University of Szeged

Department of Computer Science

The Chooser-Picker games

PHD Dissertation- Theses

Written by:
András Csernenszky

Supervisor:
András Pluhár
Associate Professor

Szeged
2011

Abstract

The main goal of this work is to understand Picker-Chooser (or Chooser-Picker) games and Beck's conjecture as deeply as possible. The text has three main parts.

At first we examine the complexity of Picker-Chooser(P-C) and Chooser-Picker(C-P) games. Here we found that it is NP-hard to decide the winner for both P-C and C-P games [24]. Then we discuss the Picker-Chooser version of well-known games, to explore the differences and similarities among the various types. The examined games are the C-P 4×4 *Tic-Tac-Toe*, the P-C version of generalized Shannon switching game, the C-P version of the k-in-a-row and some of the C-P, M-B (Maker-Breaker) and P-C Torus games. We improve a little on the "Erdős-Selfridge" theorem for C-P games, although a gap remains this and the conjectured form [21].

Secondly, we solve with the Chooser-Picker 7-in-a-row game. This game is quite interesting because the last really valuable result for the 8-in-a-row game (by playing on infinite board the 8-in-a-row game the second player can achieve a draw), was made more than 30 years ago. Since then all attempts to solve the 7-in-a-row was unsuccessful. The thesis deals with the Chooser-Picker version of the same problem. In that section we prove that the Chooser-Picker 8-in-a-row and the Chooser-Picker 7-in-a-row game is a Picker win. The proof is a bit lengthy and a non- trivial case study. After we sketch some idea how can we deal with the original (M-M or M-B) version of this game [22].

Finally we will discuss the P-C diameter games. Here we found a very interesting result that how different result is given by the Maker-Breaker version and the Picker-Chooser version [2, 23]. As we show the Picker-Chooser version restores the probabilistic intuition, just like the acceleration of the game.

Chapter 1

Definitions, a conjecture and some new tools

1.1 The weak version of the games

There can be defined the weak version of the positional games [6], where the second player wins if he/she can achieve a draw. It means that the first player do not have to be afraid of (and defend against) that the second player occupies a winning set. Here the first player is called Maker, and the second is called Breaker. It is easy to see the following statement, see [7].

Statement 1.1. *If the Breaker wins in the weak version of the game, then the strong version is draw.*

1.2 Chooser-Picker and the Picker-Chooser games

Studying the very hard clique games, Beck [6] introduced a new type of heuristic, that proved to be a great success. He defined the *Picker-Chooser* or shortly P-C and the *Chooser-Picker* (C-P) versions of a Maker-Breaker game that resembles fair division, (see [70]). In these versions Picker takes an unselected pair of elements and Chooser keeps one of these elements and gives back the other to Picker. In the Picker-Chooser version Picker is Maker and Chooser is Breaker, while the roles are swapped in the Chooser-Picker version. When $|V|$ is odd, the last element goes to Chooser. Beck obtained that conditions for winning a Maker-Breaker game by Maker and winning the Picker-Chooser version of that game by Picker coincide in several cases. Furthermore, Breaker's win in the Maker-Breaker and Picker's win in the Chooser-Picker version seem to occur together.

The study of these games gives invaluable insight to the Maker-Breaker version. For some hypergraphs the outcome of the Maker-Breaker and Chooser-Picker versions is the same [6, 21]. In all cases it seems that Picker's position is at least as good as Breaker's. It was formalized in the following conjecture.

Conjecture 1.2. *If Maker (as the second player) wins the Maker-Breaker game, then Picker wins the corresponding Picker-Chooser game. If Breaker (as the second player) wins the Maker-Breaker game, then also Picker wins the Chooser-Picker game.[21]*

It is necessary for the Chooser-Picker Games infinite version the following restriction: At the beginning Chooser can select a bounded subset of the board, where they will play. Because if they play on the infinite board, then Picker could select points far from each other, and it is a trivially winning strategy for Picker.

