ONLINE SIGNATURE VERIFICATION AND HANDWRITING CLASSIFICATION

Ph.D. dissertation

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PUBLICATIONS

Some ideas, tables and figures have appeared in the following publications:

AFHA2011

János Csirik, Zoltán Gingl and Erika Griechisch: *The Effect of Training Data Selection and Sampling Time Intervals on Signature Verification*, Proceedings of the First International Workshop on Automated Forensic Handwriting Analysis, pp. 6–10

DOI: `http://dx.doi.org/10.1.1.369.5581`

CIARP2011

Horst Bunke, János Csirik, Zoltán Gingl and Erika Griechisch: *Online signature verification method based on the acceleration signals of handwriting samples*, Lecture Notes in Computer Science, Volume 7042, Progress in Pattern Recognition, Image Analysis, Computer Vision, and Applications, pp. 499–506

URL: `http://forums.graphonomics.org/showthread.php?t=194`

IGS2013

Erika Griechisch, Muhammad Imran Malik and Marcus Liwicki: *Online Signature Verification using Accelerometer and Gyroscope*, Proceedings of 16th International Graphonomics Society Conference, pp. 143–146.

DOI: `http://dx.doi.org/10.1109/ICDAR.2013.82`

ICDAR2013

Erika Griechisch, Muhammad Imran Malik and Marcus Liwicki: *Online Signature Analysis based on Accelerometric and Gyroscopic pens and Legendre Series*, Proceedings of 12th International Conference on Document Analysis and Recognition, Washington, DC, USA, August 25-28, pp. 374–378

DOI: `http://dx.doi.org/10.1109/ICDAR.2013.82`

ICFHR2014

Erika Griechisch, Muhammad Imran Malik and Marcus Liwicki: *Online Signature Verification Based on Kolmogorov-Smirnov Distribution Distance*, Proceedings of 14th International Conference on Frontiers in Handwriting Recognition, Crete, Greece, September 1-4, 2014, pp. 738–742

DOI: `http://dx.doi.org/10.1109/ICFHR.2014.129`

ICDAR2015

Erika Griechisch and Erika Bencsik: *Handedness Detection of Online Handwritings based on Horizontal Strokes*, Proceedings of 13th International Conference on Document Analysis and Recognition, pp. 1272–1277

DOI: `http://dx.doi.org/10.1109/ICDAR.2015.7333953`

IGS2017

Erika Bencsik and Erika Griechisch: *The frequency of occurence of handwriting features in online male and female handwriting*, Proceedings of 18th International Graphonomics Society Conference pp. 169–172

We have seen that computer programming is an art,
because it applies accumulated knowledge to the world,
because it requires skill and ingenuity, and especially
because it produces objects of beauty.

— Donald E. Knuth [1]

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Erika Griechisch
16th May 2018

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ACRONYMS

AccSigDb AccSigDb2011

4NSigComp2010 ICFHR 2010 Signature Verification Competition

4NSigComp2012 ICFHR 2012 Signature Verification Competition

ANN Artificial Neural Network

CER Crossover Error Rate

DFKI Deutsche Forschungszentrum für Künstliche Intelligenz GmbH

DPM Dynamic Programming Matching

DWT Discrete Wavelet Transformation

DTW Dynamic Time Warping

EDD Electrostatic Detection Device

EER Equal Error Rate

ESDA ElectroStatic Detection Apparatus

FAR False Acceptance Rate

FHEs Forensic Handwriting Experts or Examiners

FRR False Rejection Rate

GMM Gaussian Mixture Model

GyroSigDb GyroSigDb2012

HMM Hidden Markov Models

HOG Histogram of Oriented Gradients

IAM-OnDB IAM On-Line Handwriting Database

IR Infrared

K-NN K-Nearest Neighbors

KS test Kolmogorov-Smirnov test

KS distance Kolmogorov-Smirnov distance

LBP Local Binary Pattern(s)

LLR Log-likelihood ratio

LR Likelihood ratio

MLP Multilayer Perceptron

NFI Netherland Forensic Institute

OCR Optical Character Recognition

PCA Principal Component Analysis

RBF Radial Basis Function

RF Random Forest

RNN Recurrent Neural Network

SigComp2009 ICDAR 2009 Signature Verification Competition

SigComp2011 ICDAR 2011 Signature Verification Competition

SigWiComp2013 ICDAR 2013 Competitions on Signature Verification and Writer Identification for On- and Offline Skilled Forgeries

SigWiComp2015 ICDAR 2015 Competitions on Signature Verification and Writer Identification for On- and Offline Skilled Forgeries

SVM Support Vector Machine

SVMs Support Vector Machines

UV Ultraviolet

Part I

HISTORY, DEFINITIONS AND DATABASES

1 INTRODUCTION

HISTORY OF FORENSICS

Authentication has always played an important role in our world, since the identification of a person is essential. Generally different types of biometric authentication are applied to determine a person's identity. Figure 1.1 summarizes the most common ones. Some of them have been discovered to be unique in the last few decades, such as DNA and iris recognition and they provide more accurate results than the earlier methods did (e.g. fingerprint, signature). Hence they are more difficult to forge.

Since some people commit crimes it is an important task to identify the person who commited the crime or detect alteration in documents for several reasons. Preliminary methods for carrying out such examinations existed, but at the beginning it was not "scientific". The first science-based forensic book was called "Washing away wrongs", published in 1248 by a Chinese author called Song Ci. This book describes how to distinguish drowning from strangulation using medical knowledge.

