruction of $\mathbf{A}$
$\mathcal{G}_{\mathbf{A}}(i)$	the largest gradient of the solutions of $\mathbf{A}\mathbf{x} = \mathbf{0}_N$ with respect to x_i
$\mathcal{U}_{\mathbf{A}}(i)$	the grayscale uncertainty of the x_i pixel with a given $\mathbf{A}$ projection matrix
$\Omega_{\mathbf{A}, \mathbf{b}}$	set of reconstructions
$E(\Omega_{\mathbf{A}, \mathbf{b}}, i)$	the expected value of x_i in $\Omega_{\mathbf{A}, \mathbf{b}}$
$\sigma(\Omega_{\mathbf{A}, \mathbf{b}}, i)$	the standard deviation of x_i in $\Omega_{\mathbf{A}, \mathbf{b}}$

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Introduction

The goal of tomographic reconstruction is to discover the inner structure of objects using their projections in a slice-by-slice manner. Most often, the projections are produced by X-rays. Measuring the attenuation of the beams passing through the object can provide information about the density of the materials along the paths of the beams. Collecting such projection data from many angles, one can produce an image showing the inner structure of the object [40, 49]. Computed Tomography (CT) is a widely used tool in different fields, such as medical imaging, non-destructive testing (NDT) [11], material characterization, crystallography, etc. to analyze the interior of the subject of investigation.

Discrete Tomography (DT) [41, 42] uses the prior information that the cross-section image to be reconstructed contains only a few different intensities which are known in advance. Exploiting this prior knowledge, a smaller set of projections is sufficient for a reconstruction with an acceptable quality.

Binary Tomography (BT) is a more restricted variant of DT. In this case, every single pixel of the image to be reconstructed can take only two different intensities. In practice, objects corresponding to these images must be made of a homogeneous material. Based on this fact, one pixel in the cross-section image holds the information on the presence or absence of the material. BT is most commonly used in the fields of industrial NDT and electron microscopy [3, 10].

This thesis is the summary of the Author's research in the field of CT and BT. Our main aim was to improve reconstruction quality by developing novel algorithms and improving previous approaches in the research fields of selecting the most informative projection angles, automatic selection of the tube voltage of a CT scanner, and binarizing already reconstructed CT slices using Convolutional Neural Networks. The structure of the dissertation is as follows.

In *Chapter 1*, we detail some fundamentals that are indispensable for understanding the different research topics in the thesis. Here we assemble the formulations of different reconstruction approaches and the reconstruction methods belonging to them. Some quality measurements are given for later evaluations.

As we said before, BT reconstructs binary images from a low number of their projections. Often, there is a freedom in how these projections can be chosen, which can significantly affect the quality of reconstructions [78, 115]. In *Chapter 2*, we apply sequential feature selection methods to propose new offline projection selection algorithms to find the "most informative" projection set based on a blueprint image. Using various software phantom images, we show that these methods outperform the previously published projection selection algorithms. Based on the results of these previous algorithms, we propose two strategies for reducing the amount of data needed for binary tomographic reconstructions. We study how the direction dependency changes by lowering the resolution of an image. We point out how to specify the most informative angles for the original image using its downscaled version. We also show how to predict the final acceptable resolution. Applications of the proposed strategies are also mentioned.

In many applications, incomplete projection data holds insufficient information for the correct reconstruction of the original object. The low number of projections leads to a lack of information and uncertainty in the reconstructions. In practice, this means that the pixel values of the reconstruction are not uniquely determined by the measured data and thus can have variable values. In *Chapter 3*, we modify one of our offline sequential projection selection methods and a previously published online adaptive angle selection algorithm [38] to use global uncertainty [119] to select

projection angles. Besides the projection data, we assume no further knowledge of the image to be reconstructed. Based on the experience gained during experimenting with binary uncertainty, we provide a theoretically proven uncertainty metric that can be used for measuring the variability of pixel values in grayscale reconstructions. The uncertainty values are based on linear algebra and measure the slopes of the hyperplane of solutions in the algebraic formulation of tomography. The methods can also be applied for any linear equation system that satisfies a given set of conditions. Using the uncertainty measure, we derive upper and lower bounds on the possible pixel values in tomographic reconstructions. We also show that the perturbed reconstructions can directly be applied for calculating the difference between the original object of study (ground-truth image) and its reconstructions, and how it could be used to predict the reconstruction error that may arise during the reconstruction. We give practical validation and analysis to the results.

Since CT is a widely used X-ray based imaging modality in radiodiagnostics and NDT, for medical considerations as well as for economic and environmental reasons, it is desirable to reduce the dose of radiation and to optimize the energy used for acquiring X-ray projection images. Especially in the case of elongated objects, using a constant energy spectrum radiation may not provide realistic information about the interior. In *Chapter 4*, we provide an adaptive tube voltage selection method, which determines the proper amount of radiation energy on-the-fly during the acquisition, based on projection information. By simulation toolbox experiments on software phantom images, we show that this adaptive approach can produce better quality CT images, simultaneously consuming less energy.

Recently, deep learning approaches, especially Convolutional Neural Networks (CNNs) [58], have achieved tremendous success in various fields, such as classification, segmentation, denoising, super-resolution, and removing CT noise related to low dose. In *Chapter 5*, we apply CNNs to select projections in order to reconstruct the original images from their sinograms with the smallest possible error. The training of neural networks is generally a time-consuming process, but after the network has been trained, the prediction for a previously unseen input is fast. We train CNNs using sinograms as input and the desired, algorithmically determined k-best projections as labels in a supervised setting. We achieve a significantly faster projection selection and only a slight decrease in the quality of the reconstructed images. Then, we provide a U-Net [99] based segmentation algorithm for CT slices of microarchitected specimens, as a part of a bigger project. We compare our approach to U-Net and other non-CNN based methods and show that our approach gave the closest results to the groundtruth images in most of the cases.

In *Chapter 6*, we summarize the results achieved in the thesis.

Chapter 1

Fundamentals

1.1 Formulations of the reconstruction problem

The aim of tomography is to reconstruct the 2D slices of an object using its projections. In *transmission tomography*, X-ray passes through the object of investigation from several directions while a certain amount of photons are absorbed. A detector line measures the intensity of the traversed radiation. Passing through different materials, the degree of absorption varies based on the characteristics of the materials. This is called the *linear attenuation coefficient* of a material. This coefficient depends on the applied energy and the type of the material. Based on the Beer-Lambert law [44], if I_0 is the initial intensity of the monochromatic X-ray, and it crosses a homogeneous material with Δx thickness and μ linear attenuation coefficient, then the exiting intensity can be given with the following formula

$$I = I_0 \cdot e^{-\mu \Delta x} , \quad (1.1)$$

where the intensity can be replaced with the number of photons. Inasmuch as the X-ray passes through materials with different $\mu_1, \mu_2, \dots, \mu_n$ linear attenuation coefficients and $\Delta x_1, \Delta x_2, \dots, \Delta x_n$ thicknesses, Eq. (1.1) can be changed as follows

$$I = I_0 \cdot e^{-\sum_{i=1}^n \mu_i \Delta x_i} . \quad (1.2)$$

In the mathematical sense, the material density of a 3D object's 2D slice in the (x, y) point can be described using a bivariate $f(x, y)$ function. The only information one has about these slices is the sum of the material densities from certain angles, i.e., instead of the $f(x, y)$ function, only its line integrals are known, taken from certain directions. The task of the reconstruction is to produce the $f(x, y)$ function from its line integrals. These line integrals are called *projections*. Let $g(s, \theta)$ denote the projection of an $f(x, y)$ function from the θ direction, where θ is fixed. The $g(s, \theta)$ is the Radon transform of the $f(x, y)$ function and denoted as $[Rf](s, \theta)$. For parallel beam tomography, the projections can be expressed as the Radon transform of the object that is to be reconstructed. The Radon transform is defined as:

$$[Rf](s, \theta) = g(s, \theta) = \int_{-\infty}^{\infty} f(s \cos \theta - u \sin \theta, s \sin \theta + u \cos \theta) du , \quad (1.3)$$

where s indicates the distance from the origin and u is the parameter for positioning the line that is determined by the (s, θ) pair (see Fig. 1.1b for illustration).

The projections can be placed one after another in a 2D coordinate system by ascending θ values on the vertical axis, where the projection values are replaced with grayscale intensities. This image is called a *sinogram* (see Fig. 1.2). The size of the sinogram depends on the density of projection sampling. Thus, in essence,

the task of image reconstruction is to produce the original function (or at least its best possible approximation), which is basically the construction of the $f(x, y)$ image from its sinogram.

FIGURE 1.1: (a) Parallel beam geometry. (b) Acquisition of a projection.

There are two main types of reconstruction approaches: analytic and algebraic. In the next two sections, we discuss both approaches along with the reconstruction algorithms that will be used in the thesis. Before getting into the details, it is important to mention that we use parallel beam geometry for all types of reconstructions. This means that we map the projection of a function (object) from a certain direction using parallel projection rays (Fig. 1.1a).

FIGURE 1.2: (a) A simple object. (b) Its sinogram, acquired on the $[0^\circ, 179^\circ]$ integer angle interval.

1.1.1 Analytic approach

In the case of analytic approaches, no prior information is given. A simple reconstruction method, *backprojection*, can be achieved by evenly backprojecting all the projection values from the right direction and summing the right projection values for all the pixels. This is the basis for the most commonly used reconstruction techniques. Let us assume that we have a finite number of projections of an object. We need to simply run the projections back through the image to obtain a rough approximation to the original. The projections will interact constructively in regions

that correspond to the sources in the original image. This is the previously mentioned "backprojection". An immediately apparent problem is the blurring (star-like artifacts) that occurs in the reconstructed image (see Fig. 1.3a). To avoid this blurring, one could choose another simple reconstruction technique, which is described by the *Projection-Slice* (Central Slice or Fourier Slice) Theorem:

Theorem 1. *The 1D Fourier-transform of the $f(x, y)$ function's projection at angle θ is equal to a section of the $f(x, y)$ function's 2D Fourier-transform frequency space that is covered by a line passing through the center of this space in θ direction.*

Theorem 1 provides a reconstruction method, which is called the *Fourier reconstruction technique*. The essence of this method is as follows: taking the 1D Fourier-transform of the projections they specify lines in the function's 2D Fourier-transform. If one takes all the angles between 0 and π , one can get all the points of the original function's 2D Fourier-transform. Applying a 2D inverse Fourier-transform in this, one can also get back the original function. Although the Fourier reconstruction is mathematically elegant, it may suffer from different problems. The 2D inverse Fourier-transform is computationally expensive. Still more significant is that an infinite model of the Fourier-transform has to be discretized and in practice, only a finite number of projections can be taken.

Filtered Backprojection (FBP) The optimal way – in the noiseless case – to eliminate the previously mentioned blurring effect that occurs during a simple backprojection process is through a ramp filter. The ramp filter is a high pass filter that does not permit low frequencies that cause blurring to appear in the image. Thus, in this case – as opposed to the Fourier reconstruction technique – not the original projections' inverse Fourier-transforms are backprojected during the reconstruction, but the functions that are created by filtering the original projections in the frequency space (see Fig. 1.3b). This method is called *filtered backprojection* [49], which is the combination of the simple backprojection and the Fourier reconstruction methods. It is one of the oldest and most popular analytic reconstruction algorithms that was developed. In general, for an accurate reconstruction, this method needs hundreds of projections and cannot take prior information to improve the reconstruction process.

FIGURE 1.3: The reconstruction of the Fig. 1.2b sinogram after applying backprojection (a) and filtered backprojection (b).

1.1.2 Algebraic approach

Due to the small number of projections, typically available in DT, the reconstruction is mainly solved by one of the algebraic approaches. The idea is to describe the

connections between projections and pixels using equations. Assuming that the size of the image to be reconstructed is $n \times n$ ($N = n^2$), the reconstruction problem can be described as a system of equations (see Fig. 1.4 for illustration)

$$\mathbf{Ax} = \mathbf{b}, \quad \mathbf{A} \in \mathbb{R}^{m \times N}, \quad \mathbf{x} \in \mathbb{R}^N, \quad \mathbf{b} \in \mathbb{R}^m, \quad (1.4)$$

where

- $\mathbf{x}$ is the vector of all N unknown image pixels,
- $\mathbf{b}$ is the vector of all m measured projection values (m is the total number of projection lines used),
- $\mathbf{A}$ describes the projection geometry with all $a_{i,j}$ elements giving the length of the line segment of the i th projection line through the j th pixel.

Therefore, the task of the continuous reconstruction is to find an $\mathbf{x} \in \mathbb{R}^N$ reconstruction such that $\mathbf{Ax} = \mathbf{b}$. If we want to further limit this formulation into the binary reconstruction problem, we have to add a new restriction to $\mathbf{x}$, which is $\mathbf{x} \in \{0, 1\}^N$. This way $\mathbf{x}$ is forced to contain only $\{0, 1\}$ values. Although the basic formulation is given for the 2D case of tomography, the results are directly applicable to the 3D case of tomography as well. Some of our proposed methods strongly rely on the non-negativeness of the projection matrix, the projection values, and the reconstructed pixels. This assumption can be made in transmission tomography since there are no materials with negative density or projection rays with line segments of negative lengths. We also will benefit on the fact that the plane of solutions in Eq. (1.4) is a linear hyperplane in $\mathbb{R}_{\geq 0}^N$.

It can be noted, that the stability of the reconstruction is connected to the condition number of the $\mathbf{A}$ matrix. Unfortunately, classical tools for analyzing the condition number are hard to carry out due to the large size of projection matrices. Therefore, alternative approaches are needed. Furthermore, in real scenarios, the projection data is often perturbed with noise. As a consequence, the reconstruction problem may happen to have no exact solution. We note that the reconstruction model in Eq. (1.4) also covers the DT model that works on lattice sets, i.e., finite subsets of $\mathbb{Z}^2$ (in that case $\mathbf{A}$ is a 0-1 matrix describing the connection between the points and lattice lines). It is known that the general reconstruction problem on lattice sets is NP-hard and unstable (implying that an entirely different solution can be returned), which can also cause problems in finding an exact solution of Eq. (1.4) [29, 30].

Therefore, in BT, often an approximate solution of Eq. (1.4) is satisfactory. The Algebraic Reconstruction Technique (ART) is the first published method that approximated the object to be reconstructed by solving a system of equations [35]. ART is based on the Kaczmarz-method. For details about the method see [48]. All the different algebraic algorithms use ART as a basis, where we start out from a given initial image and approximate the correct reconstruction by iteratively subtracting the back-projected error of the intermediate state from itself. In general, these methods provide continuous reconstructions with real pixel values and (with the proper setup) can produce images which are closest (in the Euclidean sense) to the initial image.

Simultaneous Iterative Reconstruction Technique (SIRT) The difference between ART and SIRT is that SIRT backprojects the error of the system of equations simultaneously during one iteration. In a single iteration, it determines the error for all

FIGURE 1.4: Equation system-based representation of the parallel-beam projection geometry.

the rays passing through a certain pixel. Still, it modifies the pixel value only once, namely with the average of the backprojected errors. It converges slower than ART but provides smoother results. When a binary image is needed as the output, we will use the thresholded version of the SIRT (TSIRT), which differs from the result of the SIRT algorithm in a way that

$$x_i = \begin{cases} 0 & \text{if } x_i < 0.5, \\ 1 & \text{otherwise.} \end{cases}$$

This modification is applied to the output image of the SIRT reconstruction [37, 40, 113]. We refer to the SIRT algorithm as *bounded SIRT* when $\mathbf{x} \in [0, 1]^N$.

In Fig. 1.5, one can see the difference between FBP and bounded SIRT in the reconstruction quality, in the case of small number of projections.

FIGURE 1.5: Reconstruction of the Fig. 1.2a image. (a) and (b) show the reconstructions with FBP using 4 and 8 projections, respectively. (c) and (d) show the reconstructions with bounded SIRT using 4 and 8 projections, respectively.

Simultaneous Algebraic Reconstruction Technique (SART) This technique aims to combine the benefits of ART and SIRT. Therefore, SART provides fast convergence and images with good quality. The main difference is that the projection errors are not propagated evenly, but using a Hamming window, which propagates the error in the middle of the image with bigger weights than in the borders of the image.

The weights are proportional to the distance between the propagated part and the border of the image. For more details, see [5, 49].

Conjugate Gradient Least Squares (CGLS) CGLS is considered a CG-type method. In this case, instead of Eq. (1.4), the following equation should be solved:

$$\mathbf{A}^T \mathbf{A} \mathbf{x} = \mathbf{A}^T \mathbf{b}, \quad \mathbf{A} \in \mathbb{R}^{m \times N}, \quad \mathbf{x} \in \mathbb{R}^N, \quad \mathbf{b} \in \mathbb{R}^m. \quad (1.5)$$

This form ensures that the equation system always has a solution. It has a faster convergence than the SIRT-type methods [100, 109].

1.2 Measurement of the reconstruction quality

The Relative Mean Error (RME) [60] is often used in previous works [115, 116] to measure the reconstruction error when a blueprint (ground truth) image was available for BT cases. RME is the total number of different pixels between the original and the reconstructed binary images divided by the number of object pixels in the original image, which is defined as:

$$RME(\mathbf{x}^*, \mathbf{x}^S) = \frac{\sum_i |x_i^* - x_i^S|}{\sum_i x_i^*}, \quad (1.6)$$

where $\mathbf{x}^*$ is the original (blueprint) and $\mathbf{x}^S$ is the reconstructed image from angle set S . Values of 0 will correspond to perfect reconstructions, and higher values denote a more significant error. RME is a widely accepted formula for measuring the difference between binary images. We use RME in Chapters 2, 3, and 5.

In the case of (non-binary) CT images, the quality of the reconstructions can be evaluated by the Signal-to-noise ratio (SNR). SNR is sometimes quantified in decibels (dB) of signal power relative to the noise power, though in the imaging field the concept of "power" is sometimes taken to be the intensity of a certain pixel and the noise could be equivalent to the standard deviation. Therefore, the SNR is given as the ratio of the mean gray value inside the object μ_o , and its standard deviation σ_o :

$$SNR(\mathbf{x}) = \frac{\mu_o}{\sigma_o}. \quad (1.7)$$

When the noise is high, the SNR is lower. Thus, the higher the value is, the better the quality. This measurement method will be used in Chapter 4.

Chapter 2

Offline Projection Selection Algorithms and their Scale Invariance

2.1 Introduction

Ideally, in the noiseless case, the exact reconstruction needs at least $\frac{m\Pi}{\sqrt{2}}$ projections, when the number of detectors is m . Taking so many projections, however, is not always possible (e.g., missing wedge problem [83], dynamic tomography [16]) or can cause unwanted effects. Furthermore, we often cannot take more than a few (say, up to 18) projections in NDT, due to physical and/or time considerations. This brings new approaches of reconstruction alive ensuring accurate results from fewer projections. For example, in BT, we use the prior information that the object consists of a single material, and the gaps are filled with air. In some cases, this constraint can lead to accurate results, even from just 4–8 projections.

In the paper [78], the authors noted that when using a low number of projections, the accuracy of a binary reconstruction can depend on the projection angles. Further investigated in [115], this phenomenon was found to depend on the structure of the object of study. Thus, further attention was paid on how the “most informative” projections can be found. Projection selection methods can be classified as online and offline ones. In this chapter, we will focus on the latter.

In the offline case, a blueprint image of the object to be reconstructed is given (which is rather typical, e.g., in industrial NDT), on which the whole projection data can be simulated, and the proper projection set can be identified. In this context, the projections are not always gathered by real physical measurements. They can be simulated using a golden standard (ground-truth or blueprint) that is often available in industrial quality testing, e.g., as a CAD model. The authors of [92] used a CAD-projector [72] to simulate the projection data from the CAD model to apply equiangular view angle selection for CT. Even though this is easier than finding the angles on-the-fly, the excessive search space of the possible projection sets makes the problem computationally challenging, as an exhaustive search is often not feasible. In [116], several approaches have been published aiming to find good projection angles for BT, all of them reducing the search space of projection sets in various ways.

In this chapter, we suggest sequential search methods [94] to find projection angles with high information content and show that these methods outperform former practices by providing better image quality. First, we work with images in their original sizes. However, dealing with lower resolution images in order to achieve higher resolution images of good quality is a general trick that is also used in DT (see [4] and its references). Therefore, as a next step we show that in BT the resolution of

the processed image can be lowered for projection selection purposes as well, and it will still provide almost as good angle sets as the original sized image does. This phenomenon opens doors to different areas as an opportunity for optimization. We also propose an approach to predict the lowest resolution that is still as informative as the original one, concerning projection selection.

2.2 Projection selection algorithms

In this section, we first recall three projection selection strategies from [116] that will serve as references for comparison with our suggested methods. The approaches given in [116] are the best performing state-of-the-art methods in this field. Then, we present our sequential search based approaches.

2.2.1 Equiangular approaches

One of the simplest methods is when the projection angles are distributed proportionally, with equiangular spacing. One can distinguish two variants. In the first case, the starting angle is positioned to 0° . As an improved version, in the second case, all the integer starting angles are analyzed between 0° and $\lceil \frac{180}{r} \rceil^\circ$, where r is the number of projections. In the end, the angle set ensuring the best quality (with the smallest RME value) is kept. The authors of [116] referred to these methods as *Naive* and *EquiAng* angle selections, respectively, and so do we in this thesis.

2.2.2 Simulated annealing

When arbitrary angles can serve as candidates for taking projections from, the problem of projection selection becomes intractable. Thus, an exhaustive search is most often no longer possible. To handle this issue, in [116], the authors presented the problem as an energy minimization task. An optimal set of angles must satisfy

$$RME(\mathbf{x}^*, \mathbf{x}^{S^*}) = \min_S RME(\mathbf{x}^*, \mathbf{x}^S), \quad (2.1)$$

where S is an arbitrary set of angles, S^* is the optimal set of angles, $\mathbf{x}^S$ and $\mathbf{x}^{S^*}$ are the reconstructions from the projections with S and S^* angle sets, respectively.

Simulated annealing [73] was proposed for the minimization. In a brief description, the algorithm starts with a basic fixed-size angle set. In each iteration, a randomly chosen angle is altered. If this update results in a better angle set, it will be accepted. Otherwise, it can be only accepted with a certain probability which is constantly decreasing during the process. We refer to this method as *SA*. This approach is stochastic, i.e., every run may produce different output angle sets.

2.2.3 Proposed sequential search methods

By the observation that projections can serve as features of the image to reconstruct, we examined numerous types of feature selection algorithms, following the summary of [74]. According to this examination, the floating search methods [94] seemed to be the best options in this area. Other algorithms contain too much randomization, use exhaustive search, or tree structure which makes them hardly applicable for this task. In the following, we propose two different projection selection algorithms using sequential selection methods, and a floating search based algorithm, which can serve as a post-processing step for any types of projection selection methods.

SFS - Sequential Forward Selection The process starts with the initialization of a feature subset. Then it iteratively adds features to the initial subset. The feature to be added is chosen based on the improvement it gives to a specified measure. A particular feature is added if it provides the biggest improvement among all the available features. The process stops when no improvement happens in the evaluation measure in the last t steps, where t is a predefined threshold variable, or when all the features have been added to the set. In Alg. 1, one can see the *SFS* algorithm for our purpose. At the end of the process, the suggested angle set is stored in set S .

Algorithm 1: Angle selection with *SFS* algorithm

```

1: let  $S = \{\theta_1, \theta_2\}$  where  $\theta_1$  and  $\theta_2$  are randomly chosen integer angles
2: repeat
3:   for all angle  $\theta \in \{0, 1, \dots, 179\} \setminus S$  do
4:     calculate  $RME(\mathbf{x}^*, \mathbf{x}^{S \cup \{\theta\}})$ 
5:      $\theta_{min} \leftarrow$  angle corresponding to the smallest  $RME$  value
6:   end for
7:    $S \leftarrow S \cup \{\theta_{min}\}$ 
8: until the predefined number of projections is reached

```

SBS - Sequential Backward Selection *SBS* is the backward counterpart of *SFS*. The initial feature set contains all the features. Then, iteratively, one feature will be removed from the set, namely whose deletion causes the least loss in the evaluation measure. The process also stops when no improvement happens in the evaluation measure in the last t steps, where t is a predefined threshold variable, or all the necessary number of features has been deleted from the set. In Alg. 2, one can see the *SBS* algorithm for our purpose. At the end of the process, the suggested angle set is stored in set S .