1.3 Toolbar

1.3.1 Pairing lemma

Lemma 1.3 (Cs-P)**.** *If in the course of the (Chooser- Picker) game (or just already at the beginning) there is a two element winning set* $\{x, y\}$ *then Picker has an optimal strategy starting with* $\{x, y\}$ *.*

1.3.2 The monotonicity lemma

We mentioned that the at infinite version Chooser can select a bounded subset. In practice it means that Chooser selects a finite set $X \in V$, and they play on the *induced sub-hypergraph* that is keep only those edges $A \in \mathcal{F}$ for which $A \subset X$. More formally, given the hypergraph $(V, \mathcal{F})$ let $(V \setminus X, \mathcal{F}(X))$ denote the hypergraph where $\mathcal{F}(X) = \{A \in \mathcal{F}, A \cap X = \emptyset\}$.

Lemma 1.4. *[21] If Picker wins the Chooser-Picker game on* $(V, \mathcal{F})$*, then Picker also wins it on* $(V \setminus X, \mathcal{F}(X))$*.*

This lemma is useful tool at the next chapters, because if a bounded set S cant be partitioned into uniform sub-games, then it can be increased to S', which can be split into such sub-games. And if Picker wins on S', then also can win on S.

1.4 Some results on Chooser-Picker games

1.4.1 Complexity of Chooser-Picker positional games

Since the Maker-Breaker (and the Maker-Maker) games are PSPACE-complete, see [63], it would support both Conjecture 1.2, and the above heuristic to see that the Chooser-Picker or Picker-Chooser games are not easy, too. To prove PSPACE-completeness for positional games is more or less standard, see [63, 62, 16]. Here we can prove less because of the asymmetric nature of these games.

Theorem 1.5. *It is NP-hard to decide the winner in a Picker-Chooser game.*

Theorem 1.6. *It is NP-hard to decide the winner in a Chooser-Picker game.*

Both proofs are based on the usual reduction method. We reduce $3-\mathrm{SAT}$ to Chooser-Picker or Picker-Chooser games.

Note that Chooser-Picker games are NP-hard, even for hypergraphs (V, E), where $|A| \leq 6$ for $A \in E$.

4×4 **tic-tac-toe**

Proposition 1.7. *Picker wins the Chooser-Picker version of the* 4×4 *tic-tac-toe.*

1.4.2 Picker-Chooser version of the generalized Shannon switching game

We prove Conjecture 1.2 for the Picker-Chooser version of Shannon switching game in the generalized version as Lehman did in [41]. Let $(V, \mathcal{F})$ be a matroid, where $\mathcal{F}$ is the set of bases, and Picker wins by taking an $A \in \mathcal{F}$. Note, that this is equivalent with the Chooser-Picker game on $(V, \mathcal{C})$, where $\mathcal{C}$ is the collection of *cutsets* of the matroid $(V, \mathcal{F})$, that is for all $A \in \mathcal{F}$ and $B \in \mathcal{C}$, $A \cap B \neq \emptyset$.

Theorem 1.8. *Let* $\mathcal{F}$ *be collection of bases of a matroid on* V*. Picker wins the Picker-Chooser* $(V, \mathcal{F})$ *game, if and only if there are* $A, B \in \mathcal{F}$ *such that* $A \cap B = \emptyset$*.*

The proof closely follow the ones given by Oxley in [50] for the Maker-Breaker case.

1.4.3 Erdős-Selfridge type theorems for P-C and C-P games

The Erdős-Selfridge theorem [25] gives a very useful condition for Breaker's win in a Maker-Breaker $(V, \mathcal{F})$ game. Using a stronger condition, Beck [6] proves Picker's win in a Chooser-Picker $(V, \mathcal{F})$ game. (For the P-C version he proved a sharp result that we include here.) Let $||\mathcal{F}|| = \max_{A \in \mathcal{F}} |A|$ be the rank of the hypergraph $(V, \mathcal{F})$.

Theorem 1.9. *[6] If*

$$T(\mathcal{F}) := \sum_{A \in \mathcal{F}} 2^{-|A|} < \frac{1}{8(||\mathcal{F}|| + 1)}, \tag{1.1}$$

then Picker has an explicit winning strategy in the Chooser-Picker game on hypergraph $(V, \mathcal{F})$*. If* $T(\mathcal{F}) < 1$*, then Chooser wins the Picker-Chooser game on* $(V, \mathcal{F})$*.*

We improved on his result by showing:

Theorem 1.10. *If*

$$\sum_{A \in \mathcal{F}} 2^{-|A|} < \frac{1}{3\sqrt{||\mathcal{F}|| + \frac{1}{2}}}, \tag{1.2}$$

then Picker has an explicit winning strategy in the Chooser-Picker game on hypergraph $(V, \mathcal{F})$*.*

It is worthwhile to spell out a special case of Conjecture 1.2 for this case, that would be the sharp extension of Erdős-Selfridge theorem to Chooser-Picker games.