In the XVII century the first pathological examination reports were written, then in 1836 chemical tests were used by the English chemist James Marsh to determine arsenic as the cause of death in a murder trial (called the Marsh test), followed in 1880 by a study about the uniqueness of fingerprints [2].

A key development in forensic science occurred in the XX century when criminal and civil cases and their methods developed the most. The first crime lab was built in 1923, the polygraph was invented to detect lies in 1930, voice recording was first used as evidence in 1960 and DNA techniques appeared in 1984 – just to mention a few important steps in the history of forensic science. A new branch of forensic science was born with the appearance of the personal computers. When computers became more accessible to people, then they became tools in crime. This new branch is called computer forensic science or computer forensics.

Handwriting examination

The history of the forensic examination of handwriting and signature started in the middle of the XIX century. Inital examinations were based on graphological knowledge, but later in the XX century these two areas became separate and forensic examination moved towards the criminal discipline and developed its

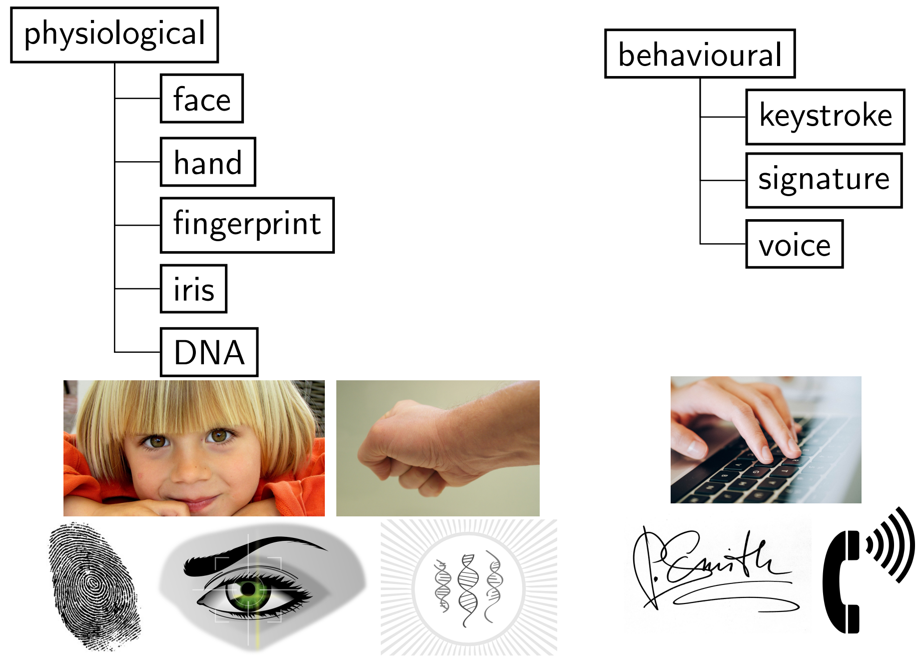


Figure 1.1: Biometrical traits

own method for detecting the authenticity of signatures or handwriting. Forensic Handwriting Experts or Examiners (FHEs) analyze, compare and evaluate handwriting samples of a particular person (e.g. a suspect) and a questioned document to decide whether the questioned document was written by the same person. The questioned documents are mostly contracts, wills, threatening letters, abusive letters, and suicide letters. They not only examine authenticity, but alteration as well, like the modification of a text, obliteration and erasure [3].

Forensic scientists have used basic measurement tools for a long time such as a metric ruler, angle meter and caliper, microscopes or magnifiers to enlarge the samples for further examination. Several new techniques and tools were invented in the last century to support the daily work of handwriting examiners. Ultraviolet (UV) and Infrared (IR) lights are useful for differentiating inks and papers, density meter shows the pressure distribution. The Electrostatic Detection Device (EDD) reveals indented impressions of handwriting which are not visible to the human eye. Figure 1.2 shows a stereo microscope and an ElectroStatic Detection Apparatus (ESDA) which is an EDD device – work tools of the FHEs.

Differences between signature and handwriting

Signing is a short and very often performed writing process, thus compared to the general and longer handwriting (such as writing notes, letters, wills, etc.), process of signing with the lot of repetition become automatic. The other main difference between signature and handwriting is its length. Signature is short, consists usually one or few ”words”, in contrast to handwriting which can include not only word, but sentence(s), line(s), one or more paragraph(s), e.g. a whole letter or will.

Figure 1.4b presents an image of an online handwritten signature from SigComp2009 dataset with the corresponding features in Figure 1.4c, while Figure 1.5 depicts an online handwriting sample from IAM On-Line Handwriting Database (IAM-OnDB).

HANDWRITING ANALYSIS

First of all, we should mention that several tasks exist related to handwriting and signature analysis. These are:

RECOGNITION. This handles character recognition and handwriting interpretation automatically.

This branch focuses on Optical Character Recognition (OCR), but also deals with the formatting and segmentation of documents which are necessary steps of a recognition system.

IDENTIFICATION. Determining of who wrote a questioned handwriting sample, based on samples from more than one writer.

An identifier’s output for a questioned sample is the writer, so this is a *multiclass classification*.

VERIFICATION. Decision about the authenticity, i.e. whether a questioned handwriting or signature belongs to a given person. Here reference samples are given for comparison taken from the corresponding person (usually called specimen).