Algorithm 2: Angle selection with *SBS* algorithm

```

1: let  $S = \{0, 1, \dots, 179\}$ 
2: repeat
3:   for all angle  $\theta \in S$  do
4:     calculate  $RME(\mathbf{x}^*, \mathbf{x}^{S \setminus \{\theta\}})$ 
5:      $\theta_{min} \leftarrow$  angle corresponding to the smallest  $RME$  value
6:   end for
7:    $S \leftarrow S \setminus \{\theta_{min}\}$ 
8: until the predefined number of projections is reached

```

SFFS - Sequential Forward Floating Search Both *SFS* and *SBS* suffer from a so-called “nesting effect”. It means that the features cannot be later discarded once selected by *SFS*. In *SBS*, the discarded features cannot be reselected. The result is that the methods are only suboptimal. The *SFFS* is a bottom-up search procedure. It starts with an *SFS* step. If the desired size of the solution is not reached yet, an *SBS* step can be applied to exclude features. A feature will be permanently excluded if this action yields the best subset found so far, otherwise the excluded feature is put back and the process continues again with the *SFS* step. This process iterates until the desired size of the feature set is reached.

SBFS - Sequential Backward Floating Search The *SBFS* is the backward counterpart of *SFFS*, i.e., it is a top-down search procedure. The basic idea is the same as it is in the case of *SFFS*, but the *SFS* and *SBS* steps are swapped.

The original *SFFS* and *SBFS* methods both contain a deviation constant, which allows the algorithm to exceed the desired size of the solution during the searching process and even to end with more or fewer features. In our case, when these algorithms are used for projection selection, we do not allow the algorithm to finish with more or fewer projections than the predefined number.

Based on the idea of *SFFS* and *SBFS*, we propose a *Refinement* algorithm. In our interpretation, the algorithm starts with an initial angle set, which can be the output of any type of projection selection algorithm, and then we apply the *Refinement*. Depending on which algorithm is used as the first step, we refer to these algorithms as *NaiveR*, *EquiAngR*, *SAR*, *SFSR*, and *SBSR*, i.e., the original name is extended with the letter *R* (*R* is for *Refinement*). The *Refinement* algorithm can be seen in Alg. 3, which is capable of refining the result of the output of any other algorithm. Therefore, we will show later on, how the already published angle selection methods can be improved by applying *Refinement* on their output angle sets, besides *SFS* and *SBS*.

Algorithm 3: Angle selection with *Refinement*

```

1: let  $S$  be the set of the actual angles - output of an angle selection algorithm
2:  $\text{fix}\Theta \leftarrow$  the last element inserted into  $S$ 
3: repeat
4:   for all angle  $\theta \in S \setminus \{\text{fix}\Theta\}$  do
5:     calculate  $\text{RME}(\mathbf{x}^*, \mathbf{x}^{S \setminus \{\theta\}})$ 
6:      $\theta_{\min} \leftarrow$  angle corresponding to the smallest  $\text{RME}$  value
7:   end for
8:    $S \leftarrow S \setminus \{\theta_{\min}\}$ 
9:   for  $\theta \leftarrow 0$  to  $179$  do
10:    calculate  $\text{RME}(\mathbf{x}^*, \mathbf{x}^{S \cup \{\theta\}})$ 
11:     $\theta_{\min} \leftarrow$  angle corresponding to the smallest  $\text{RME}$  value
12:   end for
13:    $S \leftarrow S \cup \{\theta_{\min}\}$ 
14:    $\text{fix}\Theta \leftarrow \theta_{\min}$ 
15: until the  $\text{RME}$  cannot be decreased any more.

```

As a brief explanation of Alg. 3: The last element inserted to the projection set will be fixed (Line 2). Then, the non-fixed projection angles are eliminated one-by-one, and the RME is computed (Line 4-7). We keep the angle combination with the smallest RME (Line 8). In the second part, again, all the integer angles between 0° and 179° are taken, and the RME value is computed by adding each of them to the current set of projections (Line 9-12). We insert the angle with the smallest RME value into the projection set (Line 13) and fix it (Line 14). Lines 4-14 are repeated until the RME value cannot be decreased anymore. We note that we used integer angles between 0° and 179° , but it can be changed to any angle interval. The sampling could be more dense or even sparser.

2.3 Scale Invariance in Projection Selection

In this section, we discuss two different approaches how scale invariance could be used before or during projection selection.

2.3.1 Technique (T1): Resizing the image

In industrial NDT, often hundreds or thousands of projections of the examined object are acquired, and the number of detector pixels can also be of the same magnitude. This allows us to reconstruct a high-quality image from its projections. However, if we want to apply a projection selection method on the reconstructed image, it could take an unacceptably long time due to the high resolution. To speed up this process, one can downscale the reconstructed image and then apply the projection selection algorithm just after this step.

By reducing the resolution, one may lose information, and the object in the image may deform due to structural and topological changes. For projection selection, however, we do not need that much and detailed projection information. If the image preserves the most specific structural information about its content, it is completely enough to find the most informative angles.

For downscaling, we used the Gaussian pyramid [17]. In Fig. 2.1a, G_0 is the original image. To get G_1 , a low-pass filtering is applied on G_0 . G_1 is called the “reduced” version of G_0 in that both resolution and sample density are decreased. In a similar way, G_2 is also the reduced version of G_1 . The sequence of $G_0, G_1, \dots, G_L$ is called the Gaussian pyramid, where $L + 1$ is the number of levels in the pyramid. Each value in level l is computed as a weighted average of values in level $l - 1$ within a 5×5 window:

$$G_l(i, j) = \sum_{p=-2}^2 \sum_{r=-2}^2 w(p, r) G_{l-1}(2i + p, 2j + r), \quad i = 0, \dots, C_l - 1, \quad j = 0, \dots, R_l - 1, \quad (2.2)$$

$$w(p, r) = \hat{w}(p) \hat{w}(r), \quad (2.3)$$

$$\hat{w} = \left[\frac{1}{4} - \frac{a}{2}, \frac{1}{4}, a, \frac{1}{4}, \frac{1}{4} - \frac{a}{2} \right], \quad (2.4)$$

where $l = 1, \dots, L$ is the actual level, C_l is the number of columns, and R_l is the number of rows on the l th level, $\hat{w}$ is the used kernel with $a = 0.375$. For more details about the Gaussian pyramid and the kernel see [17].

After reducing an image, the pixels hold grayscale values. We want to deal with binary images, so before applying any projection selection, we binarize the reduced images as the last step of the downscaling process, by a simple threshold $T = 0.5$.

2.3.2 Technique (T2): Summing the projection values

Another possible technique preserves the original size of the blueprint image and takes the normalized sum of every d neighboring projection value, where d will specify the magnitude of reduction:

$$D_l(i) = \frac{\sum_{j=d \cdot i+1}^{j < d \cdot i+d} D_{l-1}(j)}{d^2}, \quad i = 0, \dots, m_l - 1. \quad (2.5)$$

Here D_l represents the new projection vector and m_l is the number of the detectors on the l th level. For example, if we sum every $2nd$ neighboring projection value, we

halve the size of the projection vector and thus also the size of the sinogram. This approach can be followed in Fig. 2.1b. D_0 is the original projection vector, D_1 and D_2 are the reduced ones. We emphasize that during the backprojection, the resolution of the images is halved, therefore d^2 pixels of the original image correspond to 1 reconstructed pixel. Thus, the denominator in Eq. (2.5) is d^2 . In Fig. 2.2, one can see the approximate difference between using d and d^2 as the denominator in Eq. (2.5) (having a binary image with a disk in it). For SIRT, our goal is to produce pixel values between 0 and 1.

FIGURE 2.1: (a) Resizing the image. (b) Summing the projection values.

e.g., the RME, topological and local properties [36], moments [93], Universal Image Quality Index [121], and convexity [6] to measure the similarity in the abovementioned sense, but none of them gave reliable results. In [122], the author proposed a measure named perimetric complexity to describe the complexity of a binary digital image, which is defined as

$$PC(\mathbf{x}) = \frac{P^2}{4\pi F}, \quad (2.6)$$

where P is the sum of the inside and outside perimeters of the foreground (the unit is the length of one pixel side) and F is the area of the foreground. Eq. (2.6) serves to calculate the perimetric complexity of one object. If there are more objects in the image, their complexity values can be summed to get the complexity of the whole image. Using this measure, we can determine which is the lowest “safe” resolution where we have to stop the downscaling.

Perimetric complexity is also useful to predict how many projections one will need for a reconstruction to keep the RME value below a specified threshold. In certain applications, the 3D models of the objects are available (e.g., as a CAD file [92]). In these cases, one can tell how many projections are needed for an appropriate reconstruction even before the real projection acquisition, by simulating the projections on the model.

2.4 Test environment

We conducted several experiments on binary software phantom images. It was shown in [115] that with less projections, the result of projection selection does not significantly depend on the reconstruction technique applied. That is, the reconstruction algorithm will not influence the selected projection set seriously. We chose to use the TSIRT algebraic reconstruction technique. Our dataset consisted of 22 structurally and topologically different phantoms (see Fig. 2.3). Some phantoms were used for testing reconstruction algorithms in previous studies [7, 123], and some come from the 2D image database of the IAPR Technical Committee on Discrete Geometry (TC18)¹. We used parallel beam geometry. The distance between the beams and the detectors was set to 1 for every projection, and the number of the beams was $\lceil n_l \cdot \sqrt{2} \rceil$ to cover the whole image, where $n_l \times n_l$ is the size of the image on the l th level, in the case of testing scale invariance. The distance between the consecutive beams as well as between the consecutive detectors was equal to the length of the side of one pixel. The center of rotation was positioned to the center of the image.

For the comparison of the different projection selection algorithms, we used the software phantoms with a size of 256×256 pixels. Due to their stochastic nature, in the case of SA, SAR, SFS, and SFSR, we used the *multistart strategy* in different ways to find the best possible results. We applied exactly the same parameterization for SA that was used in [116], i.e., the random angle changes stopped after 200 iterations or when the RME decreased to zero. Then, we ran the process 5 times and chose the projection set with the lowest RME value. For SAR, the *Refinement* step was applied on all the 5 SA outputs, and again, the one with the smallest RME was chosen. In the case of SFS, we ran the process 18 times from different random integer angle pairs in order to try to avoid getting stuck into a local minimum. In this way, assuming uniform random distribution, each angle region of 5° can be covered with a high probability by at least one angle in at least one initialization. We used

¹<http://tc18.org/>

couples of angles instead of just a single starting angle since – after experiments – we noticed that in the latter case, the process could still very likely stick into a local minimum. This is the only non-deterministic part of the sequential procedures. In the followings, some slight differences may be noticeable between the RME values when one of these algorithms is used in various occasions. This is exclusively due to this property. *Refinement* was also applied on all the 18 different output angle sets to produce the result of *SFSR*.

Although it is already known (and it will be shown in Section 2.5 as well) that the methods selecting the projection angles arbitrarily may easily outperform the equiangular ones [116], we wanted to test our scale invariance theory with the *Equi-Ang* algorithm too, since this latter technique is still frequently used. For this reason, we used the same squared phantom images of Fig. 2.3 but with a side length of 16, 32, 64, 128, 256, 1024, and 2048 pixels. The first step was to select the most informative angles for the highest resolution, 2048×2048 sized phantoms. We considered these images as originals. With this, we calculated the best RME value we could obtain. We refer to this RME value as our benchmark. The next step was to perform the projection selection with the reduced images. After the angle sets were identified, we reconstructed the original sized images one more time, now with these angle sets. As the last step, we compared the obtained RME values with the benchmark. Then, to test this theory in the case of non-equangular approaches also, we repeated the experiment with the *SFSR* algorithm. This time we used phantoms of size up to 256×256 (due to time considerations), and again the highest resolution images were regarded as originals.

The TSIRT and the projection selection algorithms with their scale invariance extensions were implemented in C++, using the CUDA sdk², with GPU acceleration. The tests were performed on a machine powered with 4 NVIDIA Tesla K10 G2 8GB GPUs. According to our experiments, with a difference smaller than 0.01 in two consecutive iterations, the changes in quality are negligible. Therefore, we set this as the terminal condition the TSIRT.

2.5 Results

In Table 2.1, we present the RME values of the different projection selection methods, in the case of projection sets with 4 angles. The smallest RME values are highlighted in every row. One can notice that the *SBS* and *SBSR* are missing, that is because these algorithms provided the poorest reconstruction quality and the longest running time. Therefore, there was no reason to compare *SBS* and *SBSR* to the other methods from any point of view. It can be observed that in 18 out of 22 cases the *SFSR* performed the best or has the same RME value as some other algorithms. It is also worth to be noticed, that in the case of the rotation independent *P2*, even the *Naive* method could find the best solution, as for this image all the projections have the same information content. One can also see that although *SFSR* has the ultimate smallest average in RME, all the other algorithms with *Refinement* (*NaiveR*, *EquiAngR*, *SAR*) are really close to that result. The differences are almost negligible. It can be declared that in the case of all the different methods, *Refinement* improved the result of the input algorithm, especially by taking the equiangular approaches under examination.

²<https://www.developer.nvidia.com/cuda-zone>

FIGURE 2.3: The software phantoms used for testing.

TABLE 2.1: RME values belonging to different methods in case of 4 projections

	<i>Naive</i>	<i>NaiveR</i>	<i>EquiAng</i>	<i>EquiAngR</i>	<i>SA</i>	<i>SAR</i>	<i>SFS</i>	<i>SFSR</i>
P1	0.269	0.101	0.135	0.078	0.059	0.055	0.060	0.056
P2	0.067	0.067	0.067	0.067	0.067	0.067	0.249	0.067
P3	0.041	0.036	0.041	0.036	0.039	0.036	0.043	0.036
P4	0.237	0.072	0.152	0.074	0.079	0.072	0.067	0.062
P5	0.398	0.129	0.184	0.097	0.097	0.091	0.104	0.091
P6	0.371	0.065	0.191	0.089	0.068	0.066	0.067	0.067
P7	0.108	0.086	0.106	0.090	0.074	0.074	0.081	0.075
P8	0.160	0.120	0.154	0.147	0.137	0.119	0.123	0.119
P9	0.206	0.067	0.191	0.073	0.061	0.045	0.051	0.042
P10	0.073	0.044	0.073	0.044	0.031	0.031	0.034	0.031
P11	0.357	0.092	0.284	0.157	0.094	0.090	0.113	0.089
P12	0.008	0.008	0.008	0.008	0.011	0.008	0.010	0.008
P13	0.089	0.058	0.089	0.058	0.055	0.055	0.063	0.058
P14	0.446	0.406	0.446	0.406	0.423	0.405	0.405	0.405
P15	0.481	0.419	0.454	0.378	0.404	0.354	0.353	0.350
P16	0.516	0.478	0.516	0.478	0.483	0.478	0.483	0.478
P17	0.879	0.869	0.879	0.869	0.887	0.867	0.874	0.866
P18	0.472	0.433	0.472	0.433	0.446	0.413	0.425	0.411
P19	0.092	0.059	0.092	0.059	0.061	0.058	0.060	0.051
P20	0.105	0.036	0.105	0.036	0.037	0.037	0.081	0.036
P21	0.344	0.138	0.344	0.138	0.136	0.134	0.159	0.134
P22	0.524	0.487	0.524	0.487	0.455	0.447	0.477	0.438
Avg.	0.284	0.194	0.250	0.196	0.191	0.182	0.199	0.180

For the next type of comparison, in Table 2.2, one can follow the running times of the different methods. We notice that the original (previously published) equiangular methods are fast but provide really weak quality reconstructions. All the *Refinement*-extended algorithms performed well, but the running times are very variable. We emphasize that in the case of *SA* and *SAR*, we took 5 (as the authors of the original paper did), and in the case of *SFS* and *SFSR* 18 multistart runnings. This means that one single *SA* and *SAR* run is about 1/5 times shorter, and one single *SFS* and *SFSR* run is about 1/18 times faster than the corresponding values in Table 2.2. These scaled running times can be seen in the last row of Table 2.2. However, in most of the cases, these methods need the above detailed multistart strategy to reach the reconstruction accuracy shown in Table 2.1. As we mentioned, examining the RME values and the reconstructed images belonging to them, the difference is almost negligible when we talk about comparing the projection selection methods extended with *Refinement*. The running time of *NaiveR* and *EquiAngR* is still very fast. Furthermore, these algorithms are completely deterministic, while the results of *SA*, *SAR*, *SFS*, and *SFSR* really depend on the random initialization.

We summarize the results of Table 2.1 and Table 2.2 in Fig. 2.4. The RME values and running times are proportionally normalized between 0 and 1, for better visualization. One can draw the conclusion that applying the *Refinement* step, *NaiveR* and *EquiAngR* become very competitive against the other algorithms, considering their good quality and fast execution time.

TABLE 2.2: Running time in minutes belonging to different methods in case of 4 projections

	<i>Naive</i>	<i>NaiveR</i>	<i>EquiAng</i>	<i>EquiAngR</i>	<i>SA</i>	<i>SAR</i>	<i>SFS</i>	<i>SFSR</i>
P1	0.010	2.72	0.32	4.26	7.52	24.80	40.70	128.12
P2	0.012	1.10	0.25	1.29	6.77	12.14	33.15	96.87
P3	0.010	3.39	0.26	3.79	10.13	27.88	47.75	164.63
P4	0.006	3.51	0.31	4.87	7.29	26.70	39.45	116.18
P5	0.007	2.95	0.32	4.80	6.56	16.97	35.78	120.07
P6	0.004	4.95	0.23	6.40	7.49	23.54	40.02	139.15
P7	0.009	6.24	0.40	3.09	7.37	30.02	42.18	123.42
P8	0.006	6.38	0.22	3.01	6.63	19.69	37.17	117.98
P9	0.006	8.28	0.26	6.57	7.88	25.43	45.60	130.48
P10	0.008	8.03	0.28	8.07	8.16	22.74	45.35	140.18
P11	0.004	6.69	0.25	4.00	7.62	16.70	38.30	137.88
P12	0.006	1.26	0.36	1.57	7.27	29.32	50.32	116.80
P13	0.007	2.57	0.22	2.82	7.82	27.85	44.17	117.15
P14	0.005	3.26	0.17	3.59	5.43	18.32	29.45	89.85
P15	0.005	2.90	0.24	1.79	4.84	13.81	23.45	73.48
P16	0.004	3.29	0.19	3.61	5.64	17.30	31.08	92.13
P17	0.003	2.38	0.09	2.50	3.96	12.84	25.57	76.60
P18	0.005	6.32	0.16	6.74	5.19	19.20	28.03	94.18
P19	0.009	2.71	0.25	2.87	8.55	27.59	44.65	144.22
P20	0.008	6.86	0.13	6.94	6.75	22.36	23.48	119.75
P21	0.006	5.25	0.21	5.43	7.78	30.64	39.85	138.60
P22	0.007	4.09	0.27	4.15	6.12	19.22	30.98	104.65
Avg.	0.007	4.32	0.24	4.19	6.95	22.05	37.11	117.38
Avg.	0.007	4.32	0.24	4.19	1.39	4.41	2.06	6.52

FIGURE 2.4: Diagram for Table 2.1 and Table 2.2

TABLE 2.3: RME values belonging to different methods in case of different number of projections

	<i>Naive</i>	<i>NaiveR</i>	<i>EquiAng</i>	<i>EquiAngR</i>	<i>SA</i>	<i>SAR</i>	<i>SFS</i>	<i>SFSR</i>
4	0.357	0.092	0.284	0.157	0.094	0.090	0.113	0.089
6	0.051	0.040	0.047	0.035	0.035	0.032	0.038	0.032
9	0.031	0.017	0.020	0.014	0.014	0.012	0.014	0.013
12	0.012	0.007	0.010	0.005	0.008	0.007	0.007	0.006
15	0.008	0.002	0.005	0.003	0.005	0.002	0.003	0.002

TABLE 2.4: Running time in minutes belonging to different methods in case of different number of projections

	<i>Naive</i>	<i>NaiveR</i>	<i>EquiAng</i>	<i>EquiAngR</i>	<i>SA</i>	<i>SAR</i>	<i>SFS</i>	<i>SFSR</i>
4	0.004	6.69	0.25	4.00	7.62	16.70	38.30	137.88
6	0.010	4.38	0.29	4.73	9.37	17.87	96.60	183.80
9	0.012	6.05	0.19	7.22	8.68	21.90	174.72	241.17
12	0.011	7.44	0.14	13.01	8.38	19.64	251.25	315.43
15	0.011	8.79	0.12	10.24	8.76	41.17	331.45	403.62

For further comparison, we tested the algorithms on one of the phantoms (Fig. 2.3P11) with 6, 9, 12, and 15 projections. The RME values and the running times can be seen in Table 2.3 and Table 2.4, respectively. Fig. 2.5 and Fig. 2.6 display the same results in diagrams. If the number of projections increases, the RME values are getting more and more similar. In the case of many projections – based on the RME values – it is less relevant which strategy is chosen. Considering the running times, one can see that in the cases of *Naive*, *NaiveR*, *EquiAng*, *EquiAngR*, *SA*, and *SAR* the running times are slightly changing, while for *SFS* and *SFSR* the running time is significantly increasing with more and more projections. Here, we note again, that the running times for *SA* and *SAR*, and for *SFS* and *SFSR* should be divided by 5 and 18, respectively, to get the running time of one single run.

We mention that the equiangular methods are exhaustive searches in their own forms (in the restricted search space). *SA* is non-deterministic due to the fully randomized changes, so it is driven by chance. *SFS* and *SFSR* are deterministic, except the initialization step, i.e., with a fixed initialization they provide the same output every time.

In the case of analyzing scale invariance, first, we tested the *T1* approach (resizing the image). In Table 2.5, one can see the RME values belonging to the phantom images in case of 4 projections found by *EquiAng*. For better interpretation, the results are also visualized in Fig. 2.7. It is discernible that the RME values of the highest resolution images are the smallest, and these values increased (non-decreased) as the size of the image decreased in almost every case. The RME values, however, are really close to each other. We noticed a significant difference only in the case of size 16×16 . In the table, NaN means that reducing P15 to the lowest level erased all the object pixels, yielding a full black image, thus it had no sense to perform the tests in this case (P15 can be seen in Fig. 2.3P15). We can declare that we could reduce the phantoms radically and still achieve projection sets to reconstruct the images with almost the same quality as in the original case.

FIGURE 2.5: Diagram for Table 2.3

FIGURE 2.6: Diagram for Table 2.4

Similar behavior can be followed in Table 2.6 and in Fig. 2.8, but this time using *SFSR* as the projection selection method. It can be noticed that the RME values are not that close to each other as they were in the equiangular case since the algorithm has more freedom during the selection. Based on Fig. 2.8, it is safer to stop downscaling when the size of 64×64 is reached.

The running times belonging to Table 2.6 can be seen in Fig. 2.9. It is clearly visible that the running time decreases significantly by reducing the size of the phantoms.

We also tested both approaches in case of more than 4 projections. We mentioned that by increasing the number of projections, the RME values are getting more and more similar, independently from the projection directions we used. Thus, in the case of many projections, it is less relevant which selection strategy is chosen. Without details, we observed the same here. In the case of *EquiAng*, by increasing the number of projections it is getting less and less relevant on which resolution we stop. On the other hand, in the case of *SFSR*, even when we work with more projections, it is still important to properly identify which scale level is the lowest that still gives sufficient information.