Conjecture 1.11. *If*

$$\sum_{A \in \mathcal{F}} 2^{-|A|} < \frac{1}{2},$$

then Picker wins the Chooser-Picker game on $(V, \mathcal{F})$*.*

1.4.4 Torus games

To test Beck's paradigm we check the status of concrete games defined on the 4×4 torus, denoted by 4^2. That is we glue together the opposite sides of the grid, and consider all lines of slopes 0 and ± 1 as winning sets. For the general definition of torus games see [7]. We use a chess-like notation to refer to the elements of the board. The hypergraph of 4^2 is not almost disjoint, see e. g. the two winning sets $\{a2, b1, c4, d3\}$ and $\{a4, b1, c2, d3\}$. We can define four possible games on 4^2, those are the Maker-Maker, the Maker-Breaker, the Chooser-Picker and the Picker-Chooser versions. According to [7], the Maker-Maker version of 4^2 is a draw, and Picker wins the Chooser-Picker version, see [21]. In fact, the statement of the Maker-Breaker version implies the result for the Maker-Maker version, while the proof of it contains the proof of the Chooser-Picker version.

Proposition 1.12. *Breaker wins the Maker-Breaker version of the 4^2 torus game.*

According to Conjecture 1.2, Breaker has an easier job in the Maker-Breaker version than Chooser has in the Picker-Chooser game. For the 4×4 torus the outcome of these games are the same, although it is much harder to prove.

Proposition 1.13. *Chooser wins the Picker-Chooser version of the 4×4 torus game.*

Proof. **(sketch)** The full proof needs a lengthy exhaustive case analysis. However, some branches of the game tree may be cut by proof method of Beck's following result [6]: Chooser wins a Picker-Chooser game on $\mathcal{H}$ if $T(\mathcal{H}) := \sum_{A \in E(\mathcal{H})} 2^{-|A|} < 1$. □

It is important to remark that above we have seen an ordering due to its complexity: it is easier to get the result of the C-P case, then the M-B case, though it gives the same result. And it is far more hard to determine the P-C case then the Maker-Breaker case.

Chapter 2

The Chooser-Picker 7-in-a-row game

2.1 The k-in-a-row game

The k-in-a-row game is that hypergraph game, where the vertices of the graphs are the fields of an infinite graph paper ($\mathbb{Z}^2$), and the winning sets are the consecutive cells (horizontal, vertical or diagonal) of length k. If one of the players gets a length k line, then he wins otherwise the game is draw. Note the assuming perfect play, the winner is always the first player, or it is a draw by the strategy stealing argument of John Nash, [13]. More details about k-in-a-row games in [57, 58].

Both the Maker-Maker and the Maker-Breaker versions of the k-in-a-row for $k = 6, 7$ are open. These are wisely believed to be draws (Breaker's win) but, despite of the efforts spent on those, not much progress has been achieved.

2.2 The C-P k-in-a-row game

Before proving the C-P 7-in-a-row game, we proved the easier C-P 8-in-a-row game (by playing auxiliary games in a "Z" shaped board, what used Zetters in [32]).

Proposition 2.1. *Picker wins the Chooser-Picker version of the game 8-in-a-row on any $B \subseteq \mathbb{Z}^2$.*

Theorem 2.2. *Picker wins the Chooser-Picker 7-in-a-row game on every A subset of $\mathbb{Z}^2$.*

By applying the remedy mentioned before Lemma 1.4 at first Chooser determines the finite board S. We will consider a tiling of the entire plane, and play an auxiliary game on each tile (sub-hypergraph). It is easy to see, if Picker wins all of the sub-games, then Picker wins the game played on any K board which is the union of disjoint tiles. Let K be the union of those tiles which meet S. Since $S \subset K$, Lemma 1.4 gives that Picker also wins the game on S, too. Now we need to show a suitable tiling and to define and analyze the auxiliary games. The tiling guarantees that if Picker wins on in each sub-games then Chooser cannot occupy any seven consecutive squares on K.