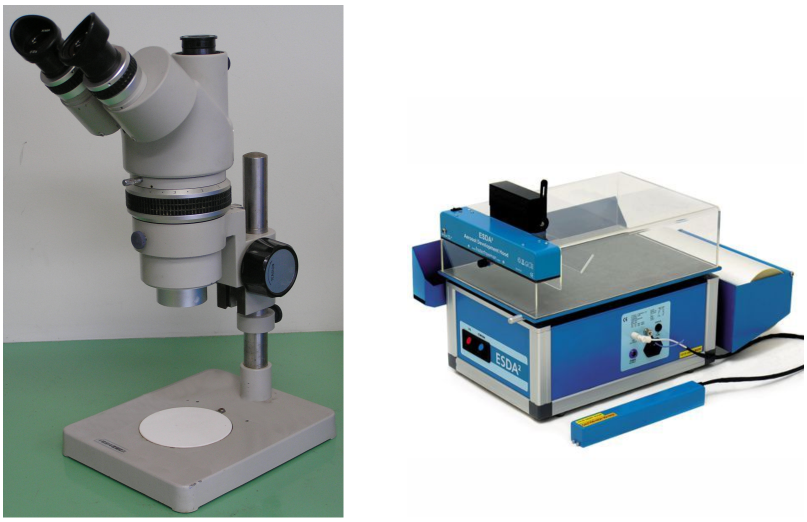


Figure 1.2: Stereo microscope and an ESDA

This is a *one-class classification* problem. A verifier's decision for a questioned sample is genuine if it is suspected that the sample belongs to the class (the given specimen), forgery if not. If the verifier is not able to make a decision, the output is *inconclusive*. In some special cases a *disguised* class is used as well. A detailed description about the type of forgeries and disguised signatures is given in Section 2.4.1 later on.

ROLE OF HANDWRITTEN SIGNATURE AND HANDWRITING

The use of signatures was first recorded in the Talmud in the IV century, complete with security procedures to prevent the alteration of documents after they were signed [4, 5].

Handwriting and especially signature became an essential mode of authentication of letters, contracts, documents, wills, etc. Nowadays with the spread of the electronic devices handwriting seems to lose some importance. However, the handwritten signature is still the most widely accepted method for identification in everyday life, in business, at banks, or in contract validation. This is why studies tackle the problem of signature verification and examine the process in detail. Usually their aim is to study the mechanics of the process and learn what features are hard to counterfeit.

Thus handwriting classification and verification is still a widely studied research area, but the way of examination of the natural handwriting is continuously developing. Primarily verification methods are changing in the last decades from traditional way (handwriting with pen or pencil on a paper) to the digitally recorded (e.g. tablet, digitizing pen) handwriting.

ACQUISITION FOR AUTOMATIC EXAMINATION

There are two basic approaches of recognizing handwriting or signatures; namely offline and online.

Offline examination systems use only the image of the handwriting or signature, thus the original sample is scanned – possibly with a high resolution. See Figure 1.3 and 1.4a for offline signature samples.

Online acquisition focuses on the dynamics of the writing process (pressure, tilt, inclination etc.), usually – depending on the device – a timestamp is recorded besides x, y coordinates so further features can be calculated using temporal information (e.g. velocity, acceleration). For a sample from ICDAR 2009 Signature Verification Competition (SigComp2009) dataset, see Figure 1.4.

Challenges

The main problem with the offline approach is that it gives lower accuracy, but the dynamic approach requires much more sophisticated techniques.

The offline and online identification or recognition systems differ in their feature selection and decision methods. Some studies analyse the consistency of the features [6], while others focus on the template feature selection [7]; some of them combine local and global features [8].

RELATED REVIEWS AND COMPREHENSIVE STUDIES

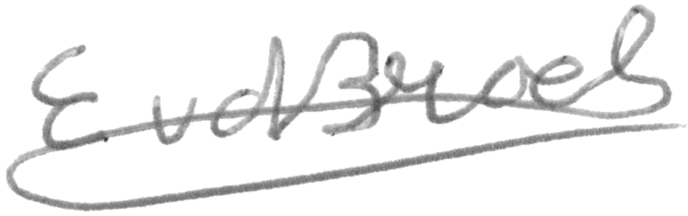
Frequently cited and often referred reviews about automatic signature verification were written by Plamondon and Lorette in 1989 [9], Leclerc and Plamondon in 1994 [10], Plamondon and Srihari in 2000 [11] and Impedovo and Pirlo in 2008 [12]. These surveys explore the studies concerning handwriting recognition, including handwriting interpretation, handwriting identification, and signature verification.

Besides these surveys, special surveys have been published in the past few years, such as that by Sanmorino and Yazid in 2012 [13], and papers on online signature verification by Zhang et al. in 2011 [14] and El-Henawy et al. in 2013 [15].

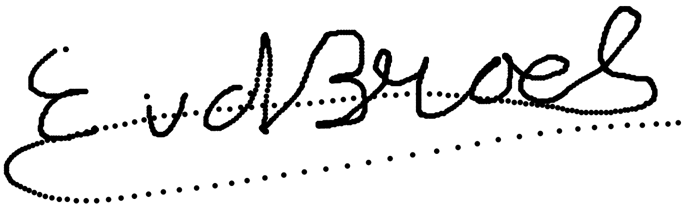
An extensive review of automatic online handwriting recognition was written by Tappert et al. in 1990 [16]. This review presents the recent digitizer technologies at that time, handwriting properties, describe the recognition problems and gives an overview of the steps (pre-processing, shape recognition and post-processing) and recent systems.