Furthermore, we also tested theory *T2* in similar experiments as it was detailed above for *T1* and deduced very similar (almost the same) properties. Thus, we do not list more detailed results to avoid information redundancy. Nevertheless, we determine that summing the projection values – i.e., reducing the number of detectors – can be applied as successfully as resizing the blueprint image.

To show how perimetric complexity can predict the degree of the acceptable reduction, in Fig. 2.10 one can see the RME values of the last column of Table 2.6 and the perimetric complexity of the phantoms with the same size, proportionally normalized between 0 and 1, for better visualization. The values corresponding to P16 and P17 are not indicated in Fig. 2.10. These are very complex phantoms (Fig. 2.3P16 and 2.3P17, respectively) with very high perimetric complexity, and it was hard to visualize them together with the other phantoms. Nevertheless, calculating the Pearson correlation shows a very strong positive correlation between the RME values and the perimetric complexity with $r = 0.845$ and – using Fisher’s z -transform to approximate normality – with the confidence interval $[0.634; 0.939]$ at 95%. This correlation phenomenon can be observed on the other scale levels, too.

2.6 Potential applications for exploiting scale invariance

In this section, we present some ideas on how to apply the methods of Section 2.3 in practice.

2.6.1 Speeding up the development of new algorithms

As every research area, projection selection is constantly evolving, which involves the development of new algorithms. If we want to run experimental tests using a new algorithm to validate its functionality, it is desirable to do the tests quickly. In the case of smaller projection or image sizes, the algorithms produce the results faster. Knowing that the reduced variant of the data is appropriate for validation, resizing the images could speed up the testing process and help the developers test the algorithms on a bigger dataset over the same amount of time.

TABLE 2.5: RME values belonging to phantoms with different sizes reconstructed from 4 projections found by *EquiAng*, in case of *T1*.

Phantoms	16	32	64	128	256	512	1024	2048
P1	0.2764	0.0896	0.0785	0.0725	0.0721	0.0721	0.0721	0.0721
P2	0.0481	0.0481	0.0481	0.0481	0.0481	0.0481	0.0481	0.0481
P3	0.0924	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248
P4	0.1808	0.1540	0.1455	0.1455	0.1430	0.1430	0.1430	0.1430
P5	0.1949	0.1729	0.1661	0.1658	0.1658	0.1658	0.1658	0.1658
P6	0.1098	0.1065	0.1020	0.1065	0.1020	0.1020	0.1020	0.1020
P7	0.0864	0.0929	0.0853	0.0857	0.0857	0.0857	0.0857	0.0853
P8	0.1796	0.1503	0.1360	0.1360	0.1360	0.1360	0.1360	0.1360
P9	0.1164	0.0582	0.0582	0.0590	0.0582	0.0582	0.0582	0.0582
P10	0.0803	0.0831	0.0503	0.0509	0.0506	0.0506	0.0459	0.0459
P11	0.2204	0.2078	0.1927	0.1592	0.1564	0.1564	0.1564	0.1564
P12	0.0046	0.0089	0.0088	0.0118	0.0046	0.0046	0.0046	0.0046
P13	0.1256	0.0842	0.0804	0.0842	0.0842	0.0783	0.0783	0.0783
P14	0.5423	0.4813	0.4933	0.4697	0.4697	0.4697	0.4697	0.4697
P15	NaN	0.3560	0.3287	0.3458	0.3424	0.3287	0.3287	0.3287
P16	0.5789	0.5588	0.5554	0.5554	0.5548	0.5548	0.5548	0.5548
P17	0.9619	0.9602	0.9528	0.9537	0.9375	0.9375	0.9375	0.9375
P18	0.4752	0.4805	0.4770	0.4679	0.4679	0.4677	0.4677	0.4677
P19	0.0878	0.1095	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878
P20	0.5647	0.1056	0.1056	0.1056	0.1056	0.1056	0.1056	0.1056
P21	0.3265	0.2641	0.2605	0.2661	0.2605	0.2605	0.2605	0.2605
P22	0.5240	0.5207	0.5082	0.5118	0.5082	0.5082	0.5082	0.5082
Average	0.2751	0.2326	0.2248	0.2234	0.2212	0.2203	0.2201	0.2200

FIGURE 2.7: RME values belonging to phantoms with different sizes reconstructed from 4 projections found by *EquiAng*, in case of *T1*.

TABLE 2.6: RME values belonging to phantoms with different sizes reconstructed from 4 projections found by *SFSR*, in case of *T1*.

Phantoms	16	32	64	128	256
P1	0.1794	0.1020	0.0600	0.0558	0.0539
P2	0.3353	0.1789	0.1128	0.0638	0.0638
P3	0.0463	0.0402	0.0362	0.0303	0.0303
P4	0.2095	0.1012	0.0778	0.0703	0.0637
P5	0.2384	0.1070	0.1194	0.0962	0.0912
P6	0.2255	0.1040	0.0862	0.0751	0.0679
P7	0.1191	0.1324	0.0779	0.0689	0.0659
P8	0.1829	0.1411	0.1441	0.1181	0.1146
P9	0.0802	0.0631	0.0432	0.0373	0.0378
P10	0.1055	0.0422	0.0402	0.0373	0.0251
P11	0.1400	0.1143	0.0982	0.0838	0.0793
P12	0.0194	0.0152	0.0111	0.0096	0.0077
P13	0.0571	0.0642	0.0533	0.0490	0.0474
P14	0.6084	0.4093	0.4076	0.3979	0.3964
P15	NaN	0.6417	0.3990	0.4125	0.3531
P16	0.5380	0.4977	0.4849	0.4876	0.4694
P17	0.9201	0.8971	0.8906	0.8621	0.8607
P18	0.5022	0.4289	0.3933	0.3839	0.3782
P19	0.0790	0.0596	0.0567	0.0482	0.0474
P20	0.5214	0.1564	0.0747	0.0372	0.0362
P21	0.1304	0.1350	0.1302	0.1308	0.1259
P22	0.6761	0.5173	0.5003	0.4646	0.4451
Average	0.2816	0.2249	0.1954	0.1827	0.1755

FIGURE 2.8: RME values belonging to phantoms with different sizes reconstructed from 4 projections found by *SFSR*, in case of *T1*.

FIGURE 2.9: Running time belonging to phantoms with different sizes, in case of $T1$. Projection selection was performed by *SFSR* with 4 projections.

FIGURE 2.10: Correlation between RME and perimetric complexity. Each point represents one phantom image.

2.6.2 Reducing the number of beam rays

Reducing the dose of the radiation during measurements is a challenging problem. In their paper [120], the authors reported the risks of medical X-rays and other low-dose radiation, while the authors of [51] proposed several strategies for CT radiation dose optimization. Based on the fact that for projection selection, we do not need a high-quality projection while it contains the main characteristics of the object. Thus, we can reduce the radiation by reducing the number of projection beams. We propose a two-step investigation process to use less radiation: first, reconstruct a lower resolution image from coarse sinograms, then – by finding the most informative projection angles – repeat the acquisition with finer sampling, but using just the informative directions.

2.6.3 Service for companies

If a company or a factory has to deal with series production and they want to apply NDT on their products, they often have to invest into a CT scanner to manage all the quality assurance steps on their own. It is not easy to be answered what type of CT scanner they would need. There are several attributes and parameters they have to consider before the purchase. A remunerative approach would be to take one piece of their product to a place equipped with a high-quality CT scanner and do the acquisition process with the highest possible resolution. After this step, the companies could send us the data, and we can provide information about the smallest resolution they need for the analysis of their product.

2.7 Summary

First, we presented sequential feature selection strategies for projection selection in BT and compared them to previously published methods. Using sequential search approaches, we proposed a *Refinement* method that can improve the resulted angle set of any type of projection selection algorithm. This way, we showed that the *SFSR* algorithm provides the ultimate best result, but all the other *Refinement*-extended algorithms can almost reach the same result. We pointed out that even a *Naive* or *EquiAng* algorithm can provide angle sets with acceptable quality when are extended with our *Refinement* method, and in a very short time. Thus, we succeeded to improve the RME values of previously published methods using *Refinement*. Moreover, considering the RME values and the running times combined, the *NaiveR* and *EquiAngR* can beat all the other algorithms in most of the cases, and in the same time, they preserve their fully deterministic nature.

Based on the experimental results provided by the different projection selection approaches, we suggested two techniques to reduce the amount of data needed to select the most informative projection angles for binary tomographic reconstructions. We showed by experiments that for projection selection purposes, the images could be downscaled to a certain scale level and the number of beam rays can also be lowered for the projection acquisition. We also showed that perimetric complexity is a good measure to automatize the reduction process and to choose the proper number of projections. The results can be useful in developing and testing new projection selection methods and in NDT.

As a final remark, we note that – as it is well-known – the spacing of projection lines can also have an impact on the reconstruction quality. However, our aim was to investigate the effect of changing the resolution (either of the image or the detector),

from the viewpoint of projection selection. The effect of using sparse projection rays (undersampling) in projection selection is a different issue.

In this chapter, the author regards the following as his main contributions:

- Adaptation of the sequential selection and search approaches into projection selection methods.
- Implementation of the methods.
- Testing and comparing them to already published algorithms.
- Suggesting two approaches to reduce the amount of data needed to select the most informative projection angles.
- Developing and testing these approaches in projection selection.
- Evaluation of the results.
- Proposing research fields where the scale invariance of projection selection could be used.

The findings of this chapter have been published in a conference proceedings [64] and in a journal paper [63].

Chapter 3

Reconstruction Uncertainty in BT and CT

3.1 Introduction

In an ideal case, the reconstruction is performed using a large number of projections. This, however, is not always possible. As we detailed in *Chapter 2*, the goal of the projection selection is to find the most informative angles, which can be done either online or offline. While the previous chapter focused only on the latter, in this chapter, we analyze in detail the former case, too. One of the main differences between these two approaches is that in the offline case all the sampled angles and the projections taken from them are known in advance (typically, when a blueprint image is available, see [116] and *Chapter 2*). In contrast, online approaches determine the successive angles during the acquisition, based on the already available projection information. Thus, the number of projections to produce is not known in advance, although, depending on the projection selection algorithm and its parameterization, an upper bound can be specified. Adaptive (online) projection selection algorithms allow performing dense sampling in the information-rich, and sparse sampling in the information-poor areas. In [38], Haque et al. proposed projection selection methods based on reconstruction error and spectral richness, and have shown an example where they used entropy-based uncertainty. The authors of [91] used the structural similarity index measure [128] to estimate the sample rotation using a CAD projector [72] toolbox. Dabravolski et al. [21] proposed a dynamic algorithm, in which new projection angles are selected by maximizing the information gain about the object, given the set of possible new angles. These previously mentioned papers provided methods for CT, but more efficient methods could be developed for BT if we directly focus only on the binary cases. The algorithm given in [9] is based on the same information gain idea as [21], but with the motivation of BT. The authors of [9] gave a heuristic approach for in-situ projection angle selection that chooses the next projection direction based on metrics calculated for a previous projection set [8].

Even with the properly selected acquisition angles, the low number of projections turns the reconstruction task into an ill-posed problem, having various possible solutions. This lack of information produces errors in the reconstructions because we cannot determine the exact pixel values.

Two research groups simultaneously investigated the theory of pixel uncertainty in discrete reconstructed images. They introduced two different approaches for measuring the variability of reconstructions [28, 119]. While [119] could be used in binary cases, [28] presents a way to apply uncertainty in DT. Unfortunately, these previous measures do not consider the uncertainty of a reconstructed image in continuous CT.

This field has a wide range of applications. It was shown that entropy-based uncertainty could be efficiently used in projection selection [38, 85]. Paper [27] presents an improved pixel update strategy by introducing a probability map that measures the classification accuracy of each pixel based on its grey value evolution throughout the iterations, which can also be applied in case of DT, but it still does not give information about the uncertainty of the projection sets and their geometry.

Although several different types of uncertainty descriptors have been invented during the last decades, they mostly focus on DT or the direct application of descriptors. The reconstruction uncertainty of the projection geometry and the variability of pixels on grayscale reconstructions – to the best of our knowledge – has never been discussed yet. Such tools could be used to analyze complex CT sinograms and reconstructed CT slices to tell how stable or accurate the reconstructions are. One of the main fields for this could be the limited angle tomography when the data are restricted to lines in a limited angular range. Typical examples of modalities where such problems arise are digital breast tomosynthesis [81, 95], dental tomography [43, 76], or electron microscopy [22].

In the first part of this chapter, we provide two projection selection methods for binary reconstruction that use the reconstruction uncertainty [119] of the available projections to choose the next angle. The first one is an offline method, which modifies the *SFSR* algorithm presented in Section 2.2.3, and the second one is an adaptive method based on [38]. Both methods select the projection angles solely using the projection data and do not use any blueprint. Compared to the paper [119], the main novelty here is that uncertainty is not directly used to improve the quality of the reconstruction, but to choose the most informative projection directions. In the second part of this chapter, we define a grayscale uncertainty measure and examine its behavior with different projection sets. First, we provide a measure for the variability of the reconstructed pixels based exclusively on the projection geometry. Then, we use this variability measure to give theoretically provable upper and lower bounds for the pixel values in the reconstructions. Finally, we show that our method can directly be applied for calculating the difference between the original object of study (ground truth image) and its reconstructions, and how it could be used to predict the reconstruction error that may arise during the reconstruction. Experimental validations are also given.

3.2 Binary uncertainty in projection selection

3.2.1 Reconstruction uncertainty

We can determine the uncertainty of the reconstruction using the results of [119] that we shortly recall here. Due to the incomplete information in the projection data, there can be several solutions of Eq. (1.4) in which the pixel values may vary. Knowing all the reconstructions we could calculate the probability of x_j ($j = 1, \dots, N$) taking the value $x_j = 1$, by

$$p_j = \frac{\mathcal{N}_b^A(x_j = 1)}{\mathcal{N}_b^A}, \quad (3.1)$$

where $\mathcal{N}_b^A$ denotes the number of solutions of Eq. (1.4) and $\mathcal{N}_b^A(x_j = 1)$ stands for the number of binary solutions with $x_j = 1$. Given the probabilities, we can determine the uncertainty of pixel x_j as

$$\mathcal{H}(x_j) = -(p_j \log_2(p_j) + (\bar{p}_j) \log_2(\bar{p}_j)), \quad (3.2)$$

FIGURE 3.1: A binary image (left) and the entropy map of its reconstruction (right).

where $\bar{p}_j = 1 - p_j$. Taking an image and indicating all the pixels with their uncertainty values, we get the *entropy map* of the reconstruction (Fig. 3.1). Furthermore, we can also calculate the global uncertainty of the whole reconstruction by summing the pixel uncertainties as

$$\mathcal{U}(\mathbf{x}) = \frac{\sum_{i=j}^N \mathcal{H}(x_j)}{\frac{1}{r} \sum_{i=1}^m b_i}, \quad (3.3)$$

where r is the number of projections. Of course, reconstruction uncertainty (both on pixel and global level) depends on the reconstruction algorithm and the number of projections applied.

Calculating Eq. (3.3) needs an unreasonable amount of processing time in practice since all the reconstructions satisfying the projection set have to be generated. Here, we will use the deterministic method proposed in [119] to approximate pixel uncertainties. This method is based on the SIRT algorithm and forces the range of the pixel intensities of the reconstructed image to be in the $[0, 1]$ interval. As an initial guess it uses an image matrix filled with 0.5 values which enables the reconstruction to converge to the “most uncertain” solution. To produce the entropy map of the solution, we substitute the probability variables in Eq. (3.2) with the pixel values of the reconstructed image. In [119], it was shown that this procedure is a good approximation of the theoretical uncertainty.

3.2.2 State-of-the-art methods

The first algorithm (*SFSR*) will form the basis for our offline approach and will be used for comparison. The subsequent three methods will be used for the comparison with our online method, and the second of them (Reconstruction error based selection) will also serve as the basis for our approach.

SFSR *SFSR* is a floating search method based offline projection selection algorithm that was proposed in Section 2.2.3. As we have seen, *SFSR* performed the best among the different algorithms (and analyzing the running was not important in this case). Therefore, it will form the basis of the global uncertainty based offline projection selection method presented in this chapter.

Spectral richness based selection One of the most successfully used adaptive samplings [25] is called level crossing sampling scheme (LCSS), where the samples are captured only when the input signal crosses one of the predefined thresholds. Thus, the samples are not uniformly spaced. The major advantage of the LCSS is that

FIGURE 3.2: Level crossing non-uniform sampling scheme for 1D signal.

less samples are required to reconstruct the analog signal (see Fig. 3.2). The spectral richness varies periodically with time and becomes maximum when the rate of change of the signal is the highest. Therefore, more samples in spectrally rich areas ensure more samples in rapidly varying regions. Spectral richness based selection adaptively selects the projections based on the spatial spectral richness of the image function. We will refer to this algorithm as *Alg1* as the authors did in the original paper [38].

Reconstruction error based selection Let $\mathbf{x}^{(k)}$ denote the reconstruction after the k th projection. Then, an error function can be defined by

$$E_k = \log_{10} \left(\sum_{j=1}^N |x_j^{(k)} - x_j^{(k-1)}|^2 \right), \quad (3.4)$$

which is a measure of how significantly the most recent reconstructed image differs from the immediate past one. Emphasis on larger values of the error function is compensated by the $\log$ function. A larger value of E_k means more informative k th projection and hence smaller step angle is defined. The pseudo-code of *Alg2* from [38] is the following:

- Step 1
  - Set an initial step size μ_{init} .
  - Take the first two projections at $\beta_0 = \beta_{init}$, $\beta_1 = \beta_0 + \mu_{init}$.
  - Compute E_0 and E_1 , associated with these two projections using Eq. (3.4).
  - Calculate the reference error function as $E_{ref} = (E_0 + E_1)/2$.
  - Let $k = 1$.
- Step 2
  - Compute the per unit change as $\nabla E = (E_k - E_{ref})/E_k$.
  - Compute the required change in the step size as $\nabla \mu = \eta_r \bar{\mu} \nabla E$, where $\bar{\mu}$ is the average step size calculated so far, and η_r is a parameter that couples ∇E and $\nabla \mu$.
  - If $\nabla \mu > \nabla \mu_{max}$ then let $\nabla \mu = \nabla \mu_{max}$, where $\nabla \mu_{max}$ is the maximum allowed change in the step size.

- Step 3
  - Update the step size: $\mu = \mu_{init} - \nabla \mu$.
- Step 4
  - Let $k = k + 1$.
  - Calculate the next projection angle by $\beta_k = \beta_{k-1} + \mu$.
  - Terminate if $\beta_k \geq 180^\circ$.
- Step 5
  - Take the projection at β_k and compute E_k by Eq. (3.4).
  - Go to step 2.

Naive approach The *Naive* approach, as the authors of [116] refer to, is simply to choose an equiangular angle set $\{i \frac{180^\circ}{r} \mid i = 0, \dots, r-1\}$, where r denotes the number of projections. In fact, this means that no adaptive projection selection strategy is applied. Therefore, this method can be regarded as a basic standard in comparing projection selection methods. This method was also used in Chapter 2.

3.2.3 Proposed offline method

In Section 2.2.3, we provided the *SFSR* algorithm for projection selection. The method being proposed here uses the same methodology. The difference is that all of the RME calculations are replaced with global uncertainty (Eq. 3.3) calculations and uncertainty is used for the optimization, i.e., no blueprint data is used for selecting the most informative angles, only the uncertainty of the reconstructed image itself. In Algorithms 4 and 5, we recall the *SFS* and *Refinement* methods with the mentioned modifications. The modified parts are indicated with red color. This affects lines 4 and 5 for *SFS*, and lines 5, 6, 10, 11, and 15 in the case of *Refinement*. From now, we refer to the modified global uncertainty based *SFSR* method as *SFSR-UNC*, and the original RME based algorithm remains *SFSR*.

Algorithm 4: Angle selection with *SFS* algorithm using global uncertainty

---

- 1: let $S = \{\theta_1, \theta_2\}$ where θ_1 and θ_2 are randomly chosen integer angles
  - 2: **repeat**
  - 3:   **for all** angle $\theta \in \{0, 1, \dots, 179\} \setminus S$ **do**
  - 4:   calculate $\mathcal{U}(\mathbf{x}^{S \cup \{\theta\}})$
  - 5: $\theta_{min} \leftarrow$ angle corresponding to the smallest **global uncertainty** value
  - 6:   **end for**
  - 7: $S \leftarrow S \cup \{\theta_{min}\}$
  - 8: **until** the predefined number of projections is reached
-

3.2.4 Proposed online method

The grade of reconstruction ambiguity can be expressed by the reconstruction uncertainty measure discussed in Section 3.2.1. While *Alg2* takes only one reconstruction into consideration when selecting the consecutive projections angles, we can modify

Algorithm 5: Angle selection with *Refinement* using global uncertainty

---

```

1: let  $S$  be the set of the actual angles - output of SFS or SBS
2:  $\text{fix}\Theta \leftarrow$  the last element inserted into  $S$ 
3: repeat
4:   for all angle  $\theta \in S \setminus \{\text{fix}\Theta\}$  do
5:     calculate  $\mathcal{U}(\mathbf{x}^{S \setminus \{\theta\}})$ 
6:      $\theta_{\min} \leftarrow$  angle corresponding to the smallest global uncertainty value
7:   end for
8:    $S \leftarrow S \setminus \{\theta_{\min}\}$ 
9:   for  $\theta \leftarrow 0$  to 179 do
10:    calculate  $\mathcal{U}(\mathbf{x}^{S \cup \{\theta\}})$ 
11:     $\theta_{\min} \leftarrow$  angle corresponding to the smallest global uncertainty value
12:   end for
13:    $S \leftarrow S \cup \{\theta_{\min}\}$ 
14:    $\text{fix}\Theta \leftarrow \theta_{\min}$ 
15: until the global uncertainty cannot be decreased any more.

```

---

this algorithm to take into account all the possible reconstructions in each iteration. For this purpose, we replace the error function Eq. (3.4) with

$$E_k = \log_{10} \left(\sum_{j=1}^N |\mathcal{H}(x_j^{(k)}) - \mathcal{H}(x_j^{(k-1)})|^2 \right), \quad (3.5)$$

and modify steps 1 and 5 of *Alg2* accordingly, to use this measure when evaluating the projection angle candidates. Then, in each iteration, the candidate ensuring the biggest decrease in reconstruction uncertainty is selected. We will refer to this algorithm as *Alg2-UNC*.

We could also follow the strategy of the previous section and evaluate the reconstruction ambiguity using global uncertainty Eq. (3.3). However, we want to stay consistent with the original error function Eq. (3.4), which is a pixel-based approach. Therefore, instead of global uncertainty, we use the entropy maps (Eq. 3.2) in Eq. (3.5). Another advantage of this approach is that the entropy map holds more information than the value of global uncertainty, the latter being an aggregated form of the former one.

3.2.5 Experimental results

We conducted experiments on binary software phantoms to show empirically that even without a blueprint, we can select nearly as informative projection sets as it is possible with the aid of the ground truth. Our image database consisted of the same 22 phantoms (see Fig. 2.3) and projection geometry properties that we used in Chapter 2 (Section 2.4). The images were reconstructed from their projection sets by a binarized and bounded version of the SIRT algorithm. The quality of the reconstruction was measured by RME (Eq. 1.6). For the offline case, the *SFSR* was already implemented in C++. We had done the modifications and ran the tests in the same environment. In the online cases, the algorithms were implemented in Matlab R2016b, and the tests ran on a machine with 2.20Ghz Intel(R) Core(TM) i7-3632QM CPU and 8GB RAM.