Each tile is a 4×8 sized rectangle and the winning sets, for the sake of better understanding, are drawn on the following four board:

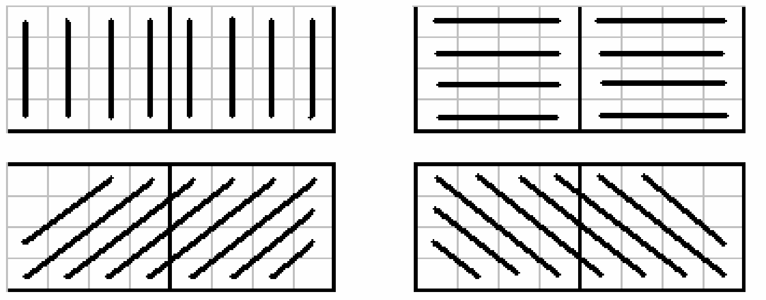

Figure 2.1: These are the winning-sets of the 4×8 rectangle. Easy to see, that there is exactly one symmetry (along the double line). Later we will make use of it.

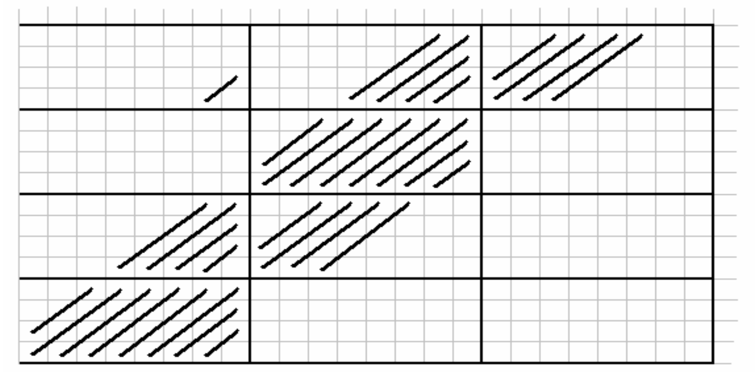

Figure 2.2: We can see, how to draw from playing on simple tile, the game played on the infinite chessboard: neither vertically, nor horizontally, nor diagonally (there is only one diagonal direction detailed) there are no seven consecutive squares without containing one winning set of a sub-game.

The key lemma for our proof is the following.

Lemma 2.3. *Picker wins the auxiliary game defined on the 4×8 rectangle.*

Remark 2.4. *We checked with brute force computer search the M-B game on the same auxiliary board, but it is a Maker win! So we cannot use the same table again, to prove that the weak version (=the Maker-Breaker version) of this game is a Breaker win. One is tempted to look for other auxiliary games, which is not going to be easy. As a rule of thumb, it always good idea to check the C-P version of these games at first.*

Chapter 3

The Picker-Chooser Diameter Game

3.1 Graph Games

Large classes of Maker-Breaker games are defined on the complete graph on n vertices. The players take the edges of the graph in turns; Maker wins iff his subgraph has a given, usually monotone, property $\mathcal{P}$, see [8, 5, 12, 17]. Balogh et al. [2] introduced the $(a : b)$ d-diameter game, shortly $\mathcal{D}_d(a : b)$, which means that Maker wins iff the diameter of his subgraph is at most d. These games turned out to be very difficult and surprising; a detailed discussion will be given in Section 3.1.1. The main result of Balogh et al. was that Maker loses the game $\mathcal{D}_2(1 : 1)$ but Maker wins the game $\mathcal{D}_2(2 : \frac{1}{9}n^{1/8}/(\log n)^{3/8})$.

This means that the acceleration of a game may change the outcome dramatically, [57]. The outcome also changes a lot when one considers the Picker-Chooser version of the game $\mathcal{D}_2(1 : 1)$. Our main result is the following theorem.

Observation.

Picker wins the P-C game $\mathcal{D}_2(1 : 1)$ on the graph K_n, if $n > 22$.