Malik defended his PhD thesis in 2015 about how to bridge the gap between existing pattern recognition methods and forensic science [17].

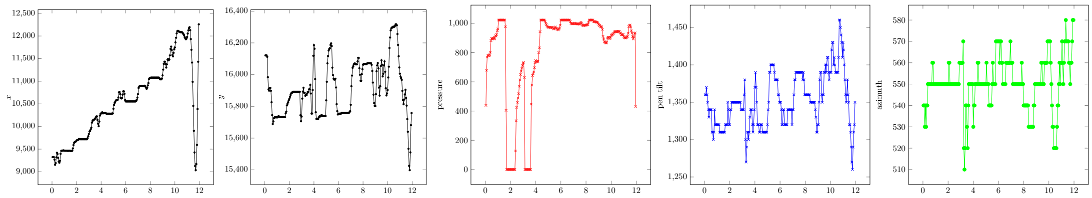
Figure 1.3: Signature of Louis XIV of France



(a) Offline signature



(b) Online signature



(c) Corresponding x, y coordinates, pressure, pen tilt and azimuth time functions

Figure 1.4: A signature sample taken from SigComp2009 database

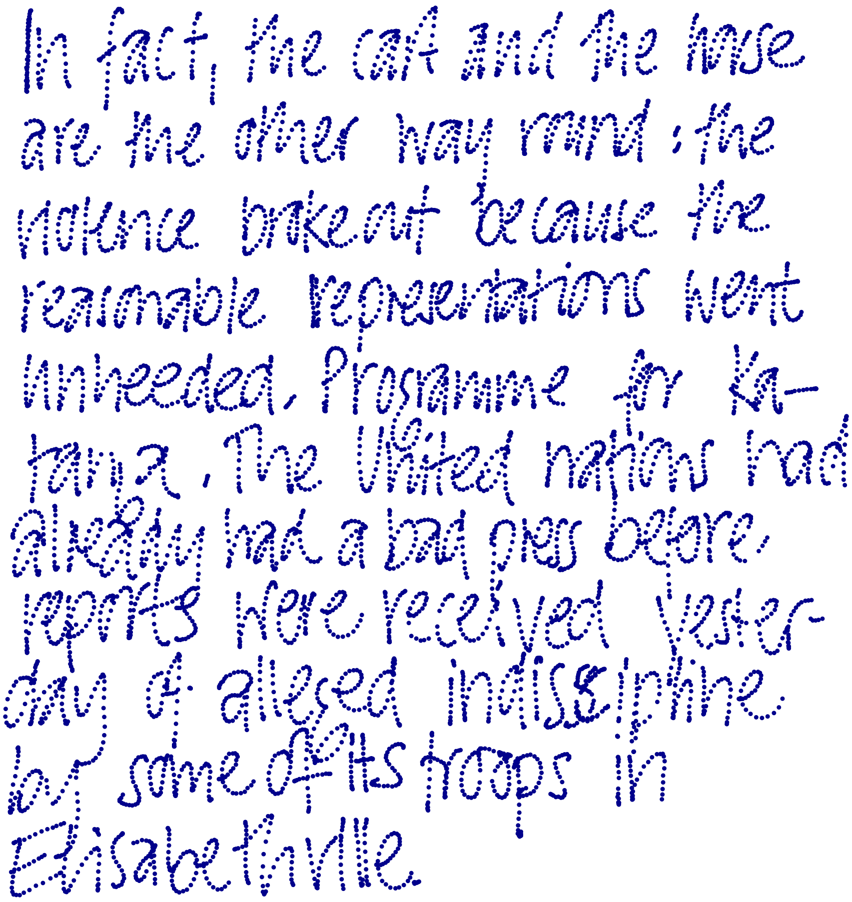


Figure 1.5: Online handwriting sample from IAM-OnDB

2 AUTOMATED ONLINE VERIFICATION

FHEs work mainly with a microscope and some special devices as it was described in Subsection 1.1.1. In addition sometimes their work is supported by computer tools to determine some geometrical features automatically. On the other hand, computer scientists have developed their techniques to verify signatures or identify writers, which are rather different from the methods that handwriting examiners use. Here we are going to describe the main aspects of automatic signature verification. Handwriting identification needs a slightly different approach as verification, but several aspects are the same.

Foremost the principal difference between forensic examination and automatic examination carried out by computer scientists is the acquisition of the samples: FHEs use the original handwriting on a paper, automatic verification can only handle digital samples. The details of online handwriting samples are described in Section 2.1.

Figure 2.1 shows the typical steps of a handwriting or signature verification system. The three main steps of automatic identification and verification are preprocessing, feature extraction, classification. Their different approaches are described in Sections 2.2, 2.3 and 2.4. Handwriting identification is slightly different at the last step: in that task the output is the suspected writer, not a ”genuine” or ”forgery” (or in some cases it is ”inconclusive”).