For the offline scenario, using the RME values, we compared the reconstructions provided by *SFSR-UNC* and *SFSR*. The results are summarized in Table 3.1. In the

TABLE 3.1: Best uncertainty and RME results provided by *SFSR-UNC* and *SFSR*

	<i>SFSR-UNC</i>		<i>SFSR</i>		difference
Phantoms	uncertainty	RME	uncertainty	RME	RME
P1	0.498686	0.054054	0.525388	0.052858	0.001196
P2	0.614922	0.053567	0.552586	0.067159	-0.013592
P3	0.293671	0.029400	0.293773	0.035786	-0.006386
P4	0.607451	0.069475	0.659946	0.060987	0.008488
P5	0.687311	0.088321	0.692301	0.089693	-0.001372
P6	0.527275	0.061344	0.509842	0.064690	-0.003346
P7	0.582135	0.089337	0.597854	0.067330	0.022007
P8	0.578104	0.116264	0.596882	0.119123	-0.002859
P9	0.310081	0.044787	0.326152	0.042202	0.002585
P10	0.350083	0.037991	0.324438	0.030669	0.007322
P11	0.526202	0.074057	0.535776	0.089109	-0.015052
P12	0.135703	0.011891	0.136618	0.007658	0.004233
P13	0.311714	0.049153	0.314140	0.054103	-0.004950
P14	1.324257	0.416702	1.337183	0.404755	0.011947
P15	2.337384	0.425000	2.481358	0.350000	0.075000
P16	1.405617	0.472347	1.430437	0.481177	-0.008830
P17	2.156736	0.873173	2.218493	0.866745	0.006428
P18	1.428904	0.413737	1.479656	0.408491	0.005246
P19	0.278831	0.052162	0.318267	0.051140	0.001022
P20	0.295304	0.037180	0.295760	0.035964	0.001216
P21	0.612189	0.127827	0.615520	0.133426	-0.005599
P22	1.598097	0.436369	1.724271	0.430816	0.005553
Average	0.793666	0.183370	0.816666	0.179267	0.004103

sixth column, one can see the difference between the RME values, given in the third and fifth column. If the value is negative, *SFSR-UNC* performed better. Otherwise *SFSR* did. The differences are rather small, and in Fig. 3.3, it is visible that the RME values of the two algorithms highly correlate.

Moving on to the online approach, we performed several simulations to find the optimal parameter setting. On this basis, for all three algorithms, we set $\mu_{init} = 10$. In the case of *Alg2* and *Alg2-UNC* we set $\eta_r = 0.8$, while in case of *Alg1*, the coupling constant η_s (having the same role as η_r has in *Alg2*) was set to $\eta_s = 0.6$. We reran each algorithm 180 times with the initial integer starting angles $\beta_{init} = 0^\circ, \dots, 179^\circ$.

Table 3.2 summarizes the results of our experiment. For each test image, we first identified the projection number belonging to the smallest RME provided by algorithm *Alg2-UNC*. This can be seen in the second column of Table 3.2. The identical fourth, sixth, and eighth columns show the RME values of *Alg2-UNC*. Then, we selected the best RME values given by *Alg1*, *Alg2*, and *Naive* using the same number of projections (see the third, fifth, and seventh columns of the table, respectively).

Thus, all four algorithms selected the same number of projections, but not necessarily with identical starting angles. The smallest RME values are highlighted in each row. It was important to check all the 180 possible integer starting angles because the list of the selected angles heavily depended on the orientation of the object.

FIGURE 3.3: RME values belonging to *SFSR-UNC* and *SFSR*TABLE 3.2: RME values belonging to phantoms produced by different projection selection methods, when *Alg2-UNC*'s best RME values were used for the comparison.

	projs.	<i>Alg1</i>	<i>Alg2-UNC</i>	<i>Alg2</i>	<i>Alg2-UNC</i>	<i>Naive</i>	<i>Alg2-UNC</i>
P1	18	NaN	0.0086	0.0117	0.0086	0.0123	0.0086
P2	18	0.0040	0.0031	NaN	0.0031	0.0030	0.0031
P3	18	0.0084	0.0079	NaN	0.0079	0.0126	0.0079
P4	17	0.0154	0.0118	0.0138	0.0118	0.0203	0.0118
P5	18	0.0249	0.0221	0.0228	0.0221	0.0304	0.0221
P6	19	0.0112	0.0103	NaN	0.0103	0.0185	0.0103
P7	19	0.0097	0.0088	0.0081	0.0088	0.0156	0.0088
P8	19	0.0113	0.0101	0.0120	0.0101	0.0133	0.0101
P9	19	0.0093	0.0082	0.0105	0.0082	0.0119	0.0082
P10	17	0.0091	0.0068	0.0075	0.0068	0.0135	0.0068
P11	19	0.0135	0.0124	NaN	0.0124	0.0170	0.0124
P12	17	0.0018	0.0010	0.0010	0.0010	0.0023	0.0010
P13	19	0.0080	0.0080	NaN	0.0080	0.0098	0.0080
P14	19	0.0742	0.0731	0.0850	0.0731	0.0864	0.0731
P15	18	0.2521	0.2219	NaN	0.2219	0.2833	0.2219
P16	20	0.2328	0.2070	NaN	0.2070	0.2356	0.2070
P17	19	0.5901	0.5793	NaN	0.5793	0.6076	0.5793
P18	18	0.0162	0.0151	0.0189	0.0151	0.0190	0.0151
P19	19	0.0061	0.0063	NaN	0.0063	0.0077	0.0063
P20	20	0.0511	0.0065	0.0188	0.0065	0.0357	0.0065
P21	18	0.0052	0.0047	0.0067	0.0047	0.0077	0.0047
P22	19	0.0435	0.0447	NaN	0.0447	0.0560	0.0447

TABLE 3.3: RME values belonging to phantoms produced by different projection selection methods, when *Alg2*'s best RME values were used for the comparison.

	projs.	<i>Alg1</i>	<i>Alg2-UNC</i>	<i>Alg2</i>	<i>Alg2-UNC</i>	<i>Naive</i>	<i>Alg2-UNC</i>
P1	17	0.0104	0.0092	0.0087	0.0092	0.0157	0.0092
P2	17	0.0043	0.0040	0.0047	0.0040	0.0082	0.0040
P3	16	NaN	NaN	0.0084	NaN	0.0126	NaN
P4	19	0.0113	0.0145	0.0126	0.0145	0.0195	0.0145
P5	17	0.0298	0.0233	0.0223	0.0233	0.0350	0.0233
P6	17	0.0152	0.0128	0.0116	0.0128	0.0224	0.0128
P7	19	0.0097	0.0088	0.0081	0.0088	0.0139	0.0088
P8	19	0.0113	0.0101	0.0120	0.0101	0.0136	0.0101
P9	16	0.0130	0.0133	0.0104	0.0133	0.0157	0.0133
P10	17	0.0091	0.0068	0.0075	0.0068	0.0153	0.0068
P11	18	0.0140	0.0133	0.0138	0.0133	0.0169	0.0133
P12	17	0.0018	0.0010	0.0010	0.0010	0.0023	0.0010
P13	17	0.0090	0.0087	0.0092	0.0087	0.0105	0.0087
P14	20	0.0802	NaN	0.0774	NaN	0.0848	NaN
P15	17	0.2625	0.2594	0.2240	0.2594	0.3010	0.2594
P16	19	0.2298	0.2234	0.2316	0.2234	0.2440	0.2234
P17	18	0.6037	0.5943	0.5969	0.5943	0.6057	0.5943
P18	17	NaN	NaN	0.0177	NaN	0.0218	NaN
P19	17	0.0064	0.0065	0.0071	0.0065	0.0115	0.0065
P20	20	0.0511	0.0065	0.0188	0.0065	0.0427	0.0065
P21	18	0.0052	0.0047	0.0067	0.0047	0.0091	0.0047
P22	18	0.0470	0.0452	0.0463	0.0452	0.0563	0.0452

The NaN entries denote the cases when none of the 180 runs of the certain algorithm finished with the same projection number as *Alg2-UNC* did. This way, we could avoid unfair comparisons. We observe that, where comparison was possible, in 10 out of 12 cases, *Alg2-UNC* performed better than *Alg2*. Against *Alg1* in 18 out of 21 cases, and compared to the *Naive* in 21 out of 22 cases, again, *Alg2-UNC* gave better results.

We also repeated the experiment so that in the first step, the number of projections was fixed by the best result of *Alg2* and *Alg1* instead of *Alg2-UNC*. For Table 3.3 and Table 3.4, we collect the results in a similar way as for Table 3.2. Again, NaN entries denote the cases when none of the 180 runs of the certain algorithm finished with the same projection number as the reference algorithm did. We can see that even using the best RMEs of *Alg2* and *Alg1* to identify the number of projections, *Alg2-UNC* still performs very well.

When dealing with online methods, running time is another important aspect of analysis. We found that *Alg1* requires less computation than *Alg2* and *Alg2-UNC* do. With the presented settings, the average running time on the phantoms of Fig. 2.3 for a fix starting angle of algorithm *Alg1*, *Alg2*, and *Alg2-UNC* was 2.34, 7.81, and 8.83 seconds, respectively, on our test machine. By changing the parameters, e.g., increasing μ_{init} , the algorithms would choose fewer projections, yielding a shorter running time but also a poorer reconstruction quality. Furthermore, by choosing a more effective implementation language (e.g., C++) all three methods could be highly speeded up, even to work on-the-fly, when there is a practical need for it.

TABLE 3.4: RME values belonging to phantoms produced by different projection selection methods, when *Alg1*'s best RME values were used for the comparison.

	projs.	<i>Alg1</i>	<i>Alg2-UNC</i>	<i>Alg2</i>	<i>Alg2-UNC</i>	<i>Naive</i>	<i>Alg2-UNC</i>
P1	20	0.0080	NaN	NaN	NaN	0.0096	NaN
P2	19	0.0039	NaN	NaN	NaN	0.0028	NaN
P3	19	0.0074	0.0086	NaN	0.0086	0.0111	0.0086
P4	19	0.0113	0.0145	0.0126	0.0145	0.0178	0.0145
P5	20	0.0206	NaN	NaN	NaN	0.0276	NaN
P6	19	0.0112	0.0103	NaN	0.0103	0.0172	0.0103
P7	20	0.0088	NaN	NaN	NaN	0.0123	NaN
P8	19	0.0113	0.0101	0.0120	0.0101	0.0125	0.0101
P9	18	0.0090	0.0092	0.0108	0.0092	0.0125	0.0092
P10	18	0.0070	0.0071	0.0079	0.0071	0.0129	0.0071
P11	19	0.0135	0.0124	NaN	0.0124	0.0160	0.0124
P12	19	0.0008	0.0010	NaN	0.0010	0.0016	0.0010
P13	19	0.0080	0.0080	NaN	0.0080	0.0108	0.0080
P14	19	0.0742	0.0731	0.0850	0.0731	0.0977	0.0731
P15	20	0.2208	NaN	NaN	NaN	0.2365	NaN
P16	19	0.2298	0.2234	0.2316	0.2234	0.2373	0.2234
P17	19	0.5901	0.5793	NaN	0.5793	0.6041	0.5793
P18	19	0.0151	0.0163	NaN	0.0163	0.0175	0.0163
P19	19	0.0061	0.0063	NaN	0.0063	0.0082	0.0063
P20	18	0.0051	0.0065	0.0272	0.0065	0.0424	0.0065
P21	18	0.0052	0.0047	0.0067	0.0047	0.0081	0.0047
P22	23	0.0301	NaN	NaN	NaN	0.0387	NaN

3.3 Grayscale uncertainty

In this section, we introduce a measurement of the uncertainty of continuous reconstructions. We provide an uncertainty measure and its various equivalent definitions. Some of these definitions will be used to ensure a practical way of calculating the uncertainty values, while other definitions are useful to illustrate the meaning of the uncertainty measure and provide proof of its theoretical background.

Knowing that the insufficient amount of information makes the problem ill-posed and leads to different possible reconstructions, we also give lower and upper bounds on the pixel values of reconstructions, based on the uncertainty measure. Then we show how our results can be used for modelling reconstruction error. Finally, we validate our theoretical results via experimental tests.

3.3.1 Definition of the uncertainty

Before getting to the formulation of the uncertainty measure, we introduce some notation and definitions.

Let $\mathcal{N} = \{1, 2, \dots, N\}$ and $\mathcal{M} = \{1, 2, \dots, m\}$. Moreover, let $\mathbf{0}_N$ denote the column vector with

$$(\mathbf{0}_N)_k = 0, \quad \forall k \in \mathcal{N}. \quad (3.6)$$

Furthermore let $\mathbf{e}_{N,i}$ denote the column vector such that

$$(\mathbf{e}_{N,i})_k = \begin{cases} 1, & \text{if } k = i, \\ 0, & \text{if } k \neq i \end{cases} \quad \forall k \in \mathcal{N}. \quad (3.7)$$

When it does not lead to confusion, we omit the size of $\mathbf{0}_N$ by just writing $\mathbf{0}$. Now we can define the core concept of the uncertainty measure.

Definition 1. For any $\mathbf{A}$ projection matrix let the i -th perturbed reconstruction of $\mathbf{A}$ be

$$\mathcal{P}_{\mathbf{A}}(i) = \arg \min_{\mathbf{x}} \{ \|\mathbf{x} - \mathbf{e}_{N,i}\| \mid \mathbf{A}\mathbf{x} = \mathbf{0}_m, \mathbf{x} \in \mathbb{R}^N \} . \quad (3.8)$$

In practice, the $\mathcal{P}_{\mathbf{A}}(i)$ can be calculated by the SIRT algorithm or the CGLS method. Technically, $\mathbf{A}\mathbf{x} = \mathbf{0}_m$ is the $\mathbf{A}\mathbf{x} = \mathbf{b}$ hyperplane of reconstructions shifted into the origin, and the $\mathcal{P}_{\mathbf{A}}(i)$ point is a reconstruction on this plane, that is the closest to $\mathbf{e}_{N,i}$ (see Fig. 3.4).

Definition 2. Let $\mathcal{G}_{\mathbf{A}}(i)$ be the largest gradient of the solutions of $\mathbf{A}\mathbf{x} = \mathbf{0}_m$ with respect to x_i , i.e.,

$$\mathcal{G}_{\mathbf{A}}(i) = \arg \max_{\mathbf{x}} \{ x_i \mid \mathbf{A}\mathbf{x} = \mathbf{0}_m, \sum_{k \in \mathcal{N} \setminus \{i\}} x_k^2 = 1 \} . \quad (3.9)$$

In a more intuitive manner, $\mathcal{G}_{\mathbf{A}}(i)$ is a point that gives us information on the slope of the hyperplane of solutions on the x_i axis. This gives a vector along which the value of x_i can change the fastest within the hyperplane of solutions.

3.3.2 The variability of the reconstructed pixels

Theorem 2. For any $\mathbf{A}$ projection matrix with $(\mathcal{P}_{\mathbf{A}}(i))_i \notin \{0,1\}$ there is a $c \in \mathbb{R}^+$ constant such that

$$\mathcal{G}_{\mathbf{A}}(i) = c \cdot \mathcal{P}_{\mathbf{A}}(i) . \quad (3.10)$$

Proof. Given an arbitrarily fixed pixel index i , for the sake of technical simplicity, let us introduce the notation $\mathbf{p} = \mathcal{P}_{\mathbf{A}}(i)$.

Assume, that $p_i \neq 0$. In this case we can also assume that $p_i > 0$. Otherwise, if $\mathbf{A}\mathbf{x} = \mathbf{0}_m$ then $\mathbf{A}(-\mathbf{p}) = -\mathbf{0}_m = \mathbf{0}_m$ and $(1 - p_i)^2 > (1 - (-p_i))^2$ leads to

$$\begin{aligned} \|\mathbf{e}_i - \mathbf{p}\|_2 &= \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (0 - p_k)^2 + (1 - p_i)^2} \\ &= \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} p_k^2 + (1 - p_i)^2} \\ &> \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} p_k^2 + (1 - (-p_i))^2} \\ &= \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (0 - (-p_k))^2 + (1 - (-p_i))^2} \\ &= \|\mathbf{e}_i - (-\mathbf{p})\|_2 , \end{aligned} \quad (3.11)$$

a contradiction with the definition of $\mathbf{p} = \mathcal{P}_{\mathbf{A}}(i)$.

We can also assume that $0 < p_i < 1$, since the points $\mathbf{p}$, $\mathbf{0}_N$, and $\mathbf{e}_{N,i}$ form a right-angled triangle in space having the $(\mathbf{0}_N, \mathbf{e}_{N,i})$ hypotenuse of length 1 (see Fig. 3.4).

Now, let δ be defined as

$$\delta = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} p_k^2} , \quad (3.12)$$

and $\mathbf{v}$ be the vector

$$\mathbf{v} = \delta \cdot \mathcal{G}_{\mathbf{A}}(i) . \quad (3.13)$$

FIGURE 3.4: Illustration of the $\mathcal{P}_A(i)$ perturbed reconstruction. FIGURE 3.5: Two cases of $v_i > p_i$. On the left: $0 < v_i < 1$; On the right: $v_i \geq 1$.

Then, we have

$$\sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} p_k^2} = \delta = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} \delta^2 \cdot (\mathcal{G}_A(i))_k^2} = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} v_k^2}, \quad (3.14)$$

and, from the definition of $\mathcal{G}_A(i)$, we get that

$$v_i = \max(x_i \mid \mathbf{Ax} = \mathbf{0}_m, \sum_{k \in \mathcal{N} \setminus \{i\}} x_k^2 = \delta^2). \quad (3.15)$$

From the definition of $\mathcal{G}_A(i)$, it follows that $v_i < p_i$ cannot hold. Assume that $v_i > p_i$, i.e., there is an $\alpha > 0$ such that $v_i = p_i + \alpha$. Now we have two cases. (For an illustration, see Figure 3.5.)

1. First, if $0 < v_i < 1$ (left side of Fig. 3.5) then we have

$$v_i = p_i + \alpha > p_i. \quad (3.16)$$

This gives us

$$\begin{aligned}\|\mathbf{e}_i - \mathbf{v}\|_2 &= \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (0 - v_k)^2 + (1 - v_i)^2} = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} v_k^2 + (1 - v_i)^2} \\ &= \sqrt{\delta^2 + (1 - (p_i + \alpha))^2} < \sqrt{\delta^2 + (1 - p_i)^2} \\ &= \|\mathbf{e}_i - \mathbf{p}\|_2.\end{aligned}\tag{3.17}$$

that is a contradiction with the definition of $\mathbf{p} = \mathcal{P}_{\mathbf{A}}(i)$.

2. In the second case, assume, that $v_i \geq 1$ (see right side of Fig. 3.5). Let

$$\mathbf{w} = \mathbf{v}/v_i.\tag{3.18}$$

Now, we have

$$\sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} w_k^2} = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} \left(\frac{v_k}{v_i}\right)^2} = \frac{\sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} v_k^2}}{v_i} = \frac{\delta}{v_i} \leq \delta\tag{3.19}$$

and by (3.18), $w_i = 1$, therefore

$$\begin{aligned}\|\mathbf{e}_i - \mathbf{w}\|_2 &= \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (0 - w_k)^2 + (1 - w_i)^2} \\ &= \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} w_k^2 + (1 - 1)^2} = \sqrt{\left(\frac{\delta}{v_i}\right)^2 + 0^2} \\ &< \sqrt{\delta^2 + (1 - p_i)^2} = \|\mathbf{e}_i - \mathbf{p}\|_2,\end{aligned}\tag{3.20}$$

This also contradicts the definition of $\mathbf{p} = \mathcal{P}_{\mathbf{A}}(i)$.

Furthermore, $\mathbf{p} = \mathcal{P}_{\mathbf{A}}(i)$ is unique, and $\mathcal{G}_{\mathbf{A}}(i)$ is also unique if the conditions of the theorem hold. We also have that p_i cannot be greater than v_i (as it would contradict the definition of v_i). Therefore, we have that $p_i = v_i$. Together with (3.14), this means that (3.10) can only hold if $\mathbf{v} = \mathbf{p}$ and $c = 1/\delta$. $\square$

The above result means that

$$\mathcal{G}_{\mathbf{A}}(i) = \frac{1}{\delta} \cdot \mathcal{P}_{\mathbf{A}}(i),\tag{3.21}$$

i.e., $\mathcal{P}_{\mathbf{A}}(i)$ can directly be used for the simple and relatively easy calculation of $\mathcal{G}_{\mathbf{A}}(i)$. This way, we can get an upper bound for the slope of the x_i variable in the space of solutions, that leads us to a local uncertainty measure.

Definition 3. Given a projection matrix $\mathbf{A}$, let the uncertainty of the x_i pixel be

$$\mathcal{U}_{\mathbf{A}}(i) = \frac{\|\mathcal{P}_{\mathbf{A}}(i)\|_2}{\|\mathbf{e}_{N,i} - \mathcal{P}_{\mathbf{A}}(i)\|_2}.\tag{3.22}$$

Note that there are two special cases that we did not discuss yet:

- If $p_i = 0$, then the hyperplane of solutions is perpendicular to the x_i axis, meaning that $v_i = 0$ and also $\mathcal{P}_{\mathbf{A}}(i) = \mathcal{G}_{\mathbf{A}}(i) = \mathbf{0}_N$. In this case, we also

FIGURE 3.6: Connection between the $\mathcal{U}_{\mathbf{A}}(i)$ uncertainty value and the $\mathcal{G}_{\mathbf{A}}(i)$ vector.

get an uncertainty value of $\mathcal{U}_{\mathbf{A}}(i) = 0$, meaning that the x_i pixel is uniquely determined by the $\mathbf{A}$ matrix.

- If $p_i = 1$, then the hyperplane of solutions is parallel to the x_i axis, meaning that either $v_i = \infty$, or v_i does not exist. In this case $\mathcal{P}_{\mathbf{A}}(i) = \mathbf{e}_{N,i}$; $\mathcal{G}_{\mathbf{A}}(i)$ is non-existent, and we get an uncertainty value $\mathcal{U}_{\mathbf{A}}(i) = \infty$ meaning that the x_i pixel can take any values independently of the other variables.

Using the above results, we can also find an interesting connection between $\mathcal{U}_{\mathbf{A}}(i)$ and the $\mathcal{G}_{\mathbf{A}}(i)$ vectors.

Lemma 1. Let $\mathbf{g}$ be a vector for a fixed i , such that $\mathbf{g} = \mathcal{G}_{\mathbf{A}}(i)$. Then $\mathcal{U}_{\mathbf{A}}(i) = g_i$.

Proof. Let $\hat{\mathbf{g}}$ be a vector, such that

$$\hat{g}_k = \begin{cases} g_k & \text{if } i \neq k \\ 0 & \text{if } i = k \end{cases}. \quad (3.23)$$

By definition $\|\hat{\mathbf{g}}\|_2 = 1$. Again, for the sake of technical simplicity, let $\mathbf{p} = \mathcal{P}_{\mathbf{A}}(i)$. This way we can get two triangles. The first one is determined by the $(\mathbf{0}_N, \mathbf{g}, \hat{\mathbf{g}})$ points, and another one by $(\mathbf{e}_{N,i}, \mathbf{p}, \mathbf{0}_N)$ (for an illustration, see Figure 3.6). These triangles are similar, and we get

$$\mathcal{U}_{\mathbf{A}}(i) = \frac{\|\mathcal{P}_{\mathbf{A}}(i)\|_2}{\|\mathbf{e}_{N,i} - \mathcal{P}_{\mathbf{A}}(i)\|_2} = \frac{\|\mathbf{p}\|_2}{\|\mathbf{e}_{N,i} - \mathbf{p}\|_2} = \frac{g_i}{1} = g_i. \quad (3.24)$$

□

According to the previous results, $\mathcal{U}_{\mathbf{A}}(i)$ is an upper bound on the slope of space of solutions on the x_i axis. It was also shown that the uncertainty values can be calculated in three different ways, i.e.,

$$\mathcal{U}_{\mathbf{A}}(i) = \frac{\|\mathcal{P}_{\mathbf{A}}(i)\|_2}{\|\mathbf{e}_{N,i} - \mathcal{P}_{\mathbf{A}}(i)\|_2} = \frac{p_i}{\delta} = g_i. \quad (3.25)$$

This means that there are more possible formulations and either $\mathcal{P}_{\mathbf{A}}(i)$ or $\mathcal{G}_{\mathbf{A}}(i)$ could be used for calculating the uncertainty. The reason for the current definition of $\mathcal{U}_{\mathbf{A}}(i)$ is that this form is less affected by the quantization error of floating-point numbers when the uncertainty is small. We must also note, that $\mathcal{G}_{\mathbf{A}}(i)$ is used for theoretical purposes, as it is useful for showing the properties of the uncertainty. In practice, the calculation of the $\mathcal{P}_{\mathbf{A}}(i)$ is computationally more effective, and sufficient for determining the uncertainty.