Theorem 3.1. *In the Chooser-Picker game $\mathcal{D}_2(1 : b)$, Picker wins if $b < \sqrt{n/\log_2 n}/4$, while Chooser wins if $b > 3\sqrt{n}$, provided that n is large enough.*

The Picker-Chooser (Chooser-Picker) games are themselves heuristics for the Maker-Breaker games. As Theorem 1.10 shows, the conditions for winning a Maker-Breaker game by Breaker and winning the Chooser-Picker version of that game by Picker coincide in several cases. Furthermore, Breaker's win in the Maker-Breaker and Chooser's win in the Picker-Chooser version seem to occur together in some cases [6]. To further explore this connection, a generalization of Theorem 1.10 for biased games is needed. No attempt is made here to get the best possible form, for our needs the following lemma will be sufficient.

Lemma 3.2. *Picker wins the Chooser-Picker $(1 : b)$ biased game on the hypergraph $\mathcal{H} = (V(\mathcal{H}), E(\mathcal{H}))$ if*

$$\frac{v}{b+1} \sum_{A \in E(\mathcal{H})} 2^{-|A|/b} < 1,$$

where $v = |V(\mathcal{H})|$.

3.1.1 Diameter and degree games

Balogh et al. [2] observed that the game $\mathcal{D}_2(1:1)$ defies the probabilistic intuition completely. Indeed, if one divides the edges of K_n among Maker and Breaker randomly, then Maker's subgraph will almost surely have diameter two. Still, Breaker has a simple pairing winning strategy for $n > 3$, [2]. First taking an edge uv, such that neither ux nor vx has been taken by Maker for any vertex x. Then if Maker takes ux, taking vx follows, and if Maker takes vx, Breaker takes ux, otherwise an arbitrary edge is taken.

However, when playing the game $\mathcal{D}_2(2:2)$, this pairing strategy is not available for Breaker. Maker wins the game $\mathcal{D}_2(2:2)$, and even more, the game $\mathcal{D}_2(2:b)$, where b grows polynomially in n, provided that n is large enough.

Theorem 3.3. *[2] Maker wins the game $\mathcal{D}_2(2:\frac{1}{9}n^{1/8}/(\ln n)^{3/8})$, and Breaker wins the game $\mathcal{D}_2(2:(2+\epsilon)\sqrt{n/\ln n})$ for any $\epsilon > 0$, provided n is large enough.*

To prove Theorem 3.1 we need to study the so-called *degree games*. Székely, Beck and Balogh et al. [71, 5, 2] showed that these games are interesting in their own right.

In such games one player tries to distribute his moves uniformly, while the other player's goal is to obtain as many edges incident to some vertex as possible. Given a graph G and a prescribed degree d, Maker and Breaker play an $(a:b)$ game on the edges of G. Maker wins by getting at least d edges incident to each vertex. We are interested only in the case of $G = K_n$. Balogh et al. [2] proved the following lemma:

Lemma 3.4. *[2] Let $a \leq n/(4\ln n)$ and n be large enough. Then Maker wins the $(a:b)$ degree game on K_n if $d < \frac{a}{a+b}n - \frac{6ab}{(a+b)^{3/2}}\sqrt{n\ln n}$.*

As we do not wish to develop the complete theory of P-C (C-P) degree games, we state only a simple form that suffices our needs

Lemma 3.5. *Let $b < n/(8\ln n)$ and n be large enough. Then Chooser wins the $(1:b)$ Chooser-Picker degree game on K_n if $d < n-1-3n/b$.*

To prove Theorem 3.1, we proved Lemma 3.2 first.

The second part of the theorem, i. e. Chooser wins if $b > 3\sqrt{n}$, comes from Lemma 3.5. Let Chooser play accordingly to that lemma, then Picker gets at most $(3n/b)-1$ edges at any vertex $x \in K_n$, so the number of vertices that are linked to x is no more than $((3n/b)-1)^2 < n-1$.

To prove the first part of the theorem implies more work. We split the vertices of the graph into three approximately equal parts, X_1, X_2 and X_3. (Let X_i be $X_{i \bmod 3}$ if $i > 3$.) The elements of X_i may be listed as $1, 2, \ldots, n/3$.[1] $E(X_i, X_j)$ denotes the edges between the sets X_i and X_j.