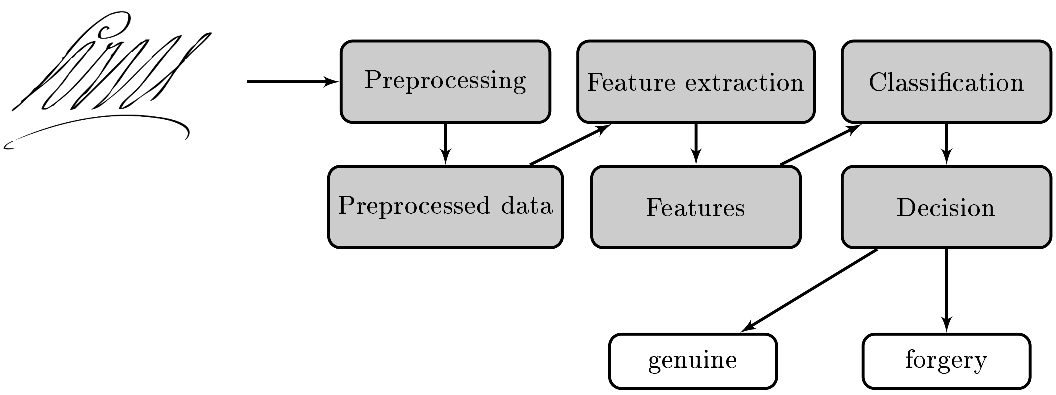


Figure 2.1: Steps of a signature verification method

Before we describe the steps in details we should provide an overview of the verification process. First of all, we usually use preliminary data to train our verifier in order to learn the differences between genuines and forgeries. During training samples are given to the verifier to adjust different parameters. After training, the evaluation (i.e. testing) phase follows. During evaluation the verifier gets questioned signatures to decide whether they are genuine or forged. The verifier has to compare the questioned signatures with given reference signatures of the suspected writer. Reference signatures show the characteristics of the given writer. This is the procedure used as well when FHEs examine forensic cases.

Sometimes we have only a few references per writer, sometimes 8 or 10, but that is a rare case in real life cases. We can observe that public datasets usally also contain only a few references, rarely 10-12 per writer (see Section 2.6 for details about databases).

Thus we normally have for each database a training set and a test set and for each writer present in the training set, it is required to have some reference signatures in order to perform a classification and decide whether a questioned signature is from the concerned writer or not. The preprocessing, feature extraction is the same for questioned samples in the test and training set, but training samples are used to revise the method (fine-tuning parameters, e.g. increase or decrease a threshold and adjust weights) and with revision to increase the performance.

ACQUISITION

Several devices such as digitizing tablets, special pens, touch screens are used to capture online handwriting data. These devices in most of the cases capture coordinate information (x, y coordinates along the horizontal and vertical axes), sometimes pen pressure and less time pen angle (azimuth, altitude) information versus time and usually have a sampling rate between 100 and 200 Hz. However, as the signing process with tablets is different from the natural way of handwriting, other devices have been created to capture the signing process.

The most widely used tools to record handwriting are produced by the German Wacom company[1]. Wacom Intous 3 devices are presented in Figure 2.2. Several studies evaluate their verifiers on databases which are acquired with Wacom tablets, see further details about these databases in section 2.6 [18–20].

Besides Wacom, another German company produces devices specifically to record handwriting signatures. Signotec[2] has recently[3] five different signature capture pads named using Greek letters: Sigma, Omega, Gamma, Delta and Alpha, see Figure 2.3 from left-to-right. Besides these pen-pads they offer solutions to record signature digitally using smartphones or tablets.

Shipping companies, post offices and other companies in daily life also use devices to record signatures. For example GLS in Hungary recently[3] uses custom-

1 www.wacom.com
2 www.signotec.com
3 February 2017

made Motorola devices with Windows CE operating systems. In 2015 the Magyar Telekom (the Hungarian telecommunication company) developed and introduced a system based on Samsung tablets with styluses and a MobilSign application with the BlackBerry Enterprise Service 12 operational background in an international cooperation. The system is able to store coordinate, pressure and inclination information. Since the introduction of this system all contracts are digital and can be signed digitally. Few months later another Hungarian telecommunication company (Vodafone) also introduced its system with more limited abilities. Their system is based on tablets and a special pen to record signatures on contracts; however the system only stores the signature in offline format without any additional (online or pressure) data. In 2017 the largest Hungarian Bank (OTP) as the first digital bank in Hungary also introduced their digital system to store contracts and record and verify signatures. Their system is based on Signotec Delta Capture pads [21].

Another device that first received attention in 2013 is the Anoto pen (see Figure 2.4). This pen is used in the banking sector with the collaboration of Deutsche Forschungszentrum für Künstliche Intelligenz GmbH (DFKI) in Germany.

Generally less known pen tablets or pens (sometimes only prototypes, not products) have been used in several studies [19, 22]; the F-Tablet was described in [23] and the Genius 4x3 PenWizard was used in [24].

PREPROCESSING

A preprocessing step is required before the feature extraction principally in order to remove noise. Preprocessing usually includes filtering, noise reduction and smoothing and it is often performed using a Fourier transform [25, 26], Gaussian functions [27] or mathematical morphology [28]. Sometimes resampling is used to improve the result [29], shorten the execution time [19] or to obtain fixed length vectors [30, 31].

Normalization

Signature normalization is also a preprocessing method, usually it involves shifting towards center mass or origin of the coordinate system, but normalized Fourier descriptor is also applied to standardize signatures [25, 26, 32–34].

Segmentation

Segmentation strongly influences the verification performance, thus a good segmentation technique can improve verification results.