There is also an important property of the uncertainty coming from the fact that the space of solutions is a linear hyperplane.

Corollary 1. *Let $\mathbf{A}$ be a projection matrix, $\mathbf{b}$ a set of projection values, and $\mathbf{x}$ and $\mathbf{y}$ two vectors such that*

$$\mathbf{Ax} = \mathbf{b}, \quad \mathbf{Ay} = \mathbf{b}, \quad \mathbf{x}, \mathbf{y} \in \mathbb{R}_{\geq 0}^N. \quad (3.26)$$

If

$$\sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (x_k - y_k)^2} = 1, \quad (3.27)$$

then

$$|x_i - y_i| \leq \mathcal{U}_{\mathbf{A}}(i). \quad (3.28)$$

This latter corollary only says that the upper bound on the slopes of $\mathbf{Ax} = \mathbf{0}_m$ also apply for $\mathbf{Ax} = \mathbf{b}$.

3.3.3 Bounds of pixel values

Using the bound on the slopes of the hyperplane of solutions, we can give bounds on the pixel values in the reconstructions. Here, we provide two possible bounds. The first one is exclusively based on the equation system of the projections, while the second one combines the first bound and the uncertainty for giving upper and lower bounds on the pixel values.

Lemma 2. *For any x_k*

$$x_i \leq \min_{j \in \mathcal{M}} \frac{b_j}{a_{j,i}}. \quad (3.29)$$

Proof. We know that $0 \leq x_j$ and $0 \leq a_{i,j}$ for any $(i, j) \in \mathcal{M} \times \mathcal{N}$, and thus $0 \leq a_{i,j}x_j$, as well. We also know that for any $i \in \mathcal{N}$ and $j \in \mathcal{M}$

$$x_i = \frac{b_j - \sum_{k \in \mathcal{N} \setminus \{i\}} a_{j,k}x_k}{a_{j,i}} \quad (3.30)$$

and from this we have

$$x_i = \frac{b_j - \sum_{k \in \mathcal{N} \setminus \{i\}} a_{j,k}x_k}{a_{j,i}} \leq \frac{b_j - \sum_{k \in \mathcal{N} \setminus \{i\}} 0}{a_{j,i}} = \frac{b_j}{a_{j,i}}. \quad (3.31)$$

Taking the minima of this upper bound for all the equations, we have the upper bound for the theorem. $\square$

Now using this bound, we can get another one by also using the uncertainty values.

Theorem 3. Let $\hat{\mathbf{x}} \in \mathbb{R}_{\geq 0}^N$ be a solution of $\mathbf{Ax} = \mathbf{b}$, and define

$$\mathcal{L}_{\mathbf{A}}(i) = \min_{j \in \mathcal{M}} \frac{b_j}{a_{j,i}} \quad (3.32)$$

and

$$\mathcal{D}_{\mathbf{A}}(i) = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (\mathcal{L}_{\mathbf{A}}(k))^2} . \quad (3.33)$$

Then, for any $\mathbf{y} \in \mathbb{R}_{\geq 0}^n$ reconstruction and any $i \in \mathcal{N}$ index

$$\hat{x}_i - \mathcal{U}_{\mathbf{A}}(i) \cdot \mathcal{D}_{\mathbf{A}}(i) \leq y_i \leq \hat{x}_i + \mathcal{U}_{\mathbf{A}}(i) \cdot \mathcal{D}_{\mathbf{A}}(i) . \quad (3.34)$$

Proof. Let i be an arbitrarily fixed index value and

$$\gamma = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (x_k - y_k)^2} , \quad (3.35)$$

and define

$$\mathbf{z} = \frac{\mathbf{y} - \mathbf{x}}{\gamma} . \quad (3.36)$$

From Lemma 2 we have that if $\mathbf{Ax} = \mathbf{b}$ then for any $i \in \mathcal{N}$

$$0 \leq x_i \leq \mathcal{L}_{\mathbf{A}}(i) = \min_{j \in \mathcal{M}} \frac{b_j}{a_{j,i}} . \quad (3.37)$$

This also means that the space of solutions is an N -dimensional hyperrectangle and for any $\mathbf{x}, \mathbf{y}$ pair of solutions

$$\gamma = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} z_k^2} = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (y_k - x_k)^2} \leq \min_{j \in \mathcal{M}} \sqrt{\sum_{k \in \mathcal{M} \setminus i} (\mathcal{L}_{\mathbf{A}}(k))^2} = \mathcal{D}_{\mathbf{A}}(i) . \quad (3.38)$$

We also have that $\mathcal{U}_{\mathbf{A}}(i) = (\mathcal{G}_{\mathbf{A}}(i))_i$. By definition

$$\sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} z_k^2} = \sqrt{\sum_{k \in \mathcal{N} \setminus \{i\}} (x_k - y_k)^2} = 1 . \quad (3.39)$$

Therefore, from Corollary 1 we have

$$z_i = y_i - x_i \leq \mathcal{U}_{\mathbf{A}}(i) . \quad (3.40)$$

This gives us

$$y_i = x_i + \gamma \cdot z_i \leq x_i + \mathcal{D}_{\mathbf{A}}(i) z_i \leq x_i + \mathcal{D}_{\mathbf{A}}(i) \cdot \mathcal{U}_{\mathbf{A}}(i) , \quad (3.41)$$

and as the hyperplane of solutions is symmetric, we also have

$$y_i \geq x_i - \mathcal{D}_{\mathbf{A}}(i) \cdot \mathcal{U}_{\mathbf{A}}(i) . \quad (3.42)$$

□

Regarding the relation between the upper bounds, we can say the following. As the upper bounds in Lemma 2 rely on mostly the upper bound on the projection

values, in practice, this bound will get to its minima after a few projections and will not be significantly lower by increasing the number of projections.

On the other hand, the bounds in Theorem 3 work differently. The $\mathcal{D}_A$ value is practically a huge constant in the upper bounds giving very loose limits with a low number of projections. However, when increasing the number of projections, the $\mathcal{U}_A$ values in the bound will tend to zero, making the bound more and more strict.

This means that in practice, the bounds of Lemma 2 are more useful with a low projection number, and Theorem 3 is more strict with higher projection counts.

3.3.4 Connection of reconstruction error and perturbed reconstructions

Assume that we have $\mathbf{x}^0$, $\hat{\mathbf{x}}$, and $\mathbf{x}^*$ as follows.

- $\mathbf{x}^0 \in \mathbb{R}^N$: is a starting point for seeking a solution.
- $\hat{\mathbf{x}}$: is a reconstruction closest to the $\mathbf{x}^0$ starting position in a euclidean manner, i.e.,

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \left(\|\mathbf{x} - \mathbf{x}^0\| \mid \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \in \mathbb{R}^N \right). \quad (3.43)$$

- $\mathbf{x}^*$: is the expected image, i.e., the image of the original object of study.

In an example scenario, we attempt to reconstruct the $\mathbf{x}^*$ image. We use the SIRT reconstruction algorithm and start its process from the $\mathbf{x}^0$. The SIRT provides the $\hat{\mathbf{x}}$ result that is on the hyperplane of solutions and also the closest point to the $\mathbf{x}^0$ starting point.

From the above definition, we define two new values. Let $\mathcal{E}_1$ be the error of the above reconstruction calculated by

$$\mathcal{E}_1 = \hat{\mathbf{x}} - \mathbf{x}^*. \quad (3.44)$$

Furthermore, we define the $\mathcal{E}_2$ weighted sum based on the perturbed reconstructions and the $\mathbf{x}^0$ base point as

$$\mathcal{E}_2 = \sum_{i=1}^N (\mathcal{P}_A(i)(\mathbf{x}_i^0 - \mathbf{x}_i^*)) . \quad (3.45)$$

Both $\mathcal{E}_1$ and $\mathcal{E}_2$ are vectors and by definition

$$\mathbf{A}\mathcal{E}_1 = \mathbf{A}\mathcal{E}_2 = \mathbf{0}_m . \quad (3.46)$$

We now show that the error of the above-defined $\hat{\mathbf{x}}$ reconstruction can directly be determined if we know the $\mathbf{x}^*$ ideal reconstruction and the $\mathbf{x}^0$ base point.

Theorem 4. *Given an equation system of the form Eq. (1.4) and $\mathbf{x}^*$, $\mathbf{x}^0$, $\hat{\mathbf{x}}$ points and $\mathcal{E}_1$, $\mathcal{E}_2$ vectors defined above, then*

$$\mathcal{E}_1 = \mathcal{E}_2 . \quad (3.47)$$

Proof. Without the loss of generality, we can assume that the equation system has the form $\mathbf{A}\mathbf{x} = \mathbf{0}_m$. We can also assume that the ideal reconstruction is placed in the origin, i.e., $\mathbf{x}^* = \mathbf{0}_N$. In this form our theorem says, that

$$\hat{\mathbf{x}} = \sum_{i=1}^N \mathcal{P}_A(i)\mathbf{x}_i^0 . \quad (3.48)$$

For the proof, let us note that the set of feasible solutions of the $\mathbf{Ax} = \mathbf{0}_m$ form a hyperplane in $\mathbb{R}^N$. Let $\mathbf{V}$ be a matrix such that

$$\mathbf{V} = (\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k) , \quad (3.49)$$

where the column vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ give us an orthogonal basis of the hyperplane of solutions. Note that we do not need to actually have the $\mathbf{V}$ matrix, we are only using it as a theoretical construct.

By the definitions of $\mathbf{e}_{N,i}$ and $\mathcal{P}_A(i)$, we have that

$$\mathbf{V}^T(\mathbf{e}_{N,i} - \mathcal{P}_A(i)) = \mathbf{0} , \quad (3.50)$$

and as such

$$\sum_{i=1}^N \mathbf{x}_i^0 \mathbf{V}^T(\mathbf{e}_{N,i} - \mathcal{P}_A(i)) = \mathbf{0} . \quad (3.51)$$

From the definition of $\hat{\mathbf{x}}$ and $\mathbf{x}^0$, we get

$$\mathbf{V}^T(\mathbf{x}^0 - \hat{\mathbf{x}}) = \mathbf{0} . \quad (3.52)$$

Finally, we notice that

$$\sum_{i=1}^N x_i^0 \mathbf{e}_{N,i} = \mathbf{x}^0 . \quad (3.53)$$

From the above equations, we get

$$\begin{aligned} \mathbf{0} &= \mathbf{V}^T(\mathbf{x}^0 - \hat{\mathbf{x}}) = \sum_{i=1}^N x_i^0 \mathbf{V}^T(\mathbf{e}_{N,i} - \mathcal{P}_A(i)) \\ &= \mathbf{V}^T \sum_{i=1}^N x_i^0 (\mathbf{e}_{N,i} - \mathcal{P}_A(i)) \\ &= \mathbf{V}^T \left(\sum_{i=1}^N x_i^0 \mathbf{e}_{N,i} - \sum_{i=1}^N x_i^0 \mathcal{P}_A(i) \right) \\ &= \mathbf{V}^T \left(\mathbf{x}^0 - \sum_{i=1}^N x_i^0 \mathcal{P}_A(i) \right) . \end{aligned} \quad (3.54)$$

Then, using Eq. (3.52) in the right-hand side of Eq. (3.54)

$$\mathbf{0} = \mathbf{V}^T \left(\hat{\mathbf{x}} - \sum_{i=1}^N x_i^0 \mathcal{P}_A(i) \right) = \mathbf{V}^T (\mathcal{E}_1 - \mathcal{E}_2) . \quad (3.55)$$

In the general case, the $\mathbf{V}$ matrix might be singular. Therefore, we cannot simply remove it from the equations by using its inverse. On the other hand, Eq. (3.54) says that the vector pointing from $\mathcal{E}_2$ to $\mathcal{E}_1$ is perpendicular to the hyperplane of solutions. As both $\mathcal{E}_1$ and $\mathcal{E}_2$ are on this hyperplane, this is only possible if $\mathcal{E}_1 = \mathcal{E}_2$. $\square$

An interesting consequence of the above result is that if we know $\mathbf{x}^0$, then $\hat{\mathbf{x}}$ can directly be calculated as

$$\hat{\mathbf{x}} = \mathbf{x}^0 + \sum_{i=1}^N x_i^0 \mathcal{P}_A(i) . \quad (3.56)$$

FIGURE 3.7: Images used for experiments. P1: Lena, P2: Cameraman, P3: a head CT slice, P4: Shepp-Logan phantom, P5: a binary shape, P6: the same phantom as P5 with a hole in it, P7: homogeneous image with value 0.5 in every pixel.

3.3.5 Experimental validation

To investigate the uncertainty, the bounds, and the errors from a practical point of view, we conducted experiments on different images. Our database consisted of 7 phantoms with different structural complexity, each with a size of 64×64 pixels and an intensity range of $[0,1]$. These images can be seen in Fig. 3.7. We used parallel beam geometry for the acquisitions of the projections. In every projection, we set the distance of the beams and detector elements to 1 pixel and used $64 \cdot \sqrt{2}$ of them to cover the whole image. The rotation center was placed into the center of the image. The reconstructions were performed using 64 different number of projections: $([2, 64] \cap \mathbb{N}) \cup \{128\}$.

There are various methods for tomographic reconstruction (*Chapter 1*). In this section, we approximate a solution of Eq. (1.4) with the CGLS technique and the bounded version of the SIRT. In general, these methods give continuous reconstructions with real pixel values and (with the proper setup) can produce images which are the closest (in the Euclidean sense) to the initial image. We used the CGLS and SIRT methods because their advantages and drawbacks made them suitable for various purposes. The CGLS, for example, has a faster convergence than the SIRT method (see, e.g., Theorem 6.23 of [109]). Thus, the former method, in practice, gives an accurate unconstrained solution for a general reconstruction in significantly fewer iterations. The SIRT, on the other hand, is easy to modify into a projected gradient method, and it can be used for solving the reconstruction problem with non-negativity constraints on the unknowns.

For the bounded SIRT, we used the previously presented C++ implementation. During the reconstruction process, the iterations of SIRT were terminated if the difference between two consecutive iterations was less than 10^{-6} or the number of iterations reached 10^6 . In the case of CGLS, we used a MATLAB implementation with double-precision floating-point numbers. The number of iterations was set to 10^4 . In each case, the iteration was stopped when the first stopping criterion was met.

FIGURE 3.8: $\mathcal{U}_A(i)$ of P1 in case of different number of projections $\{2, 4, 8, 16, 32, 64, 128\}$.

To show that the defined uncertainty formula can provide information about the variability of the reconstructed pixel values, we performed hundreds of CGLS reconstructions with randomly initialized starting images. These reconstructed images were used to calculate the standard deviation on a bigger image set. For this purpose, we used the following formulas.

Let $\Omega_{A,b}$ be a set of reconstructions such that

$$\Omega_{A,b} = \{\mathbf{x} \mid \mathbf{Ax} = \mathbf{b}, \mathbf{x} \in \mathbb{R}_+^n\} . \quad (3.57)$$

Let $E(\Omega_{A,b}, i)$ be the expected value of x_i in $\Omega_{A,b}$, i.e.,

$$E(\Omega_{A,b}, i) = \frac{\sum_{\mathbf{x} \in \Omega_{A,b}} (x_i)}{|\Omega_{A,b}|} , \quad (3.58)$$

and $\sigma(\Omega_{A,b}, i)$ be the standard deviation of x_i in $\Omega_{A,b}$, i.e.,

$$\sigma(\Omega_{A,b}, i) = \sqrt{\frac{\sum_{\mathbf{x} \in \Omega_{A,b}} |x_i - E(\Omega_{A,b}, i)|^2}{|\Omega_{A,b}| - 1}} . \quad (3.59)$$

These $\sigma(\Omega_{A,b}, i)$ standard deviation maps will be compared to the $\mathcal{U}_A(i)$ perturbed reconstructions to analyze the connection between our grayscale uncertainty formula and the pixel variability.

3.3.6 Results

First, we investigated the correlation between $\sigma(\Omega_{A,b}, i)$ and $\mathcal{U}_A(i)$, created with CGLS. For an easier representation, we selected the $\sigma(\Omega_{A,b}, i)$ and $\mathcal{U}_A(i)$ images with 2^k projections for comparison, where $k = [1, 7]$. For a certain phantom, we concatenated the standard deviation maps into one matrix for all the above-defined projection numbers, and did the same with the uncertainty maps. Then, we calculated the correlation between these “montage” images. This way, 7 comparisons were made (based on the 7 test images). The concatenated uncertainty maps – with the original intensities – belonging to P1 can be seen in Fig. 3.8a. To make the uncertainty patterns more visible, the same concatenated uncertainty maps are displayed in Fig. 3.8b with enhanced intensity values.

The Pearson correlations and the Fisher z-transformations with a 95% confidence interval can be seen in Table 3.5. The correlation values around 0.8 indicate a reliable

TABLE 3.5: The correlation between the concatenated $\sigma(\Omega_{\mathbf{A},\mathbf{b}},i)$ and $\mathcal{U}_{\mathbf{A}}(i)$ values generated from all the different number of projections using the test images in Fig. 3.7.

Phantoms	Pearson Corr.	Fisher z-trans.
P1	0.8062	[0.8021, 0.8102]
P2	0.8062	[0.8021, 0.8102]
P3	0.8068	[0.8028, 0.8108]
P4	0.8062	[0.8021, 0.8102]
P5	0.8064	[0.8023, 0.8104]
P6	0.8063	[0.8022, 0.8103]
P7	0.8067	[0.8016, 0.8098]

correspondence between the uncertainty measure and the deviation of pixel values in reconstructions. We argue that the correlation is not higher only because the pixel values in our random reconstructions are not from uniform distribution. As the generation of random reconstructions is not the goal of this study, an extended evaluation could be subject to further studies.

Using one random reconstruction from all the phantoms with a different number of projections (created with bounded SIRT), we also analyzed the range of the bounds given in Lemma 2 and Theorem 3. For these experiments, we used the complete projection angle range that we defined in Section 4.3. In Fig. 3.9, one can see the profiles of the upper bounds ($\mathcal{L}_{\mathbf{A}}(i)$) from Lemma 2. It can clearly be seen that as the projection number increases, the bounds are getting closer and closer to the x-axis. The distance between the averages can also give us information about the intensity ranges of the phantoms. Fig. 3.10 shows the profiles of the upper and the lower bounds defined in Theorem 3. In the case of a low number of projections, the bounds are far from each other, and as the projection number increases, the bounds are approximating each other.

We performed some tests to evaluate our upper and lower bounds (based on Theorem 3) in practical reconstruction scenarios. Our experiments proved that the theoretical upper and lower bounds can sufficiently limit the pixel intensities for all the phantoms with different numbers of projections. Here, we should note that the applied reconstruction algorithms are iterative methods only approximating the exact reconstruction. In our tests, reaching reconstructions of satisfactory accuracy required the bounded SIRT algorithm to run for 10^7 iterations (with all the other stopping criterion disabled).

Taking the results of Figs. 3.9 and 3.10 under consideration, using the upper bound of Lemma 2 seems to be a good choice to limit the reconstruction in the case of a small number of projections. As we increase the number of projections, it may be increasingly useful to apply the bounds provided by Theorem 3, as they converge to a unique reconstruction.

Analyzing the reconstructions with different numbers of projections, we noticed that below 59 projections, the reconstructed images were variable relative to each other, and their quality has steadily improved when approaching a larger number of projections. However, from 59 projections, the pixel intensities of the reconstructed images have not changed anymore (this applies to all the phantoms). This could mean that the equation system started to be fully determined. This phenomenon appeared several times, even during the analysis of the reconstruction errors.

FIGURE 3.9: Average upper limits from Lemma 2 in the case of different numbers of projections.

FIGURE 3.10: Average limits from Theorem 3 in the case of different numbers of projections.

TABLE 3.6: The means of $|\mathcal{E}_1 - \mathcal{E}_2|$ error maps on different projection angle ranges.

Phantoms	[2, 58]	[59, 64] $\cup$ {128}
P1	6.69E-04	4.61E-06
P2	6.74E-04	4.86E-06
P3	1.05E-03	5.86E-06
P4	8.70E-04	5.51E-06
P5	9.12E-04	5.89E-06
P6	9.20E-04	5.92E-06
P7	6.90E-04	4.54E-06

In a further investigation, we compared $\mathcal{E}_1$ (Eq. 3.44) and $\mathcal{E}_2$ (Eq. 3.45) for all the phantoms with all the different predefined number of projections (Section 3.3.5) to prove their similarity, based on Theorem 4. Table 3.6 shows the mean values of $|\mathcal{E}_1 - \mathcal{E}_2|$ on different projection angle ranges. All the values are close to zero. We remark that these extremely small values represent zeros. According to our findings, these non-zero values are present due to rounding errors in the reconstruction algorithms and the fact that the reconstructions are only approximate solutions with some tolerance. A detailed diagram can be seen in Fig. 3.11 with the absolute average $\mathcal{E}_1$ and $\mathcal{E}_2$ values of phantom P1. To emphasize the huge error drop between 58 and 59 projections, the values belonging to the last few numbers of projections are indicated in data labels for $\mathcal{E}_2$ in the diagram.

Direction dependency [115] can also be observed near to 59 projections. In most of the cases, the uncertainty ($\mathcal{U}_A(i)$) and the errors ($\mathcal{E}_1$ and $\mathcal{E}_2$) are lower in the case of odd number of projections, and always lower in the case of 57 projections than the immediately preceding and following ones. Furthermore, regardless of direction dependency, the uncertainty and the error are always smaller in the corners of the images than in the center.

3.4 Summary

In the first part of this chapter, we suggested a modified version of the offline projection selection method *SFSR* proposed in Chapter 2, and the online method *Alg2* presented in [38]. The key idea was to use reconstruction uncertainty instead of blueprint data and reconstruction error to select angles, respectively. *SFSR-UNC* was compared to the original *SFSR*, and *Alg2-UNC* to already published methods (*Naive*, *Alg1*, *Alg2*) using experimental tests on software phantom images. We found that our former method is able to provide approximately as accurate reconstruction results as methods that use blueprint images to find the optimal set of projections. In the latter case, we showed that *Alg2-UNC* can outperform the state-of-the-art algorithms and provide good alternative solutions for selecting projection angles.

In the second part, we provided a measure for the variability of the reconstructed pixels based only on the projection geometry. Then we used this variability measure to give theoretically proven upper and lower bounds for the pixel values in the reconstructions. We provided a practical analysis of the results, evaluating the correlations between the local uncertainties given in Theorem 2 and the standard deviation of a set of reconstructed images. The bounds of Lemma 2 and Theorem 3 were also

FIGURE 3.11: Absolute average values of the $\mathcal{E}_1$ and $\mathcal{E}_2$ error maps in the case of different numbers of projections belonging to phantom P1.

investigated by comparing them to each other with selected reconstructed images. We defined two tomographic reconstruction errors. We gave a theoretical and an experimental proof about the similarity of these error formulas (Theorem 4).

Grayscale uncertainty is interesting not only from a theoretical point of view but using this uncertainty measure we could tell (or at least estimate) the reconstruction error that will appear after the reconstructions. It helps us to understand how these errors are created during the reconstruction and how to eliminate them. It brings us closer to making progress in improving the algebraic reconstructions.