We will play two different games among and inside the parts. At the first game we link the points of X_i using $E(X_i, X_{i+1})$, for $i = 1, 2, 3$. At the second game we link the sets X_i with X_{i+1} playing on the edges of X_{i+1}.

[1] It can also be $\lfloor n/3 \rfloor$ and $\lceil n/3 \rceil$. In the proof we show that it works with $\lceil n/3 \rceil$, and the case $\lfloor n/3 \rfloor$ easily follows from that.

Bibliography

[1] L. V. Allis, H. J. van den Herik and M. P. Huntjens, Go-Moku solved by new search techniques. Proc. 1993 AAAI Fall Symposium on Games: Planning and Learning, AAAI Press Technical Report FS93-02, pp. 1-9, Menlo Park, CA.

[2] J. Balogh, R. Martin and A. Pluhár, The diameter game, *Random Structures and Algorithms*, Volume **35**, (2009), 369–389.

[3] J. Beck, Van der Waerden and Ramsey games. *Combinatorica* **1** (1981), 103–116.

[4] J. Beck, Remarks on positional games. *Acta Math Acad Sci Hungar* **40** (1982), 65–71.

[5] J. Beck, Deterministic graph games and a probabilistic intuition. *Combinatorics, Probability and Computing* **3** (1994), 13–26.

[6] J. Beck, Positional games and the second moment method. *Combinatorica* **22** (2) (2002) 169–216.

[7] J. Beck, Positional Games. *Combinatorics, Probability and Computing* **14** (2005), 649–696.

[8] J. Beck, Random graphs and positional games on the complete graph. Random graphs ’83 (Poznań, 1983), 7–13, *North-Holland Math. Stud.*, **118**, North-Holland, Amsterdam, 1985.

[9] J. Beck, *Combinatorial games. Tic-tac-toe theory.* Encyclopedia of Mathematics and its Applications, **114.** Cambridge University Press, Cambridge, 2008. xiv+732 pp.

[10] J. Beck, *Combinatorial games. Tic-Tac-Toe Theory.* Encyclopedia of Mathematics and its Applications, 114. Cambridge University Press, Cambridge, 2008. xiv+732 pp.

[11] M. Bednarska, and T. Łuczak, Biased positional games for which random strategies are nearly optimal. *Combinatorica* **20** (2000), 477–488.

[12] M. Bednarska, and T. Łuczak, Biased positional games and the phase transition. *Random Structures and Algorithms* **18** (2001), 141–152.

[13] E. R. Berlekamp, J. H. Conway and R. K. Guy, *Winning Ways for your mathematical plays*, Academic Press, New York 1982.

[14] B. Bollobás, *Random Graphs*. Second edition. Cambridge Studies in Advanced Mathematics, **73** xviii+498 pp.

[15] de Bruijn, ?? *Recreational Mathematics* ??

[16] J. M. Byskov, Maker-Maker and Maker-Breaker Games are PSPACE-complete. *Technical Report, BRICS Research Series* RS-04-14, Dept. Comp. Sci., Univ. Aarhus, August 2004.

[17] V. Chvátal and P. Erdős, Biased positional games. Algorithmic aspects of combinatorics (Conf., Vancouver Island, B.C., 1976). *Ann. Discrete Mathematics* **2** (1978), 221–229.

[18] J. H. Conway, *On Numbers and Games*. London: Academic Press, 1976.

[19] B. Csákány, A form of the Zermelo-von Neumann theorem under minimal assumptions. *Acta Cybernetica* **15** (2002), no. 3, 321–325.

[20] Csákány Béla, *Diszkrét Matematikia Játékok*. Polygon, Szeged, 1998.

[21] A. Csernenszky, C. I. Mándity and A. Pluhár, On Chooser-Picker Positional Games, *Discrete Mathematics* Volume **309** (2009), 5141–5146.

[22] A. Csernenszky, The Chooser-Picker 7-in-a-row-game, *Publicationes Mathematicae* **76** (2010), 431– 440

[23] A. Csernenszky, The Picker-Chooser diameter game, *Theoretical Computer Science* **411** (2010), pp. 3757–3762

[24] A. Csernenszky, R. Martin and A. Pluhár, On the Complexity of Chooser-Picker Positional Games, *submitted*.