Ansari and Kour applied uniform segmentation to obtain local characteristics [35]. During uniform segmentation, the signature is divided into a certain num-

Figure 2.2: Wacom Intuos3 graphics tablets

Figure 2.3: Sigma, Omega, Gamma and Alpha Signature Pads from signotec

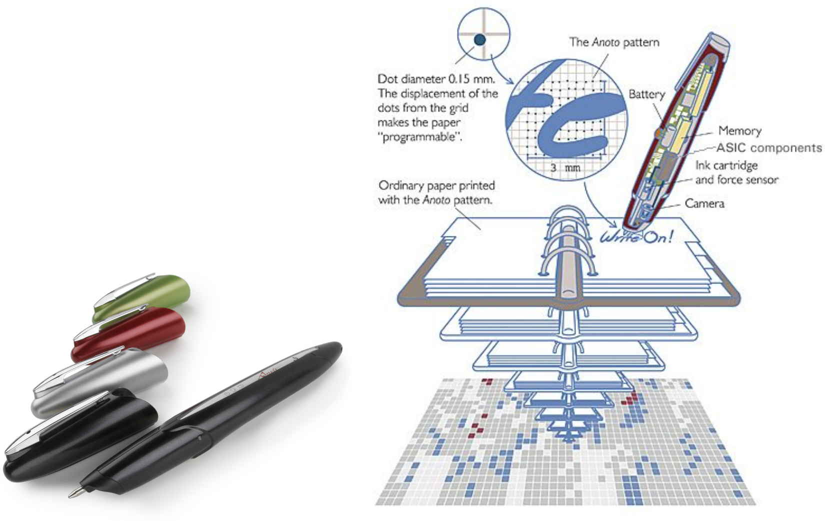


Figure 2.4: Anoto pen

ber of segments with an approximately equal number of data points in each segment, thus if the signature has T points and we divide it to S segments, then the segments would have approximately T/S points in each segment.

Segmentation techniques were performed by pen up and down signals, velocity analysis [36], Dynamic Time Warping (DTW) (more details in Section 2.4), connected components, tree structure analysis, and statistics as well.

FEATURE EXTRACTION

Many different approaches have been considered to determine which information is the most discriminative for online signature verification purposes.

Often a large intra-class variation is present, namely we can observe significant differences between genuine signatures taken from the same writer. We can distinguish two approaches: the parametric and the functional approach, also referred as global and local or regional respectively.

Parametric approach

In the parametric approach, each signature is represented by a fixed-length vector with global features derived from the data, e.g. width or height of the signature, average speed, length of the signing process. The signatures are represented by a vector that contains the selected global features. Several different distance metrics are used to compute distance between the questioned signatures and the corresponding reference signature(s), more details in Subsection 2.4.2.

Functional approach

In the functional approach we distinguish local or regional features. Local features are mostly time functions which typically occur in online verification, due to the varying size of the data that mostly depends on the length of the signing process (e.g., x_t, y_t the x, y coordinates or p_t pressure at time t). The regional features characterize different regions of the signature, typically the time functions are transformed to sequence vectors [37].

For a comparison of time-dependent signals, two widely used techniques have been applied for a long time. DTW measures the distance between two vectors with different sizes. It aligns two time functions and finds a distance between them. Hidden Markov Models (HMM) is a statistical Markov Model where the probability distribution of the features is used to build a model for each author.

Recently Neural Networks [38–40] and Support Vector Machines (SVMs) [41, 42] are used as well. More details about these classifiers can be found in the next section.

Feature analysis

The global approach is often used in offline signature verification as well. Nguyen et al. extracted global features in offline signature verification [43], Ohyama et al. extracted global gradient features [44], Yilmaz et al. used global Local Binary Pattern(s) (LBP) and Histogram of Oriented Gradients (HOG) features [45].

Richiardi et al. examined more than 150 global and 39 local features and determined the most discriminative features in online signature verification [7]. The method was applied to a subset of the MYCT it found x, y coordinates, pressure, path tangent angle, azimuth, elevation, velocity along the x and y axes the most discriminative local features, total time, maximal pressure, point of maximum pressure, duration of positive velocity in x are the most discriminative global features. Yanikoglu and Kholmatov in [46] and [47] found Δx and Δy the best features. Ketabdar et al. examined several global features to find the optimal features with respect to Bayes classifier [48].

Fierrez-Aguilar et al. evaluated 100 global features based on the Mahalanobis distance and 14 local features (7 discrete time functions and their derivatives) based on HMM and ranked the features according to the inter-used class separability to select the best ones [49]. Some studies have been carried out to apply both global and local information [31, 36].

CLASSIFICATION

Signature verification is a special classification problem. In the basic scenario we have only a few reference signatures from a writer and the questioned signatures. The goal is to decide whether the questioned signature belongs to the writer who wrote the reference signatures or not. This scenario is called a one-class classification.

Type of forged signatures

Rarely we have fake (forged) signature samples to train our verifier. Forgeries can be classified into several classes based on the quality and authenticity of the signature. Studies work with different types of forgeries, but sometimes the studies use different types of forgeries even they use the same term.

Malik and Liwicki in [50] summarized how the different forgeries appear in different studies and suggested a terminology which clarifies terms and is also meaningful to most FHEs.

FORGED or simulated forgery. The forger only knows the name of the authentic author, but not the actual signature.

SIMPLE FORGERY a forgery where actual signatures are known, but a forgery is produced without any practice.

SKILLED FORGERY same as simple forgery, but it is produced after practice.

We should also mention here the *disguised signature*, the willfully distorted signature of the writer which is usually modified to later hide or deny identity.

Distance-based classification

Distance-based classification is mostly used in the parametric approach, where a fixed-length vector represents the signature and each coordinate of this vector is a global feature of the corresponding signature.

The mostly commonly used classifiers are based on the *Euclidean distance* [51], *Mahalanobis distance* [52–55] (which takes into account the correlation), *Dynamic Programming Matching (DPM)* [44] or use majority.