In this chapter, the author regards the following as his main contributions:

1. Projection selection using uncertainty

- Modifying the already implemented offline and online algorithms to embed the uncertainty as the optimization measure (implementation of these modifications).
- Testing and comparing them to already published algorithms.
- Evaluation of the results.

2. Grayscale uncertainty

- Implementation of certain formulas.
- Experimental validation/proof of the formulas.
- Evaluation of the results.

The findings of this chapter have been published in three conference proceedings [66, 67, 118] and are submitted as a journal paper [117].

Chapter 4

Adaptive Automatic Tube Voltage Selection for CT

4.1 Introduction

CT is one of the most frequently used modalities in radiodiagnostics and NDT. However, undergoing multiple high radiation dose examinations may cause adverse health effects, even lethal diseases in the human body [79]. Besides, high radiation consumes a lot of energy. In both human studies and NDT, it is a reasonable goal to use as low energy and radiation as possible for the acquisition of projections, while still preserving the required reconstruction quality. When using a constant energy spectrum, the investigation of elongated objects may be problematic. Using too low energy, the photons cannot penetrate the object in the direction of elongation. On the other hand, too high energy can cause the photons to traverse the object without significant attenuation, in the direction perpendicular to the elongation of the object. A solution could be to use different energy levels when producing projections from different directions.

For example, let us assume that we have an elongated box-shaped object of a homogeneous material. Based on the Beer-Lambert law [44], if monochromatic X-ray passes through this object in the direction of elongation, it has to travel a longer distance, i.e., the absorption will be bigger, and a smaller amount of photons will reach the detector line. In contrast, in the direction perpendicular to the elongation of the object, this distance is shorter, i.e., the absorption will be smaller and more photons can get to the detector line (see Fig. 4.1 for illustration). For a better understanding, if one takes a $4 \times 4 \times 20$ mm sized box made of PVC (Polyvinyl chloride), then using a monochromatic X-ray with 20 keV energy and 1000 photons, the expected number of exiting photons would be 0.11 in the direction of elongation, and 160.16 perpendicular to that direction. In the case of increasing the energy to 40 keV, the expected number of exiting photons would be 232.24 and 746.77, respectively (see Table 4.1). These values are calculated based on the linear attenuation coefficients from [70].

Automatic Tube Current Modulation (ATCM) and Automatic Tube Voltage Selection (ATVS) are well-researched areas. In most of the cases, the CT scanners contain intelligent scanning technologies, which can automatically recommend tube current and voltage settings to provide the lowest radiation dose and high image quality. For this purpose, widely used scanners are CARE kV and CARE Dose4D from Siemens Medical Solutions, Forchheim, Germany [62], and the scanners of GE Healthcare, with the previously mentioned similar embedded technologies. Papers [19, 31, 69, 108] show examples for using ATCM and/or ATVS combining with novel iterative reconstruction techniques to achieve good image quality with a low dose.

In [77], a detailed list is given about the CT parameters that influence the radiation dose. There, the author claims that variations in tube potential should not be

FIGURE 4.1: Photons cannot traverse the object (on the left). Photons completely traverse the object without or only with a small attenuation (on the right).

TABLE 4.1: The expected number of output photons based on Beer-Lambert law.

Photons IN	Energy [keV]	Length [mm]	Photons OUT	OUT/IN
1000	20	4	160.16	0.16
1000	20	20	0.11	0
1000	40	4	746.77	0.75
1000	40	20	232.24	0.23

considered for pure dose reduction purposes except in the case of CT angiography. However, numerous articles investigate the effects of using only low tube voltage level or ATVS for different radiation tasks (see, e.g., [98, 102, 104, 112, 124]). They prove that optimizing tube voltage itself can reduce the radiation dose and it can increase the image contrast for structures with a high effective atomic number, such as calcium and iodine. Moreover, with low voltage level, more energy-efficient acquisitions could be performed. From the viewpoint of the lifetime of the X-ray tube, it is also beneficial to use lower energies.

Several studies focus on CT parameter optimization using different quality measures like transmission, linearity, signal-to-noise ratio, contrast-to-noise ratio, sharpness, etc. [59, 96, 97]. Using the proper combination of these measures could lead to even better optimized CT parameters.

The aim of this chapter is to provide an adaptive tube voltage selection method. In this context, “adaptive” means that the tube voltage will be re-selected during the acquisition, projection-by-projection. To the best of our knowledge, this is the first approach that attempts to select tube voltage on-the-fly based only on projection (sinogram) information. This approach could be effectively applied, especially in the case of elongated objects.

FIGURE 4.2: Two sinograms of the same rectangle-shaped object, acquired on lower (left) and higher (right) energy levels, and the transmission values of certain parts.

4.2 Proposed method

First, we need a figure of merit to quantitatively evaluate the radiation used, more precisely, the projection images produced. After experimenting with different quality measures, we narrowed down the list of possible options and chose *transmission*, which describes the ratio between the minimal and the maximal X-ray intensity at the detector. CT standards, which are still in development, suggest transmission values of 14% (ISO 15708) or 10% (prEN 16016) at the path of highest absorption for optimal scan quality. However, depending on the scanning and reconstruction problem, optimal transmission can be even lower or higher. In our experiments, 20% proved to be a good choice.

In Fig. 4.2, one can see two sinograms (before the logarithmic transformation) of the same rectangle-shaped object ($[0^\circ, 179^\circ]$ integer angle interval). The sinogram on the left was acquired on a lower, and the one on the right on a higher energy level. Based on the previous statement that 20% transmission value could be optimal, the middle part of the first sinogram could hold useful data created using lower energy, while at the beginning and at the end of the sinogram, the higher energy (sinogram on the right) provides correct projections.

Our proposed voltage selection method is described in Alg. 6. In the beginning, we define a couple of parameters. The optimal transmission is 0.2 (20%), as we mentioned before. Furthermore, a tolerance value is associated with this optimum. By empirical investigations, we set this value to 0.25, meaning that the transmission is still acceptable if it is not lower than $0.2 \cdot 0.75$ and not greater than $0.2 \cdot 1.25$. We also need to define tube voltage levels and an angle set for the acquisitions (see more details in the experimental section). As a first part (Lines 2-7 of Alg. 6), we acquire the first projection on all the different energy levels. Then the energy level with the closest transmission value to the optimum will be chosen for further acquisitions. In the second part (Lines 9-19 of Alg. 6), we acquire a projection on the previously chosen energy level. If its transmission value is in a tolerance distance to the optimum, then we stay on the current energy level. Otherwise, if it is out of the acceptable range, then we step to a higher/lower energy level for the next projection. This second part is repeated until the predefined number of projections is reached. This way, based on the shape of the object, the resulting sinogram may contain projections from different energy levels. Thus, one can collect the most informative projections (energy

levels) belonging to the given angles. We will refer to the sinogram obtained this way as the *adaptive sinogram*.

It would be ideal if the sum of intensities at the detectors were equal for all individual projections. However, since we are using different energy levels for different projections, this is not always the case. This can be an issue, during the reconstruction, since reconstruction algorithms typically assume a constant energy spectrum for acquiring the projections. To resolve this issue, in Alg. 7, we propose a sinogram equalization method. In the pseudo-code, $\odot$ and $\oslash$ stands for the pointwise product and division, respectively. First, we create a mask, which covers the region(s) of the sinogram where the shadow of the object is present (Line 1). Then, we take the sum of the pixels covered by the mask and determine its maximum value (Lines 2-3). A scaling vector is created by taking the ratio between the maximum value and the previous pixel sum (Line 4). Finally, the scaling vector is used to re-scale the *adaptive sinogram* (Line 5). After applying Alg. 7 on the *adaptive sinogram*, the sum of the intensities will be close to each other, in all projections, and thus, with this approach one can approximate the ideal case. We will refer to the sinogram created by the consecutive application of Alg. 6 and Alg. 7 as the *corrected-adaptive sinogram*.

Algorithm 6: Voltage selection

Require: $\mathbf{tr}_{\text{opt}}$ - optimal transmission; $\mathbf{tr}_{\text{tol}}$ - tolerance for the transmission level; TVL - set of predefined tube voltage levels; $\mathbf{S}$ - set of predefined angles

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1:  $\mathbf{tr}_{\text{opt}} \leftarrow 0.2, \mathbf{tr}_{\text{tol}} \leftarrow 0.25$ 
2: for all  $lvl \in \text{TVL}$  do
3:   acquire projection from starting angle on  $lvl$ 
4:    $\text{closest}_{lvl} \leftarrow lvl$  with the closest transmission value to  $\mathbf{tr}_{\text{opt}}$ 
5:    $\text{closest}_{pr} \leftarrow$  projection acquired on  $\text{closest}_{lvl}$ 
6:    $\text{closest}_{tr} \leftarrow$  transmission value of  $\text{closest}_{pr}$ 
7: end for
8:  $\text{sinogram}_{\text{opt}} \leftarrow \text{sinogram}_{\text{opt}} \cup \text{closest}_{pr}$ 
9: for all angle  $\theta \in \mathbf{S}$  do
10:   $\text{actual}_{pr} \leftarrow$  acquire projection from angle  $\theta$  on  $\text{closest}_{lvl}$ 
11:   $\text{sinogram}_{\text{opt}} \leftarrow \text{sinogram}_{\text{opt}} \cup \text{actual}_{pr}$ 
12:  if  $|\mathbf{tr}_{\text{opt}} - \text{closest}_{tr}| \geq \mathbf{tr}_{\text{opt}} \cdot \mathbf{tr}_{\text{tol}}$  then
13:    if  $\mathbf{tr}_{\text{opt}} - \text{closest}_{tr} < 0$  then
14:       $\text{closest}_{lvl} \leftarrow$  decrease  $\text{closest}_{lvl}$  to the closest lower level in TVL
15:    else
16:       $\text{closest}_{lvl} \leftarrow$  increase  $\text{closest}_{lvl}$  to the closest higher level in TVL
17:    end if
18:  end if
19: end for

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Algorithm 7: Sinogram equalization

Require: $\text{sinogram}_{\text{adapt}}$

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1:  $\text{sinogram}_{bw} \leftarrow$  thresholded  $\text{sinogram}_{\text{adapt}}$ 
2:  $\text{vector}_{obj} \leftarrow$  column sum of  $(\text{sinogram}_{\text{opt}} \odot \text{sinogram}_{bw})$ 
3:  $\text{vector}_{max} \leftarrow$  column maximums of  $\text{vector}_{obj}$ 
4:  $\text{vector}_{scale} \leftarrow \text{vector}_{max} \oslash \text{vector}_{obj}$ 
5:  $\text{sinogram}_{\text{corr-adapt}} \leftarrow \text{sinogram}_{\text{adapt}} \odot \text{vector}_{scale}$ 

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4.3 Test environment

To conduct experiments, we used the GATE simulation software [46, 47], an advanced open-source software developed by the international OpenGATE collaboration and dedicated to numerical simulations in medical imaging and radiotherapy (see similar simulation tools [12, 18]). During the simulations, we used a polychromatic source, which can keep us closer to real applications. To simulate a perfect energy spectrum, we used Spekcalc [86–88], which is an executable for calculating the X-ray emission spectra from tungsten anodes such as those used in diagnostic radiology and kV radiotherapy X-ray tubes. In our experiments, the difference between the peak and minimum energy was set to 30keV (the default of the software) for each energy level, except the ones where the peak energy was too low to set the minimum to be 30keV smaller.

We studied 6 different software phantom models shown in Fig. 4.3. P1 is a perfect sphere on which we show how the transmission behaves if an object is not elongated, P2-P5 are elongated binary objects (with different length-width ratio and/or inner structure), and P6 is a self-made approximation of the well-known Forbild hip phantom¹. For the reconstructions, we took the middle slices of these objects (see Fig. 4.6). In the GATE software, Phantoms P1-P5 were virtually made of PVC. This decision was based on the fact that the PVC is the world’s third-most widely produced synthetic plastic polymer (after polyethylene and polypropylene) and this material was available in the simulation software. P6 consists of the materials detailed in the description of the Forbild hip phantom. All the wire-framed objects in Fig. 4.3 are solid, the cyan colored objects are holes, and in the case of P6, the yellow color distinguishes the solid objects inside each other.

During the simulations in GATE, parallel geometry was assumed. The length of the simulated detector line was 72mm, and its height was 0.1mm. The width of one single detector was set to 0.4mm. This ensures a precision of 0.4mm on the x-axis and 0.1mm on the y-axis. Equivalently, it means 180 projection rays in each direction which suitably covers images of size 128×128 , in each direction. We collected 285 projections on the 0° - 179° interval, based on the Nyquist–Shannon sampling theorem. The rotation time was 1 sec/projection. In GATE, the tube current cannot be controlled, only the number of photons to emit, which was set to 200 000 in our case. A simulation could take a long time, depending on the number of simulated photons. We chose the value 200 000 based on empirical tests, it provided an acceptable consensus between running time and quality. After acquiring the projections, the reconstructions were performed using the Filtered Backprojection (FBP) technique. The quality of the reconstructions was evaluated by the Signal-to-noise ratio (SNR). For more information about FBP and SNR, see Chapter 1.

4.4 Results

We predefined 15 different energy levels. In Table 4.2, we present the SNR values of the reconstructed images belonging to sinograms of the 15 different energy levels. To ensure a fair and realistic comparison with the *corrected-adaptive sinograms* later, Alg. 7 was also executed on these 15 sinograms (Alg. 6 was not applied yet). The highest (best) SNR values are highlighted in every row to indicate which energy levels provided the best quality. We will refer to the sinograms of these reconstructions as the *original best sinograms*. The last row shows the SNR values of the reconstructed

¹<http://www.imp.uni-erlangen.de/phantoms/index.htm>

FIGURE 4.3: The 3D models of the used phantoms. P1: a sphere, P2: an elongated cuboid, P3: a wider cuboid, P4: a P2 sized cuboid with one small hole in it, P5: a P2 sized cuboid with five small (equal sized) holes in it, P6: an approximation of the Forbild hip phantom.

images belonging to the *corrected-adaptive sinograms* (created using Alg. 6 and 7). In the case of P1, one can see that the *corrected-adaptive* and the *original best* SNRs are the same since the sphere is the perfect opposite of an elongated object. It means that tube voltage selection is unnecessary in this case. The *corrected-adaptive* SNRs of P2-P5 are better than that of the *original best* reconstructions. Taking P6 under examination, the SNR of the *corrected-adaptive* reconstruction is almost as good as that the *original best sinogram* can provide.

Based on the polychromatic spectra of the energy levels, their mean energies could also be determined. We note that the mean energy belonging to the highest energy level (15th) is lower than the mean energy of the preceding one (14th) (see the second column of Table 4.2). The reason behind it is that based on the energy spectrum, on the 15th energy level, the number of low energy photons are lot higher. These photons are attenuating with a higher probability, resulting in a lower mean energy.

In Fig. 4.4, we present the transmission values of P2 on the different energy levels. In this case, the acquisition started against the shorter side, moved continuously towards the elongated one, and continued until it reached the opposite shorter side. The *Opt. trans.* line shows where the transmission should be for an optimal sinogram (based on our experiments). In this case, Alg. 6 chose the 9th energy level (30-60 keV) first, then it decreased to the 5th energy level (10-40 keV) and then increased back to the 9th. It is reasonable because, in the beginning, it needed higher energy to traverse the phantom, and as it approached the elongated side, less and less energy was sufficient. In the case of P1, this diagram would contain almost straight lines. Fig. 4.5 shows the difference between the sinograms of P2. Fig. 4.5a is the *original best* of P2 acquired on the energy level highlighted in Table 4.2, Fig. 4.5b is the *adaptive sinogram* created with Alg. 6, and Fig. 4.5c is the *corrected-adaptive sinogram* after applying Alg. 7 on Fig. 4.5b. The reconstructed phantoms can be seen in Fig. 4.6, using the

TABLE 4.2: The SNR values of the reconstructed slices on different energy levels [keV] and (in the last row) the SNR values of the reconstructed slices of the *corrected-adaptive sinograms*.

Energy	Mean E.	P1	P2	P3	P4	P5	P6
2 - 20	15.7	1.874	3.204	2.604	3.309	3.593	1.074
3 - 25	19.3	3.156	4.900	4.080	5.033	5.373	1.095
3 - 30	22.1	4.550	8.880	6.782	8.697	9.392	1.110
5 - 35	24.5	5.236	9.820	7.544	10.021	9.894	1.119
10 - 40	26.8	6.162	11.236	8.756	11.018	10.719	1.124
15 - 45	28.8	6.310	10.648	8.796	10.335	9.924	1.119
20 - 50	31.7	6.548	11.149	9.242	10.889	10.689	1.042
25 - 55	35.6	7.357	12.728	10.290	12.376	11.412	0.957
30 - 60	40.1	7.797	13.237	10.598	12.875	11.051	0.883
35 - 65	44.9	7.157	10.621	9.634	10.298	9.417	0.803
40 - 70	49.9	6.178	8.896	7.994	8.835	7.872	0.722
45 - 75	54.9	5.321	7.387	6.567	7.291	6.658	0.632
50 - 80	59.9	4.408	5.992	5.337	5.987	5.451	0.540
55 - 85	64.4	3.742	5.125	4.414	5.073	4.618	0.462
60 - 90	60.3	3.049	4.082	3.481	4.023	3.640	0.378
<i>corr.adapt</i>	<i>corr.adapt</i>	7.797	14.175	10.729	13.757	11.610	1.111

corrected-adaptive sinograms.

Besides the slightly improved SNR values, another advantage of the adaptive voltage selection strategy is lower energy consumption, based on the fact that from certain angles, even a lower energy level could ensure optimal transmission. In the first and second row of Table 4.3, one can see the cumulated energies for creating the *original best* and the *corrected-adaptive sinogram*, respectively (using the mean energies of the different energy levels). Note that sinogram correction is simply achieved by executing Alg. 7, i.e., it does not affect cumulated energy. As we mentioned before, P1 is the perfect opposite of an elongated object. Therefore, the cumulated energy is the same in both cases. In all the other cases, the required energy for acquiring the *corrected-adaptive sinograms* is lower than it is needed for the *original best sinograms*. The percentages of the energy savings are shown in the third row. A significant energy reduction can be observed.

4.5 Summary

In this chapter, we proposed an adaptive Automatic Tube Voltage Selection method complemented with a sinogram equalization algorithm for CT. The method adjusts the tube voltage on-the-fly during the acquisition based on the transmission value of the previous projection. This method relies only on the projection data (and its transmission values) and does not require any further information. After producing the *adaptive sinograms*, an equalization is performed to get the *corrected-adaptive sinograms*. By experimental tests on software simulated phantoms using the GATE toolbox, we found that our method is able to produce optimized sinograms and by that ensuring lower energy consumption and slightly better reconstruction quality.

FIGURE 4.4: The transmission values of the scan of P2.

FIGURE 4.5: The sinograms of P2. (a) the *original best*, (b) the *adaptive* and (c) the *corrected-adaptive* one.TABLE 4.3: The cumulated energies needed for the *original best* and the *corrected-adaptive sinograms*, and the percentage of the saved energies, respectively.

	P1	P2	P3	P4	P5	P6
<i>orig. best</i> [keV]	11428.5	11428.5	11428.5	11428.5	10146	7638
<i>corr. adapt.</i> [keV]	11428.5	8930.4	10395	8930.4	8776.7	6906.1
Saved energy [%]	0	21.86	9.04	21.86	13.5	9.58

FIGURE 4.6: The reconstructed middle slices of the phantoms.

The presented method could be utilized in industrial NDT, and it has potential even in medical cases.

In this chapter, the author regards the following as his main contributions:

- Suggesting transmission for tube voltage selection.
- Design and implementation of the proposed methods.
- Creating the test phantoms and generating the sinograms in the simulation software.
- Testing the proposed methods.
- Evaluation of the results.

The findings of this chapter have been published in a conference proceedings [65].

Chapter 5

Convolutional Neural Networks in BT

5.1 Introduction

In *Chapter 2* and *Chapter 3*, we provided a detailed overview of the available approaches for finding the most informative angles in order to achieve binary tomographic reconstructions of acceptable quality. All of these algorithms focus on solving the problem of projection selection using procedural algorithms. Recently, deep learning approaches, especially CNNs have achieved tremendous success in various fields, such as classification [58], segmentation [99], denoising [126], super-resolution [54, 107] and removing low-dose related CT noise [52]. Although neural networks are widely used in image processing tasks, we are not aware of any studies focusing on the problem of projection selection using machine learning algorithms.

Furthermore, tools of machine learning can be utilized not just in the reconstruction but also in analyzing CT images. For a concrete example, the production of complex customized structures using 3D printing has drawn a significant interest both in industry and research due to its high flexibility and wide range of application. A large spectrum of scientific developments has led to a better understanding of the underlying physical processes and has contributed to the improvement of additive manufacturing (AM) (e.g. [80, 103]). However, a complete description of the AM processes and products remains a challenge. One particularly interesting research area is the material characterization of the final products, whose mechanical properties highly depend on the AM process parameters. Slight changes in these parameters may lead, for example, to surface roughness (e.g. [23, 26, 110]), undesirable porosity or other microstructural defects (e.g. [32, 53, 125]). Unfortunately, the experimental mechanical testing in AM is limited by the high cost of printing and the need for large statistical repetitions of the same specimens. Thus, the robust estimation of the mechanical characteristics of the final AM products through computational methods is an important area of research. A straightforward approach to evaluate the macroscopic (effective) mechanical characteristics of these structures is the direct numerical simulation (DNS) of a virtual experiment. Another one is numerical homogenization, which is advantageous due to its low computational cost. One recently proposed method is to perform a numerical analysis on a statistically representative geometrical model determined from a CT of final components [68].

In the first part of this chapter, we show that neural networks are capable of solving a complex task like projection selection without any reconstruction step (and to significantly decrease the running time of this process for the online scenario). In the second part, we provide a U-Net based segmentation algorithm for binarizing CT slices of different 3D printed specimens as the part of an industrial project, which aims to propose an alternative direct image-based computational framework

for the linear material characterization of imperfect microarchitected AM products, demonstrating the possibility of an efficient CT-based analysis of defective AM geometries using the high-order Finite Cell Method (FCM) [24]. We also show how well some other thresholding techniques perform in this particular case when CT slices are highly affected by different artifacts. In this thesis, we detail only the segmentation step, all the other parts and tasks of the project are out of our scope.

5.2 Artificial and Convolutional Neural Networks in general

Before getting into the details of the proposed methods, we introduce some important fundamentals about the Artificial and Convolutional Neural Networks (ANNs and CNNs). ANNs are based on the biological structure and behavior of the human brain, and the main application field for them is in Machine Learning. ANNs have provided an efficient and reliable tool for statistical pattern recognition and are capable of learning many tasks in the diverse field of computer science, and they are also applied frequently to other related disciplines. The ANNs are graph-based models, in which artificial neurons arranged in layers that communicate with each other through non-linear activation functions. Between the layers, numerous types of different activation functions could be used. Mainly, these functions are evaluated element-wise on the input matrix. The activation functions that we used are the ReLU (Rectified Linear Unit) and sigmoid functions. ReLU (Eq. 5.1, depicted in Fig. 5.1) is the default activation function recommended for use with most feed-forward neural networks. Applying this function to the output of a linear transformation yields a non-linear transformation. Because rectified linear units are nearly linear, they preserve many of the properties that make linear models easy to optimize with gradient-based methods.

$$relu(x) = \max\{0, x\} \quad (5.1)$$

FIGURE 5.1: The ReLU function.

The logistic sigmoid function (Eq. 5.2, depicted in Fig. 5.2) is commonly used to produce the ϕ parameter of a Bernoulli distribution because its range is $(0, 1)$, which lies within the valid range of values for the ϕ parameter. It helps us to get a smoother or more continuous range of values between 0 and 1.

$$\sigma(x) = \frac{1}{1 + \exp(-x)} \quad (5.2)$$

FIGURE 5.2: The logistic sigmoid function.