[25] P. Erdős and J. L. Selfridge, On a combinatorial game. *J. Combinatorial Theory Series A* **14** (1973), 298–301.

[26] S. Even and R. E. Tarjan, A combinatorial problem which is complete in polynomial space. *J. Assoc. Comput. Mach.* **23** (1976), no. 4, 710–719.

[27] A. S. Fraenkel, Complexity of Games. *Combinatorial games* (Columbus, OH, 1990), 111–153, Proc. Sympos. Appl. Math., **43**, Amer. Math. Soc., Providence, RI, 1991.

[28] A. S. Fraenkel, Combinatorial games: selected bibliography with a succinct gourmet introduction. *The Electronic Journal of Combinatorics* Dynamic Surveys (2009) 88pp.

[29] D. Gale, H. Kuhn and A. Tucker, Linear programming and the theory of games, in T. Koopmans, ed., *Activity Analysis of Production and Allocation*, John Wiley and Sons, New York, (1951) pp. 317–329.

[30] D. Gale, The game of Hex and the Brouwer fixed-point theorem. *American Mathematical Monthly* **86** (1979), no. 10, 818–827.

[31] J. J. Gik, *Sakk és matematika*. Gondolat, Budapest (1989).

[32] R. K. Guy and J. L. Selfridge, Problem S.10, *Amer. Math. Monthly* **86** (1979); solution T.G.L. Zetters **87** (1980) 575–576.

[33] A. W. Hales and R. I. Jewett, Regularity and positional games. *Trans Amer. Math. Soc.* **106** (1963) 222–229; M.R. # 1265

[34] D. Hefetz, M. Krivelevich, M. Stojaković and T. Szabó, Planarity colorability and minor games. *SIAM Journal on Discrete Math* **22** (2008), 194–212.

[35] F. Harary, Is Snaky a winner? *Geombinatorics* **2** (1993) 79–82.

[36] H. Harborth and M. Seemann, Snaky is a paving winner. *Bull. Inst. Combin. Appl.* **19** (1997), 71–78.

[37] M. Hegyháti, Colorability of mixed hypergraphs and their chromatic inversions, manuscript.

[38] R. Hochberg, C. McDiarmid and M. Saks, On the bandwidth of triangulated triangles. *Discrete Mathematics* **138** (1995) 261–265.

[39] J. P. Jones, Some undecidable determined games. *Internat. J. Game Theory* **11** (1982), no. 2, 63–70.

[40] M. Krivelevich, Positional games and probabilistic considerations, Oberwolfach Report 20/2007 16–17.

[41] A. Lehman, A solution of the Shannon switching game. *J. Soc. Indust. Appl. Math.* **12** 1964 687–725.

[42] Hall, Marshall, Jr. Distinct representatives of subsets. *Bull. Amer. Math. Soc.* **54**, (1948). 922–926.

[43] J. F. Nash, Noncooperative games. *Ann. of Math.* **54** (2), (1951) 284–295.

[44] J. F. Nash, The bargaining problem. *Econometrica* **18**, (1950) 155–162.

[45] J. F. Nash, Two-person cooperative games. *Econometrica* 21, (1953) 128–140.

[46] J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior.* Princeton, NJ: Princeton University Press, 1944.

[47] Neukomm Gyula, Egy híres sakkprobléma matematikai megfejtése. *Magyar Sakkvilág* 1941 December.

[48] K. Noshita, Union-Connections and Straightforward Winning Strategies in Hex. *ICGA Journal*, 2005 28(1): cover, 3–12.

[49] R. Nowakowski eds. *Games of No Chance*

[50] J. G. Oxley, *Matroid Theory.* New York: Oxford University Press, 1992.

[51] C. H. Papadimitriou, *Computational Complexity* Addison-Wesley Publishing Company, Inc., 1994.

[52] O, Patashnik, Qubic: $4 \times 4 \times 4$ tic-tac-toe. *Math. Mag.* **53** (1980), no. 4, 202–216.

[53] Y. Peres, *Game Theory, Alive.* electronic lecture note (created 2008.01.24 2:54:38)

[54] A. Pluhár, Játékelmélet, *egyetemi jegyzet (2010)*

[55] A. Pluhár, Generalized Harary Games. *Acta Cybernetica* **13** no. 1, (1997) 77–83.