The Euclidean distance is calculated as

$$D(Q, R) = \sqrt{\sum_{i=1}^{n} (q_i - r_i)^2},$$

where $R = (r_1, \ldots, r_n)$ denotes the reference signature of the corresponding writer, $Q = (q_1, \ldots, q_n)$ is the questioned signature and r_i, q_j are global features extracted from the signatures.

We can use a weighted or normalised Euclidean distance to determine the importance of the different features:

$$D(Q, R) = \sqrt{\sum_{i=1}^{n} w_i (q_i - r_i)^2},$$

where w_i denotes the weight of the i^{th} global feature.

The Mahalanobis distance of a questioned and the reference signature can be calculated as follow

$$D(Q, R) = \sqrt{(Q - R)^T \cdot C^{-1} \cdot (Q - R)}$$

where C is the covariance matrix of the R and Q feature vectors.

Dynamic Time Warping (DTW)

DTW first appeared in speech recognition [56] in 1978 and it is capable of measuring the distance between two (not necessary temporal) sequences which have different lengths. DTW aligns in a non-linear way the signals to each other, computes a distance matrix based on the matchings, and calculates the shortest path from the left-bottom to the right top corner of the matrix.

The DTW algorithm finds the best non-linear alignment and a corresponding "warping path" between two vectors such that the overall distance between them is minimised. Besides the warping path we get the DTW distance between the $\mathbf{u} = (u_1, \ldots, u_n)$ and $\mathbf{v} = (v_1, \ldots, v_m)$ vectors. The distance (and the warping path) can be calculated in $\mathcal{O}(n \cdot m)$ time.

We can construct, iteratively, a $C \in \mathbb{R}^{(n+1)\times(m+1)}$ distance matrix in the following way:

$$
\begin{aligned}
C_{0,0} &= 0, C_{i,0} = +\infty, C_{0,j} = +\infty, \quad i = 1, \ldots, n; j = 1, \ldots, m \\
C_{i,j} &= |u_i - v_j| + \min\left(C_{i-1,j}, C_{i,j-1}, C_{i-1,j-1}\right), \\
&\qquad i = 1, \ldots, n; j = 1, \ldots, m,
\end{aligned}
$$

where $|u_i - v_j|$ denotes the absolute difference between the coordinate i of vector $\mathbf{u}$ and coordinate j of vector $\mathbf{v}$.

After the calculation of the whole C matrix, $C_{n,m}$ tells us the DTW distance between the vectors $\mathbf{u}$ and $\mathbf{v}$, and the warping path. The warping path is the shortest path from $C_{0,0}$ to $C_{n,m}$. It is constructed iteratively as the DTW distance: at point (i, j) the direction of the shortest path depends on the minimal choosen value among the three values $C_{i-1,j}, C_{i,j-1}, C_{i-1,j-1}$. Here $C_{i-1,j}$ represents horizontal direction, $C_{i,j-1}$ represents vertical direction, and $C_{i-1,j-1}$ is diagonal direction.

Figures 2.5.a, b and c represent the best matches, the shortest path and the density matrix (derived from distance matrix) with shortest path respectively for $\mathbf{u} = (5, 10, 6, 5, 2, 1)$ and $\mathbf{v} = (2, 6, 3, 11, 7, 2, 1, 0)$ vectors. $C_{0,0}$ is the bottom left corner, $C_{n,m}$ is the top right corner of the matrix. Here the horizontal direction is rightward, the vertical direction is upward, diagonal is rightward and upward all at once.

The DTW algorithm has several modifications, e.g., weighted DTW [57, 58], correlation optimized [59], and constrained by Sakoe-Chiba Band [60] or Itakura Parallelogram [61].

Wirotius et al. applied a polynomial approximation to improve DTW distance-based verification [62], Cho et al. applied a modified DTW computation to reduce time complexity [63], Faundez-Zanuy combined vector quantization (VQ) with DTW [64]. Khalil et al. applied an enhanced DTW distance in online signature verification [65], Putz-Leszczyńska and Kudelski also used DTW to compare signatures and introduced an artifical hidden signature instead of the template signature [66].

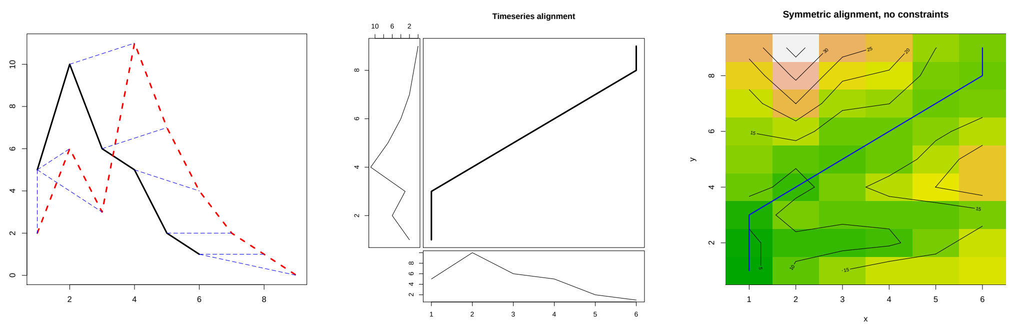
Hidden Markov Models

HMM is a statistical model that was first described by Baum and Petrie in 1966 [67]. A $\lambda = (A, B, \pi)$ HMM consists of

- n states, $S = \{s_1, \ldots, s_n\}$.
- m possible observations, $V = \{v_1, \ldots, v_m\}$,
- an $A = a_{ij} = \mathbf{P}(X_{n+1} = j \mid X_n = i)$, state transition probability distribution matrix,
- a $b_i(k) = \mathbf{P}(o_t = v_k \mid X_t = i)$, probability distribution in state i,
- and π initial state distribution, where $\pi_i = \mathbf{P}(X_1 = i)$.