A general ANN consists of three different types of layers: input, hidden, and output. The anatomy of the layers can be seen in Fig. 5.3.

FIGURE 5.3: Fully connected ANN. Every neuron in a certain layer is connected to all the other neurons in the neighboring layer(s). The different colors denote the three main parts of the network.

The type of a layer is based on the operation that it performs. The number, types and sequence of the layers form the architecture of a network. The most often used layer types are Dense- and Convolutional layers. A Dense layer feeds all outputs from the previous layer to all its neurons, each neuron providing one output to the next layer. It is the most basic layer in neural networks. A Convolutional layer's parameters consist of a set of learnable filters (or kernels), which have a small receptive field but extend through the full depth of the input volume. During the forward pass, each filter is convolved across the width and height of the input volume, computing the dot product between the entries of the filter and the input and producing a 2-dimensional activation map of that filter. As a result, the network learns filters that activate when they detect some specific type of feature at some spatial position in the input. Some other layers that we used during our work are pooling, upsampling and merge layers. A pooling layer computes the maximum (or in some cases the average) of the convolved features, the upsampling layer is a simple layer with no weights that will double the dimensions of the input, and the merge or concatenation layer takes inputs and concatenates them along a specified dimension.

CNNs are different from the structure of other neural networks [58] since CNNs are mainly applied in the field of image processing and can also handle different types of input (e.g., image, video, voice). CNNs generally make use of convolutional layers (2-dimensional, in the case of binary images) in which the convolutional neurons respond to a predefined window of perception. Each neuron has its kernel, and these kernels are convolved with the image data. After computation with every possible position with its step size, a pooling layer is applied. This is necessary to

reduce the parameter space and to make the features translation invariant. After the desired number of convolutional and pooling layers, the collected activation values are flattened and connected to a dense neural network, which attempts to solve a classic machine learning task (e.g., classification or regression) [34].

Although CNNs are very effective tools in various fields of image processing, there was a major breakthrough in the field of image segmentation, a novel architecture named U-Net was proposed by Ronneberger et. al. [99]. U-Net is based on the fully convolutional networks [106], and its architecture was modified and extended to work with fewer training images and to yield more precise segmentations. The architecture consists of a contracting path to capture context and a symmetric expanding path that enables precise localization. Unlike in the contracting path, in the expanding path, the pooling operations are replaced by upsampling operators. Hence these layers increase the resolution of the output. The expansive pathway combines the feature and spatial information through a sequence of these up-convolutions and concatenations with high-resolution features from the contracting path. The expansive path is more or less symmetric to the contracting path, and yields a u-shaped architecture (hence the name).

5.3 Proposed projection selection algorithm using CNNs

In this section, our main aim is to find the most informative projection angles belonging to an object (software phantom), based only on its sinogram (projections) using CNNs. Therefore, we propose a CNN-based offline projection selection algorithm for solving this task (see Chapter 2 for a recall of offline projection selection). We detail the workflow of the method and how the architecture is built and parameterized. The tests and the results for our approach are also provided.

5.3.1 Method and CNN architecture

Fig. 5.4 summarizes the main steps of our method. The architecture of our model is implemented in Keras [20] and empirically parameterized. It consists of 3 convolutional layers with (5,5), (3,3), (3,3) kernel sizes, respectively, followed by two fully connected dense layers with 500 and 180 units. We use ReLU activations as the nonlinearities, aside from the last layers' sigmoid function. Between the convolutions we apply maximum pooling with a size of (3,3). *Adam* (adaptive moment estimation) [55] was used for the optimization, corresponding to the available directions.

First, the CNN takes a sinogram (180×91) as input (from the training database). After the 3 convolutional layers, a dense classifier connected to it outputs 180 activation values optimizing the MSE (Mean Squared Error) as the loss-function:

$$MSE(y, \hat{y}) = \frac{1}{N_e} \sum_{i=1}^{N_e} (y_i - \hat{y}_i)^2, \quad (5.3)$$

where y is the label and $\hat{y}$ is the predicted value, and N_e is the number of examples. One label is the one-hot encoded [39] variant of the most informative angles belonging to the actual sinogram. Therefore, one gets a 180-dimensional vector as one label, where all the informative angles are denoted with a single 1 bit and all the others with a 0. The predicted value also preserves this format. The only difference between y and $\hat{y}$ is that $\hat{y}$ can contain non-integer score values on a $[0,1]$ scale. Next, these output values are thresholded to get the minimum number of projections required for each entity. Since we might end up with more than the required number of

FIGURE 5.4: Our proposed pipeline. The input for the CNN is a sinogram, while the output of the network is 180 activation values (one for each direction) upon which we threshold and use K-means clustering. Afterwards, we reconstruct the images and calculate the RME.

projections, a K-means clustering is applied to determine the exact angles to be used in the reconstruction process. We found that the output values of the sigmoid units responsible for angles close to the ground-truth are relatively high, which makes clustering them a necessary step. For example, often the angles 89-90-91-92 are essential for precise reconstruction, but only one of them should be chosen. Lastly, calculating the RME between the original image and the images reconstructed using the 3 methods explored here (labels, predictions, and equiangular projections) gives an estimate of the effectiveness of the selection procedure.

5.3.2 Test environment

Our image database consisted of 8983 phantoms (icons) of varying structural complexity, each with size 64×64 pixels. 3427 images come from the *MPEG7 CE Shape-1 Part B*¹ database [61], 1400 images from *Game-icons.net*² and 4156 from *Flaticon*³. To create the training dataset, we performed a modified version of the *SFS* (Sequential Forward Selection) method (*Chapter 2, Section 2.2.3*). We started the algorithm without initial angles, which resulted in even the first two angles being selected by the algorithm (the same way as it would choose in the case of a bigger number of angles). Since the first two angles were not randomly chosen, we ran this algorithm only once, instead of the proposed 18 Multistart. The reason for these changes is that the *SFS*'s running time was much too long for 8983 images. Furthermore, with this approach, we obtained the sequence of the most informative angles, which was a useful property while generating the training data. They turned out to be feasible owing to the fact that many of the experiments needed to be done with a different number of label projections (e.g. 4-8). This way, we could reuse the already selected angles even if we needed less than 8 projections. The labels stored the information

¹http://www.imageprocessingplace.com/root_files_V3/image_databases.htm

²<https://game-icons.net/>

³<https://www.flaticon.com/>

in descending order, with the first containing the angle with the most valuable information for minimizing the reconstruction error. This way of selecting projections resulted in having only a local optimum with the most informative angles instead of calculating a global one. However, in our case, the former was also as applicable to the given task as the latter.

For the label projection selection, we used the same setup as we did in *Chapter 2* (Section 2.4), except for the above-mentioned changes in the *SFS* algorithm. For the validation of our CNN, the reconstructions were performed using the thresholded version of the *skimage* python package's SART algorithm. The output of SART is an image with intensity values around 0 and 1. The quality of the reconstructions using the predictions was measured with the RME.

5.3.3 Evaluation

Since our input for training consists of sinograms, which can be considered 2D images, we decided to apply CNNs. The sinograms extracted from the original images were 91 pixels wide for each projection direction, which formed a 180×91 sized image. The intensity ranges laid between 0-91, so we normalized all of our data by dividing by 91. Reducing the input shape by a scaling factor was also experimented with, but we did not notice any improvement regarding the accuracy. Based on this observation, we thought about increasing the size of the input parameters, which we also explored using 32760 and 65160 points of data as the source of the training set. These trials were prone to overfitting (when the gap between the training error and test error is too large) despite the strict regularization and normalization methods (dropout [111], batch normalization [45], early-stopping [34]) applied. To sum up, 180 by 91 pixels seems to be the optimal size for training the networks, as smaller input features decreased the accuracy of the reconstructions, while larger input spaces led to overfitting (not to mention the increase in training time and memory requirements).

Standard evaluation methods used for scoring neural network predictions might be misleading in the case of a task such as the one presented here. The reason for this is that a label mostly depends on the basic geometrical properties of the input image and it might generate some artifacts resulting from degree-favouritism (selection of common projection angles such as 0° or 90°), or it might find equiangular projections to be the best predictions. The latter might be regarded as the closest one to every possible label projection, but this produces sub-optimal reconstructions. Thus, the images were rotated randomly to eliminate or lessen the effect of the problem mentioned above. The main purpose of the projection selection procedures is to outperform the equiangular approaches by choosing the required angles in order to achieve a lower reconstruction error. To further investigate this issue and to get a better understanding of the selected projections, our method was compared to both the algorithmically selected projection angles, and to the equiangular angle set $\{i \frac{180^\circ}{r} \mid i = 0, \dots, r-1\}$, where r denotes the number of projections used in each case. We will simply refer to the former projection selection method (the modified *SFS*) as *Label* (as it was the training objective of our neural nets) and the latter as *Naive*, following the authors of [116]. The proposed method will be denoted as *CNN*.

Here, 10-fold cross-validation [56] was used during the training of our network, and we based our RME values and other statistical calculations on these runs.

TABLE 5.1: Average of RME and Standard Deviation values calculated from 10 runs for the three different approaches. The significance values were computed pairwise for *Naive-Label*, *Label-CNN* and *Naive-CNN* and the significant statistical differences are presented column-wise with the symbols of $\bullet/\circ$.

4 projections	<i>Label</i>	<i>CNN</i>	<i>Naive</i>
RME	0.3817 $\circ$	0.4015 $\circ$	0.4912 $\circ$
STD	0.2078	0.2189	0.2903
6 projections	<i>Label</i>	<i>CNN</i>	<i>Naive</i>
RME	0.3196 $\circ$	0.3430 $\circ$	0.3866 $\circ$
STD	0.1723	0.1723	0.2341
8 projections	<i>Label</i>	<i>CNN</i>	<i>Naive</i>
RME	0.2746 $\bullet$	0.2940 $\bullet$	0.3128 $\bullet$
STD	0.1842	0.2174	0.2312

5.3.4 Results

In Table 5.1, we present the RME and Standard Deviation (STD) values of the different methods with 4-6-8 angles, respectively. The $\bullet/\circ$ symbols highlight whether the differences are statistically significant ($\bullet$) or not ($\circ$), using a t-test with a significance level of 0.05. The results are statistically significant only with 8 projections. The RME values computed from the *Label* projection angles are naturally the smallest of the three, followed by our *CNN* approach. The equiangular approach produces the highest RMEs, meaning that the reconstructed images differed from the original ones the most using the *Naive* angle set. Using our 10-fold cross-validation test evaluations, we analyzed the RME values obtained using the 3 methods with 4 projections. We also present our findings in Table 5.2. In 37 cases our *CNN* managed to predict a set of projections that was as good as the *Label*. We note here that these values are from the test set containing examples never encountered during the training of the model. We should add that the more angles we have, the closer we will be to the equiangular approach in terms of minimizing the RME. Some of the reconstruction results with the RME values can be seen in Fig. 5.5, produced with 8 projections. The first row shows a case where the *CNN* selected the most informative projections. Reconstruction from labels produced the smallest RME in the second row. The last row shows an image where the equiangular set gave the best score.

TABLE 5.2: The numbers of images on which the 3 distinct methods gave the smallest RME values are shown along the diagonal. The other cells show how many times the two approaches gave the same RME value.

4 projections	<i>Label</i>	<i>CNN</i>	<i>Naive</i>
<i>Label</i>	4676	37	19
<i>CNN</i>	37	3096	14
<i>Naive</i>	19	14	1141

FIGURE 5.5: (a) Reconstruction from 180 projections. Results of (b) the *Label*, (c) the *CNN* approach and (d) the *Naive* approach. The values under the figures are the RME values compared to (a).

5.4 Segmentation of CT slices using CNNs

In this section, our goal is to provide a CNN based segmentation algorithm to binarize CT slices, which task is part of a bigger industrial project, aimed at performing a numerical analysis on a statistically representative geometrical model determined from a CT of final components. Here, we detail how the architecture of the model is constructed and how our segmentation algorithm works and performs. Furthermore, we compare our results to other segmentation approaches.

5.4.1 Motivation

In the scope of this work, the geometrical model for numerical analysis is obtained via CT. The CT-images provide an extensive description of the process-induced morphological differences, thus, making the material characterization more accurate. However, it is well known that different artifacts can be encountered in the CT-images [14] (see 5.8b and 5.11b as example CT slices of different specimens). Ring artifacts, noise, beam hardening, etc. can obscure geometrical borders and make numerical analysis difficult. Furthermore, the presence of metal objects in the CT-scan causes severe metal artifacts. They arise due to the high attenuating material properties of the metal itself and the metal edges [15, 71]. The question of artifact reduction for better reflection of the real objects is a separate area of research. Many different methods, e.g., projection completion strategies, multidimensional adaptive filtering, metal deletion technique, were proposed ([14, 50, 90] and the literature cited

therein). Because of these artifacts, the general image processing segmentation algorithms did not perform very well on this particular segmentation task. For example, global thresholding performed well only in certain parts of the slices, adaptive thresholding amplified the noise, thresholding by variational minimax optimization [101] gave a relatively good result but overestimated the regions filled with material. Instead of the classical thresholding approaches, we proposed to use CNNs for binarizing the CT slices on account of their ability to learn how to overcome the presence of artifacts while obtaining a suitable geometrical model for numerical analysis. A typical use case of CNNs is to provide image data (on a pixel level) to the network as input and, based on this information, to perform classification. This is especially suitable for the purposes of this work.

5.4.2 Architectures and the workflow of the segmentation

For the segmentation of the CT-image slices, first, we trained a U-Net, using the original source code⁴ (implemented in Tensorflow [1]), provided by the authors. Inside the architecture, ReLU activations are utilized, except for the last layer, which has a sigmoid function. For optimization, *Adam* with a binary cross-entropy (BCE) loss function is employed (learning rate was set to 10^{-5}). The BCE is defined as follows

$$BCE(y, \hat{y}) = -(y \cdot \log(\hat{y}) + (1 - y) \cdot \log(1 - \hat{y})) , \quad (5.4)$$

where y is the ground-truth and $\hat{y}$ is the predicted value.

Depending on the task, 3-4 original slices (meaning less than 1 percent of the whole database) have been manually segmented to get golden standards (labels). The training data is then formed by these few original CT-slices together with their corresponding golden standard binary masks. The test dataset consists of the non-segmented slices. To increase the number of training images, the input data is cut into pixel patches of size $n \times n$, where n is chosen from the interval $[50, 100]$ such that the remainder of the integer division $image\_width/n$ is the minimal possible. After the classification, the CT slices are reassembled from the patches. Since some false negatives and false positives may appear on the outer parts of the patches, they are padded on all sides. The specimen may contain some powder inside, which has a very similar intensity to the foreground pixels, so it might cause wrong classification results on a pixel level. To overcome this problem, some patches in which powder is present are shown twice to the neural network.

The output of the network is a probability distribution, where the outputted values are expected to be very close to 0 or 1, so they can be interpreted as binary pixel values. As a post-processing step, a global thresholding was applied with different threshold values, and the qualitatively best has been chosen as the final binarized result. The inside-outside areas are indicated by these binary slices for further analysis.

After the examination of the results, given by the U-Net, we wanted to improve its performance. Therefore, we started to change the number of layers and the size of kernels in the U-Net architecture. All the different modifications we did were based on empirical tests. We modified some parts of the architecture, re-trained the model, generated the predictions and analyzed the resulted binary images. Then, we kept changing various parts based on the outcome. At the end of this process, the original architecture got modified quite significantly. An overview of the convolutional layers used in our modified U-Net can be seen in Fig. 5.6. If one follows the

⁴<https://lmb.informatik.uni-freiburg.de/people/ronneber/u-net/>

diagram carefully, it can be seen that it is more like a 'W' shaped architecture than a 'U'. After numerous experimental tests, this model was the most suitable one for a certain interval of the CT slices, and it will be seen, that in some cases, this architecture provided better results, using exactly the same parameterization that was used for U-Net.

Fig. 5.7 summarizes the main steps of the above-proposed segmentation approach, using our modified U-Net.

FIGURE 5.6: The type of the operation and the number of convolutional kernels are noted above the boxes. The height, width and depth of the boxes are representative of the convolutional layers' output shape.

FIGURE 5.7: Our proposed pipeline. The input for the CNN is a CT slice patch, while the output of the network is the thresholded patch.

5.4.3 Evaluation

Our segmentation approach was validated on two additively manufactured metal components.

Inconel®718 Specimen 300. Fig. 5.8a shows a sample of a microporous metallic structure produced by the selective laser melting. The base mechanical properties of Inconel®718 are provided by Siemens AG. A part of the microstructure of one of the similar specimens was subjected to a CT. The overall domain size is $600 \times 414 \times 496$ voxels with an interval of $13.08\mu\text{m}$ in every direction. Due to the high contrast of Hounsfield units, the presence of trapped powder and alternating intensity differences, and metal artifacts in the resulting images (see Fig. 5.8b), the CT-scan is segmented using our segmentation technique detailed in Section 5.4.2. After running a flood-fill algorithm on the segmentation, a full 3D voxel model is constructed in Fig. 5.9.

FIGURE 5.8: (a) Inconel®718 Specimen 300. (b) Axial slice of the CT-Scan for Specimen 300.

FIGURE 5.9: Specimen 300: Voxel model after segmentation using the modified U-Net.

Unfortunately, for this specimen, the complete golden standard image set was not given. Furthermore, because of the presence of the trapped powder, the specimen's CT scans differed significantly from the 3D CAD model on which it was based. Therefore, in this case, we cannot analyze the quality of the segmentation from the image processing but only from a physical point of view. In this case, the directional Young's modulus is calculated on the discretized version ($100 \times 69 \times 124$ cells) of the thresholded specimen, using a technique called embedded uniaxial numerical tensile experiment (embedded DNS). Young's modulus is a mechanical property that measures the stiffness of a solid material. It defines the relationship between stress (force per unit area) and strain (proportional deformation) in a material in the linear elasticity regime of a uniaxial deformation. Then, this value was compared to the directional module $E_{zz} = 34,882.85$ [MPa] of the fully resolved specimen with 1,064 million DOFs for the high-order Voxel-FEM computation. The convergence curves (see Fig. 5.10) show that the last embedded computation with the polynomial degree of $p = 5$ leads to 11.3% error relative to an overkill solution provided by Voxel-FEM. Therefore, this discretization – and consequently our segmentation – is considered acceptable and useful for further numerical tests.

FIGURE 5.10: Specimen 300: Convergence of the directional Young's modulus E_{zz}

Inconel®718 Specimen 600. In this paragraph, a sample 600 of Inconel®718 is analyzed (see Fig. 5.11a). Due to a larger space between individual laser tracks, there is no powder trapped within the specimen. Moreover, the process parameters for this setup are easier to control. Therefore, the microstructure does not present a large variety of defects and is more reproducible. Nevertheless, the metal artifacts still remain present in the CT-scan (see Fig. 5.11b). Our CNN segmentation technique was applied to treat these defects and obtain the reliable 3D voxel model depicted in Fig. 5.12. The CT-scan consisted of 371 CT slices, and the dimension of one slice was 400×800 pixels.

FIGURE 5.11: (a) Inconel@718 Specimen 600. (b) Coronal slice of the CT-Scan for Specimen 600.

FIGURE 5.12: Specimen 600: Voxel model after segmentation using the modified U-Net.

For this specimen, a full golden standard image set was given. Therefore, this specimen could be analyzed from physical and from image processing point of view as well by comparing our segmentations to other thresholding approaches.

As the physical investigation, for the calculation of Young's modulus, two different discretizations were studied. The first one had $100 \times 46 \times 100$ finite cells, and the second one consisted of $200 \times 92 \times 200$ cells. Again, the embedded simulations were compared to a high-order Voxel-FEM solution with $E_{xx} = 23,584.62$ [MPa] of a fully resolved specimen with 1 428 159 774 DOFs for $p = 4$. Fig. 5.13 summarizes the relative error in Young's modulus E_{xx} for the two discretizations. The number of nodes and processes are the same as for specimen 300. The first discretization is considered acceptable as it provides a 9.74% error with respect to the overkill solution, which means that this discretization – and consequently our segmentation – is considered acceptable and useful for further numerical tests as well as it was in the case of the previous specimen.

FIGURE 5.13: Specimen 600: Convergence of the directional Young's modulus E_{xx}

Moving forward to the image processing field, during the evaluations, extremely big differences appeared in the qualities of certain CT slice intervals. After further analysis, we noticed that serious intensity changes can be observed on the horizontal axis of the CT slices. In Fig. 5.14a, one can see the vertical mean intensities of the first slice (Fig. 5.14c). We could say that it is close to the expected optimal intensity distribution. In contrast, Fig. 5.14b shows a quite different intensity distribution belonging to the very middle (186.) slice (Fig. 5.14d). The intensities are really high in the middle part of the slice and really low in the outer parts. So the intensity is quite varying. Mainly this phenomenon made this segmentation task difficult. It will be visible in the results later in Section 5.4.4.

For evaluating the performance of different segmentation techniques, numerous different approaches were proposed in the last decades [2, 75, 105, 114, 127]. We based our quantitative performance evaluation on a related research. In [105], the authors presented 40 segmentation techniques, and then ranked them based on 5 different criteria: misclassification error, edge mismatch, region nonuniformity, relative foreground area error, and shape distortion penalty via Hausdorff distance. We decided to use 4 measurements for this task. We used three propositions of [105]: misclassification error, relative foreground area error, and shape distortion penalty via Hausdorff distance. Instead of the edge mismatch, we used the mean-square distance figure of merit introduced by Pratt [89] because we found it a better measure for analyzing edges. We did not use the region nonuniformity because it does not require ground-truth information, which could be misleading in this case. Here we define the used quality descriptors:

FIGURE 5.14: The variance of the vertical mean intensities of different slices and the slices themselves. Left: Slice no. 1 - first, Right: Slice no. 186 - middle.

Misclassification error (ME): it reflects the percentage of background pixels wrongly assigned to foreground, and conversely, foreground pixels wrongly assigned to the background. For the two-class segmentation problem, *ME* can be simply expressed as

$$ME = 1 - \frac{|B_O \cap B_T| + |F_O \cap F_T|}{|B_O| + |F_O|}, \quad (5.5)$$

where B_O and F_O denote the background and foreground of the original (ground-truth) image, B_T and F_T denote the background and foreground area pixels in the test image, respectively, and $|\cdot|$ is the cardinality of the set.

Pratt's Figure of Merit (PFOM): a simple mean-square distance figure of merit, mainly used for evaluation of enhancement/thresholding edge detectors:

$$PFOM = 1 - \frac{1}{\max\{I_I, I_A\}} \sum_{i=1}^{I_A} \frac{1}{1 + \alpha d^2(i)}, \quad (5.6)$$

where I_I and I_A are the numbers of ideal and actual edge points, respectively, $d(i)$ is the pixel miss distance of the i th edge detected, and α is a scaling constant chosen to be $\alpha = \frac{1}{9}$ to provide a relative penalty between smeared edges and isolated, but offset edges.

Relative foreground area error (RAE): it compares the areas, as obtained from the segmented image compared to the reference image:

$$RAE = \begin{cases} \frac{A_0 - A_T}{A_0} & \text{if } A_T < A_0, \\ \frac{A_T - A_0}{A_T} & \text{if } A_0 \leq A_T, \end{cases}$$

where A_0 is the area of the reference image, and A_T is the area of the thresholded image.

Shape distortion penalty via Hausdorff distance (HD): The Hausdorff distance can be used to assess the shape similarity of the thresholded regions to the ground-truth shapes. Recall that, given two finite sets of points, say ground-truth and thresholded foreground regions, their Hausdorff distance is defined as

$$HD(F_O, F_T) = \max\{d_H(F_O, F_T), d_H(F_T, F_O)\}, \quad (5.7)$$

where $d_H(F_O, F_T) = \max_{f_O \in F_O} d(f_O, F_T) = \max_{f_O \in F_O} \min_{f_T \in F_T} \|f_O - f_T\|$ and $\|f_O - f_T\|$ denotes the Euclidean distance of two pixels in the ground-truth and thresholded objects.