[56] A. Pluhár, *Positional Games on the Infinite Chessboard.* Ph.D. dissertation, Rutgers University 1994.

[57] A. Pluhár, The accelerated k-in-a-row game. *Theoretical Computer Science* **270** (2002), 865–875.

[58] A. Pluhár, Pozíciós játékok, Szigma, vol 3-4, Pécs, 2007, 111–130

[59] P. Hein, Vil de laere Polygon? *Politiken newspaper*, Denmark, 26 December 1942.

[60] Norman Do, How to Win at TicTacToe. *The Australian Mathematical Society, Gazette*, Volume **32** Number 3, July 2005, 151–161.

[61] O, Patashnik, Qubic: 444 tic-tac-toe. *Mathematical Magazine* **53** (1980), no. 4, 202–216.

[62] S. Reisch, Hex ist PSPACE-vollständig. *Acta Inform.* **15** (1981), no. 2, 167–191.

[63] T. J. Schaefer, On the complexity of some two-person perfect-information games. *J. Comput. System Sci.* **16** (1978), no. 2, 185–225.

[64] Székely Gábor, Paradoxonok a véletlen matematikájában, *Műszaki Könyvkiadó*, Budapest (1982).

[65] S. Shelah, Primitive recursive bounds for van der Waerden numbers. *J. Amer. Math. Soc.* **1** (1988), no. 3, 683–697.

[66] N. Sieben, Hexagonal polyomino weak $(1, 2)$-achievement games. *Acta Cybernetica* **16** (2004), no. 4, 579–585.

[67] N. Sieben, Snaky is a 41-dimensional winner. *Integers* **4** (2004), G5, 6 pp.

[68] H. Steinhaus, The problem of fair division, *Econometrica* **16** (1948) 101–104.

[69] J. Spencer, Randomization, derandomization and antirandomization: three games. *Theoretical Computer Science* **131** (1994), no. 2, 415–429.

[70] H. Steinhaus, *Matematikai kaleidoszkóp.* Gondolat Budapest (1984).

[71] L. A. Székely, On two concepts of discrepancy in a class of combinatorial games. Finite and Infinite Sets, Colloq. Math. Soc. János Bolyai, Vol. 37, North-Holland, 1984, 679–683.

Társszerzői nyilatkozat

Kijelentem, hogy ismerem Csernenszky András PhD fokozatra pályázó ***The Chooser-Picker games*** című disszertációját.
A disszertációban szereplő közös eredményekre vonatkozóan kijelentem, hogy a következő eredményekhez való hozzájárulásunk oszthatatlan:

- **A. Csernenszky, C. I. Mándity and A. Pluhár, On Chooser-Picker Positional Games**, *Discrete Mathematics* Volume **309** (2009), 5141–5146.
- **Csernenszky, R. Martin and A. Pluhár, On the Complexity of Chooser-Picker Positional Games**, *submitted to Integers, Electronic Journal of Combinatorial Number Theory*

Szeged, 2011-05-30, ...

dr. Pluhár András

Társszerzői nyilatkozat

Kijelentem, hogy ismerem Csernenszky András PhD fokozatra pályázó ***The Chooser-Picker games*** című disszertációját.
A disszertációban szereplő közös eredményekre vonatkozóan kijelentem, hogy a következő eredményekhez való hozzájárulásunk oszthatatlan:

- **A. Csernenszky, C. I. Mándity and A. Pluhár, On Chooser-Picker Positional Games**, *Discrete Mathematics* Volume **309** (2009), 5141–5146.

2011.05.30, Csizmárikné dr. Tóth Erzsébet [signature]
..........., ...
dátum, aláírás

Coauthor's declaration

I hereby certify that I am familiar with the thesis of the applicant Mr ***András Csernenszky*** entitled ***The Chooser-Picker games***
Regarding our joint results referred to in this thesis, the following ones were obtained as the result of joint contribution by the applicant and myself:

- **Csernenszky, R. Martin and A. Pluhár, On the Complexity of Chooser-Picker Positional Games**, *submitted to Integers Journal*

30/05/2011 ....
date signature