5.4.4 Results

Table 5.3 shows how the different thresholding methods performed based on the four different criteria. In the first column, one can see the thresholding approaches we used for the comparison. These methods are: Otsu method [82], local adaptive thresholding [33], variational minimax optimization based thresholding [101], fuzzy C-means [13], and the original U-Net. All the performance measures are adjusted so that their scores vary from 0 (for an entirely correct segmentation) to 1 (for an entirely erroneous case). Columns 2-5 contain the average scores over all the slices belonging to different quality measures in case of different segmentation techniques. To obtain an average performance, the authors of [105] suggested using the arithmetic average of the normalized scores defined above. The sixth column of Table 5.3 shows this average value. In every column, the smallest value is indicated with bold characters. One can see that U-Net and the proposed architecture can significantly outperform all the other techniques. In two of four measures, the U-Net performed better, while in the other two cases, the proposed architecture did. Taking the last (average) column into account, overall, the proposed approach seems to be the best one.

The slice-by-slice average scores belonging to the different thresholding methods are depicted in Fig. 5.15. One can see that the lanes between the trend lines are quite consistent. It can also be observed that significantly different score values appear for the inner slices than for the outer ones. This phenomenon is based on the varying intensities, which appear moving towards the middle slices, and then changing back as we are getting closer to the last slice as we discussed previously in Section 5.4.3.

TABLE 5.3: Comparison of different thresholding approaches using different quality measures

Techniques	$PFOM$	RAE	ME	HD	Average
Adaptive	0.334	0.327	0.162	0.597	0.355
Otsu	0.225	0.205	0.133	0.591	0.289
Fuzzy C-means	0.212	0.188	0.132	0.592	0.281
Variational	0.134	0.117	0.134	0.600	0.246
U-Net	0.105	0.093	0.084	0.559	0.210
Proposed	0.059	0.032	0.093	0.579	0.191

FIGURE 5.15: The mean values of the different quality measures belonging to the different segmentation techniques.

To compare the U-Net and the proposed architecture from another perspective, U-Net's total number of parameters is 1 940 817, while in the case of the proposed architecture, this number is only 566 657. This means that the proposed one consists of almost four times fewer parameters and can still provide at least as good results as the U-Net can, moreover, in most of the cases it can outperform U-Net.

5.5 Summary

In the first part of this chapter, we trained a CNN to select projections for the accurate reconstruction of BT images. After training, the prediction was achieved using K-means clustering to get a smaller set of projection data. After comparing the procedural algorithm and the results obtained using a neural network, we observed

only a small increase in the RME values compared to the reconstructions from *Label* projections (but performance-wise we achieved a notable improvement with the former). We found that CNNs can be applied to projection selection tasks by training in a multilabel classification scenario. To the best of our knowledge, this was the first attempt to predict projections of binary images for reconstruction using CNNs and to perform projection selection without any reconstruction step.

In the second part, we proposed CNNs for solving a particular segmentation problem, as part of a bigger project. First, we used U-Net for the segmentations, then modified its architecture to improve its performance. Two specimens were given for the experimental tests. In the case of the first specimen, only the physical experiments could be done due to the lack of golden standard thresholded CT slices. For the second specimen, the complete ground-truth image set was given, therefore, both physical and image processing analyzes were possible. Physical investigations showed that in the case of both specimens, the segmentations were acceptable and reasonable for further numerical tests. Furthermore, our segmentation result featuring the second specimen was compared to state-of-the-art segmentation techniques. Using different segmentation quality measures, we showed that our proposed model is smaller and can perform better than the U-Net can in certain cases.

In this chapter, the author regards the following as his main contributions:

1. Projection selection with CNNs

- Providing the theoretical background for projection selection.
- Proposing CNNs for projection selection and the elaboration of its basis.
- Modifying the projection selection algorithm, and generating the train and test databases.
- Comparing the proposed method to previously published methods.
- Evaluation of the results.

2. Segmentation with CNNs

- Proposing CNNs for solving the segmentation problem.
- Generating the train and test databases.
- Applying U-Net on the data.
- Modifying U-Net to get the proposed architecture.
- Implementation and optimization of certain parts of the above-mentioned method used in the experiments.
- Comparing the segmentation results to state-of-the-art methods using different quality measures.
- Evaluation of the results.

The findings of this chapter were in part published in a conference proceedings [84] and in a journal paper [57].

Chapter 6

Final Conclusions

First, in this dissertation, we presented sequential feature selection strategies for projection selection in BT and compared them to previously published methods. We proposed an algorithm called *Refinement*, as a post-processing step for different projection selection methods. With experimental tests, we showed that *SFSR* provides the best angle sets, especially in the case of few projections, and even equiangular approaches could perform almost as well as *SFSR* when they are extended with *Refinement* (*NaiveR* and *EquiAngR*) and still preserving their fast execution time. Based on the experimental results provided by the different projection selection approaches, we suggested two strategies to reduce the amount of data needed to select the most informative projection angles for binary tomographic reconstructions. We showed by experiments that for projection selection purposes, the images can be downscaled to a certain scale level and the number of beam rays can also be lowered for the projection acquisition. We also showed that perimetric complexity is a good measure to automatize the reduction process and to choose the proper number of projections. The results can be useful in developing and testing new projection selection methods.

Then, we proposed the modified versions of an offline- and an online projection selection method in BT. The key idea was to use reconstruction uncertainty to select angles. We compared these methods to state-of-the-art algorithms using experimental tests on software phantom images. We found that our offline method is able to provide approximately as accurate reconstruction results as methods that use blueprint images to find the optimal set of projections. In the online case, we showed that our method can outperform the state-of-the-art algorithms and provide good alternative solutions for selecting projection angles. Based on our experiments with reconstruction uncertainty in BT, we provided a measure for the variability of the reconstructed pixels in CT, based only on the projection geometry. Then we used this variability measure to give theoretically proven upper and lower bounds for the pixel values in the reconstructions. We also showed that the perturbed reconstructions can directly be applied for calculating the difference between the original object of study (ground-truth image) and its reconstructions, and how it could be used to predict the reconstruction error that may arise during the reconstruction. We gave practical validation and analysis to the results.

Next, we proposed an adaptive Automatic Tube Voltage Selection method complemented with a sinogram equalization algorithm for CT. The method adjusts the tube voltage on-the-fly during the acquisition based on the transmission value of the previous projection. This method relies only on the projection data and does not require any further information. After producing the adaptive sinogram, an equalization is performed. By experimental tests on software simulated phantoms using the GATE toolbox, we found that our method is able to produce optimized sinograms and by that ensuring lower energy consumption and slightly better reconstruction

quality. The presented method could be utilized in industrial NDT, and it has potential even in medical cases.

Finally, we trained a CNN to select projections for the accurate reconstruction of BT images. After training, prediction is achieved using K-means clustering to get a smaller set of projection data. After comparing the procedural algorithm and the results obtained using a neural network, we observed only a small increase in the RME values compared to the reconstructions from label projections (but performance-wise we achieved a notable improvement with the former). We found that CNNs can be applied to projection selection tasks by training in a multilabel classification scenario. To the best of our knowledge, this was the first attempt to predict projections of binary images for reconstruction using CNNs and to perform projection selection without any reconstruction step. Also in the field of machine learning, we used U-Net for the segmentation of CT slices as the part of a bigger project, and proposed some modifications on its architecture to improve its performance. We compared our results to state-of-the-art segmentation techniques. Using different segmentation quality measures, we showed that the proposed architecture consists of almost four times less parameters and can still provide at least as good results as the U-Net can, moreover, in most of the cases it can over-perform U-Net.

Appendix A

Summary in English

The goal of tomographic reconstruction is to discover the inner structure of objects using their projections in a slice-by-slice manner. Most often, the projections are produced by X-rays. Measuring the attenuation of the beams passing through the object can provide information about the density of the materials along the paths of the beams. Collecting such projection data from many angles, one can produce an image showing the inner structure of the object. Computed Tomography (CT) is a widely used tool in different fields, such as medical imaging, non-destructive testing (NDT), material characterization, crystallography, etc. to analyze the interior of the subject of investigation.

Discrete Tomography (DT) uses the prior information that the cross-section image to be reconstructed contains only a few different intensities which are known in advance. Exploiting this prior knowledge, a smaller set of projections is sufficient for a reconstruction with an acceptable quality.

Binary Tomography (BT) is a more restricted variant of DT. In this case, every single pixel of the image to be reconstructed can take only two different intensities. In practice, objects corresponding to these images must be made of a homogeneous material. Based on this fact, one pixel in the cross-section image holds the information on the presence or absence of the material. BT is most commonly used in the fields of industrial NDT and electron microscopy.

This thesis is the summary of the Author's research in the field of CT and BT. Our main aim was to improve reconstruction quality by developing novel algorithms and improving previous approaches in the research fields of selecting the most informative projection angles, automatic selection of the tube voltage of a CT scanner, and binarizing already reconstructed CT slices using Convolutional Neural Networks.

A.1 Key points of the thesis

The findings of the research can be divided into four thesis groups. Table A.1 gives the connection between the results and the publications of the Author.

In the first thesis group, I proposed different sequential projection selection algorithms and analyzed their behavior under serious resolution down-scaling. These results were published in a conference proceedings [64], and in a journal paper [63].

- I/1. I adapted and implemented sequential selection and search approaches to projection selection methods with almost fully deterministic behavior. I compared these methods to already published algorithms and showed with experimental tests that my algorithms could outperform the already existing approaches in this field.

- I/2. I suggested two approaches to reduce the amount of data needed to select the most informative projection angles, and also examined how changing the scale of blueprint images and number of detectors affect the performance of projection selection algorithms. I developed and tested these approaches with the previously proposed projection selection methods. After the evaluation of the results, I proposed three different application fields where the scale invariance of projection selection could be used.

In the second thesis group, I embedded reconstruction uncertainty into projection selection algorithms and showed how grayscale uncertainty could be used in CT. These results were published in three conference proceedings [66, 67, 118], and are submitted as a journal paper [117].

- II/1. I modified already implemented offline and online projection selection algorithms to embed the uncertainty as the optimization measure. I tested and compared them to already published algorithms. I showed that these algorithms can select the most informative projection angles by relying only on the reconstruction uncertainty, and even in the offline case, they do not require any blueprint data during the selection process.
- II/2. I gave experimental validation to our results in the field of defining the grayscale uncertainty. I showed how the grayscale uncertainty could be used in the reconstruction of phantom images with continuous intensity interval and its correlation with the standard deviation of reconstructed pixel intensities, and how one can predict the upper and lower boundaries of the intensity values in practice. I also showed how the perturbed reconstructions could be used in practice to predict the reconstruction error that may arise during the reconstruction, which was followed by the evaluation of the results.

In the third thesis group, I proposed a transmission-based adaptive Automatic Tube Voltage Selection method for CT. The results of this thesis group were published in a conference proceedings [65].

- III/1. I proposed an adaptive Automatic Tube Voltage Selection method complemented with a sinogram equalization algorithm for CT. I proposed to use only the transmission value of the sinogram to adjust the tube voltage on-the-fly during the acquisition and thus, to rely only on the projection data. By experimental tests on software simulated phantoms using the GATE toolbox, I found that this method is able to produce optimized sinograms and by that ensuring lower energy consumption and slightly better reconstruction quality. I pointed out that the presented method could be utilized in industrial non-destructive testing, and it has potential even in medical cases.

In the fourth thesis group, I proposed a CNN based offline projection selection algorithm and modified the well known U-Net architecture for the segmentation of CT images. The results of this thesis group were published in a conference proceedings [84], and in a journal paper [57].

- IV/1. I proposed CNNs for projection selection to show that the most informative angles could be chosen without performing any reconstruction. I provided

Appendix B

Summary in Hungarian

A tomográfiai rekonstrukció célja különböző objektumok belső felépítésének feltárása a vetületeinek szeletről-szeletre történő felhasználásával. Legtöbb esetben a vetületek röntgensugárzás segítségével állnak elő. Az objektumon áthaladó sugarak elnyelődésének mérése ad információt az anyag sűrűségéről a sugarak pályáján. Az ilyen vetületi adatoknak a több irányból való begyűjtésével előállítható egy kép, mely az objektum belső struktúráját ábrázolja. A számítógépes tomográfia (Computed Tomography, CT) egy széleskörűen használt eszköz a legkülönbözőbb tudományos területeken, mint például az orvosi képalkotás, ipari nemroncsoló tesztelés (NDT), különböző anyagok karakterisztikájának jellemzése, kristallográfia, stb.

A diszkrét tomográfia (Discrete Tomography, DT) azon prior információt használja ki, mely szerint a rekonstruálandó szelet csak néhány különböző intenzitást tartalmazhat, amelyek előre ismertek. Ezen prior tudás kihasználásával kevesebb vetület is elegendő lehet elfogadható minőségű rekonstrukciók végrehajtásához.

A bináris tomográfia (Binary Tomography, BT) egy sokkal korlátozottabb variánsa a DT-nek. Ebben az esetben a rekonstruálandó kép pixelei csak két különböző intenzitásértéket vehetnek fel. A gyakorlatban tehát ez azt jelenti, hogy az objektumoknak, melyekről ilyen technikával készül rekonstrukció, homogén anyagból kell lenniük. Ezekre a tényekre alapozva, a vetületi szelet egy pixele az adott anyag jelenlétéről vagy hiányáról tárol információt. A BT-t az ipari NDT és elektron mikroszkópia területein alkalmazzák a leggyakrabban.

Ezen értekezés a Szerző CT és BT területén végzett tudományos munkásságát foglalja össze. A kutatás fő célja az volt, hogy javítsa a rekonstrukció minőségét új algoritmusok fejlesztésével és már meglévő eljárások továbbfejlesztésével az alábbi területeken: leginformatívabb vetületi szögek kiválasztása, a CT szkennerek csőfeszültségének automatikus megválasztása, és rekonstruált CT szeletek binarizálása konvolúciós neuronhálók (CNNs) segítségével.

B.1 Az eredmények tézisszerű összefoglalása

Az értekezés eredményei négy csoportba sorolhatók. Az eredmények és a kapcsolódó publikációk viszonyát a [B.1](#) táblázat foglalja össze.

Az első téziscsoportban különböző szekvenciális vetületválasztó eljárásokat javasoltam, és vizsgáltam ezen módszerek viselkedését a felbontás csökkentésének függvényében. A téziscsoport eredményei egy konferencia kiadványban [\[64\]](#) és egy folyóirat publikációban [\[63\]](#) jelentek meg.

I/1. Átültettem és implementáltam szekvenciális választó és kereső módszereket, hogy vetületválasztásra megfelelőek legyenek, mindemellett majdnem teljesen

megőrizve a determinisztikus viselkedésüket. Összehasonlítottam ezen módszereket más, már publikált eljárásokkal, és kísérleti tesztekkel bemutattam, hogy az eljárásaim képesek túlteljesíteni az eddigi eljárások eredményeit ezen kutatási területen.

- I/2. Két megközelítést javasoltam a leginformatívabb vetületi szögek kiválasztásához szükséges adatok mennyiségének csökkentésére, és vizsgáltam, hogy a blueprint képek felbontásának illetve a detektorok számának csökkentése hogyan befolyásolja a vetületválasztó módszerek teljesítményét. Kidolgoztam és teszteltem ezeket a megközelítéseket az előzőleg javasolt vetületválasztó eljárásokkal. Az eredmények kiértékelése után ajánlottam három kutatási területet, melyen hasznosíthatók lehetnek a skálafüggetlenséggel kapcsolatos eredményeim.

A második téziscsoportban beépítettem a rekonstrukciós bizonytalanságot különböző vetületválasztó eljárásokba és bemutattam, hogy hogyan alkalmazható a szürkeárnyaltos bizonytalanság a számítógépes tomográfiában. A téziscsoport eredményei három konferencia kiadványban [66, 67, 118] jelentek meg, továbbá benyújtásra került egy folyóirat cikk [117] is.

- II/1. Módosítottam meglévő offline és online vetületválasztó eljárásokat, mely módosítás során beépítettem a rekonstrukciós bizonytalanságot, mint optimalizálási értéket. Teszteltem és összehasonlítottam ezen algoritmusokat meglévő eljárásokkal. Bemutattam, hogy ezek az algoritmusok képesek kiválasztani a leginformatívabb vetületi irányokat csupán a bizonytalanság értékére támaszkodva, és hogy offline esetben sem igényel az eljárás blueprint adatot a választás során.
- II/2. Kísérleti validálást nyújtottam a szürkeárnyaltos bizonytalanság definiálása terén elért eredményeinkhez. Megmutattam, hogy a szürkeárnyaltos bizonytalanság hogyan használható szürkeárnyaltos fantom képek rekonstruálásakor. Megvizsgáltam a bizonytalanság kapcsolatát a rekonstruált képek pixelintenzitásainak szórásértékével. Bemutattam, hogy hogyan lehet előre meghatározni egy adott kép pixelintenzitásainak alsó és felső korlátját a gyakorlatban. Emellett példát adtam rá, hogy a perturbált rekonstrukciókat miként lehet használni a gyakorlatban, hogy megjósoljuk a hibát, amely a rekonstrukció alatt jelenhet meg.

A harmadik téziscsoportban javasoltam egy transzmissziós információ alapú adaptív automatikus csőfeszültség-választó eljárást CT felvételek elkészítéséhez. A téziscsoport eredményei egy konferencia kiadványban [65] jelentek meg.

- III/1. Egy transzmissziós információ alapú adaptív automatikus csőfeszültség választó eljárást javasoltam, egy szinogram kiegyenlítő algoritmussal egybekötve, melyet CT felvételek előállítása közben lehet alkalmazni. Javasolataim alapján csupán a vetületekből kinyerhető transzmisszió nagyságát felhasználva megválasztható a csőfeszültség. A GATE eszköztárban szoftveresen szimulált fantomokon végzett kísérletekkel mutattam be, hogy az ajánlott eljárás képes optimalizált szinogramok előállítására, így biztosítva a kisebb energiafogyasztást és mérsékelt jobb rekonstrukciós minőséget.

Appendix C

Publications of the Author

Journal publications

- Gábor Lékó and Péter Balázs. “Detecting Steel Cord Discontinuities in Tire Tread X-Ray Images: A Preliminary Study”. *International Journal of Circuits, Systems and Signal Processing (NAUN)* 11 314-318 (2017), ISSN: 1998-4464. SJR: Q4
- Gábor Lékó and Péter Balázs. “Scale Invariance in Projection Selection Using Binary Tomography”. In: *Fundamenta Informaticae* 172.2 (2020), pp. 129–142. DOI: [10.3233/FI-2020-1897](https://doi.org/10.3233/FI-2020-1897). IF: 1.204. SJR: Q3
- Nina Korshunova, John Jomo, Gábor Lékó, Daniel Reznik, Péter Balázs and Stefan Kollmannsberger. “Image-based material characterization of complex microarchitected additively manufactured structures”. In: *Computers & Mathematics with Applications* 80 (11 2020), pp. 2462–2480. ISSN: 0898-1221. DOI: [10.1016/j.camwa.2020.07.018](https://doi.org/10.1016/j.camwa.2020.07.018). IF: 3.370. SJR: Q1
- László G. Varga, Gábor Lékó, and Péter Balázs. “Grayscale Uncertainty and Errors of Tomographic Reconstructions Based on Projection Geometries and Projection Sets”. In: *Submitted to Journal of Combinatorial Optimization - SI: Combinatorial Optimization in Imaging Sciences* (2020). IF: 0.843. SJR: Q2

Full papers in conference proceedings

- Gábor Lékó and Péter Balázs. “Sequential Projection Selection Methods for Binary Tomography”. In: *Computational Modeling of Objects Presented in Images. Fundamentals, Methods, and Applications*. Ed. by Reneta P. Barneva et al. Vol. 10986. Cham: Springer International Publishing, 2019, pp. 70–81. ISBN: 978-3-030-20805-9. DOI: [10.1007/978-3-030-20805-9_7](https://doi.org/10.1007/978-3-030-20805-9_7).
- Gábor Lékó, Péter Balázs, and László G. Varga. “Projection Selection for Binary Tomographic Reconstruction Using Global Uncertainty”. In: *Image Analysis and Recognition*. Ed. by Aurélio Campilho, Fakhri Karray, and Bart ter Haar Romeny. Vol. 10882. Cham: Springer International Publishing, 2018, pp. 3–10. ISBN: 978-3-319-93000-8. DOI: [10.1007/978-3-319-93000-8_1](https://doi.org/10.1007/978-3-319-93000-8_1). CORE: C
- Gábor Lékó, Szilveszter Domány, and Péter Balázs. “Uncertainty Based Adaptive Projection Selection Strategy for Binary Tomographic Reconstruction”. In: *Computer Analysis of Images and Patterns*. Ed. by Mario Vento and Gennaro Percannella. Vol. 11679. Cham: Springer International Publishing, 2019, pp. 74–84. ISBN: 978-3-030-29891-3. DOI: [10.1007/978-3-030-29891-3_7](https://doi.org/10.1007/978-3-030-29891-3_7). CORE: B

- Gergely Pap, Gábor Lékó, and Tamás Grósz. “A Reconstruction-Free Projection Selection Procedure for Binary Tomography Using Convolutional Neural Networks”. In: *Image Analysis and Recognition*. Ed. by Fakhri Karray, Aurélio Campilho, and Alfred Yu. Vol. 11662. Cham: Springer International Publishing, 2019, pp. 228–236. ISBN: 978-3-030-27202-9. DOI: [10.1007/978-3-030-27202-9_20](https://doi.org/10.1007/978-3-030-27202-9_20). CORE: C
- Gábor Lékó and Péter Balázs. “Transmission Based Adaptive Automatic Tube-Voltage Selection for Computed Tomography”. In: *Combinatorial Image Analysis*. Ed. by Tibor Lukić et al. Vol. 12148. Cham: Springer International Publishing, 2020, pp. 199–208. ISBN: 978-3-030-51002-2. DOI: [10.1007/978-3-030-51002-2_14](https://doi.org/10.1007/978-3-030-51002-2_14). CORE: C
- László G. Varga, Gábor Lékó, and Péter Balázs. “Grayscale Uncertainty of Projection Geometries and Projections Sets”. In: *Combinatorial Image Analysis*. Ed. by Tibor Lukić et al. Vol. 12148. Cham: Springer International Publishing, 2020, pp. 123–138. ISBN: 978-3-030-51002-2. DOI: [10.1007/978-3-030-51002-2_9](https://doi.org/10.1007/978-3-030-51002-2_9). CORE: C

Other publications

- Péter Balázs, Gábor Lékó, Zoltán Ozsvár, Gábor Petrovski, Judit Szűcs, László Varga: Design and implementation of the informatical system of a 3D industrial CT scanner, Abstracts of the 11th Conference of the Hungarian Association of Image Processing and Pattern Recognition, Szováta, Romania, 24-27 January, 2017, 10 pages (in Hungarian).
- Gábor Lékó, Péter Balázs, László G. Varga: Projection selection using uncertainty in binary tomography, Abstracts of the 5th Winter School of PhD Students in Informatics and Mathematics, Debrecen, Hungary, 2-4 March, 2018, pp. 30.
- Gábor Lékó, Péter Balázs, László G. Varga: Projection selection with sequential selection methods using different evaluation measures, Volume of Short Papers of the 11th Conference of PhD Students in Computer Science, Szeged, Hungary, 25-27 June, 2018, pp. 79.
- Gábor Lékó, Péter Balázs, László G. Varga: Sequential projection selection methods and global uncertainty in projection selection, Abstracts of the 12th Conference of the Hungarian Association of Image Processing and Pattern Recognition, Debrecen, Hungary, 28-31 January, 2019, 15 pages (in Hungarian).

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