Using the model, an observation sequence $\mathbf{O} = o_1, o_2, \ldots, o_T$ is generated as follows according to [68]:

1. Choose an initial state X_1 according to the initial state distribution π
2. Set $t = 1$
3. Choose o_t according to the $b_{X_t}(k)$ the probability distribution in state X_t
4. Choose X_{t+1} according to the $a_{X_t X_{t+1}}$ state transition probability distribution for state X_t



(a) Two discrete signals and matches between the point of these signals (Signal 1: black, solid, Signal 2: red, dashed)

(b) The middle rectangle represents the distance matrix, the shortest path is marked with a solid, thick, black line. The two signals are in separate boxes (left and bottom)

(c) Density matrix – it represents the distance values with colours as a heat map, from dark red to dark green: dark red denotes very high values, dark green denotes very low values. The shortest path is marked with a solid, thick, blue line, the other lines are contour-lines.

Figure 2.5: Visualization of DTW

5. Set $t = t + 1$ and return to step 3 if $t < T$ otherwise terminate the procedure

There are three basic problems of HMMs

EVALUATION PROBLEM. Given a model λ and a sequence of observation $\mathbf{O} = o_1, \ldots, o_T$ what is the $\mathbf{P}(\mathbf{O} \mid \lambda)$ probability of the observation sequence?

DECODING PROBLEM. Given a model λ and an observation sequence $\mathbf{O} = o_1, \ldots, o_T$ what is the most probable $X_1, \ldots, X_T$ state sequence?

LEARNING PROBLEM. How to adjust the A, B, π parameters of the model in order to maximize the $\mathbf{P}(\mathbf{O} \mid \lambda)$ probability for a given $\mathbf{O}$ observation sequence ?

HMM has been successfully applied in the past few decades in speech recognition [69, 70], speech emotion recognition [71], protein modeling [72, 73], shape classification [74] and many other areas.

In the last two decades HMM has been applied in several signature verifiers as well: in 1995 Yang et al. applied the Baum-Welch algorithm for the training and classification of online signatures [75]; while in 1997 Kashi et al. compared 23 global features, local features with HMM and combined them by applying the HMM log-likelihood score [25]. McCabe applied HMM in handwriting verification [76]; while Humm et al. in 2007 combined online signature verification with speech recognition as part of an authentication system that compares Gaussian Mixture Model (GMM) and HMM [77].

Figure 2.6 depicts an HMM with 4 states and 2 possible observations. This particular type of HMM is very common in signature verification and it is called left-to-right HMM, because the states can only reach themselves or the adjacent state.

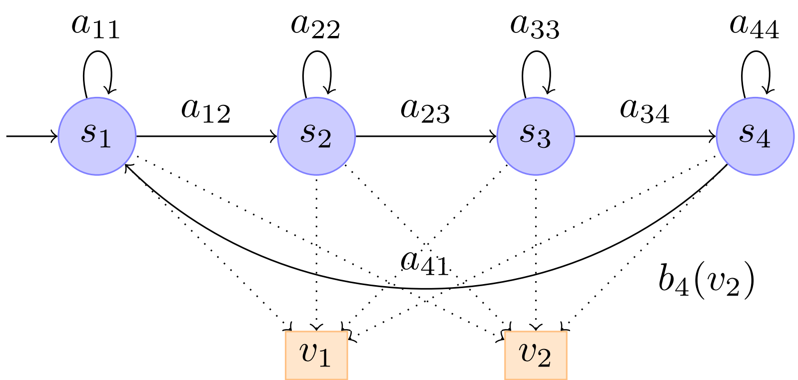


Figure 2.6: An HMM with 4 states ($s_1, \ldots, s_4$) and 2 possible observations (v_1 and v_2). Image credit: mblondel at github

Artificial Neural Networks

An Artificial Neural Network (ANN) is a model inspired by the nervous system of the brain and in computer science it is applied not only in pattern recognition to classify data, but also in data processing, robotics and function approximation. ANNs first appeared in 1943 in the work of McCulloch and Pitts [78], but since then several different models have been developed.

In general, these kind of models consist of neurons (units), their linkage (network) and weighted edges (synapses). The neurons are arranged in an input and an output layer and one or more hidden layers. Synapses run between only consecutive layers, never between neurons in the same layer From neurons in input layer synapses are only go to the hidden layer. If more hidden layers are present, then from input to the first hidden layer, from first hidden layer to the second, etc. The last layer is the output which is calculated using summation and an activation function.

During training of such ANN, it learns to adjust it's weights based on the known classes of the training data. The weights of the synapses are adjusted so as to minimize the error (the difference between the decided output and the output calculated by the network at the recent stage) on the training set.

Several heuristics exist which attempt to determine the optimal number of hidden layers, the number of neurons [79], the initial weights [80], or the best activation function and these methods take into account that too large neural network require more resources (memory, storage) and much more time to train.

Figure 2.7[4] shows a simple neural network with 4 inputs, one hidden layer with 6 neurons and an output layer with 1 neuron.

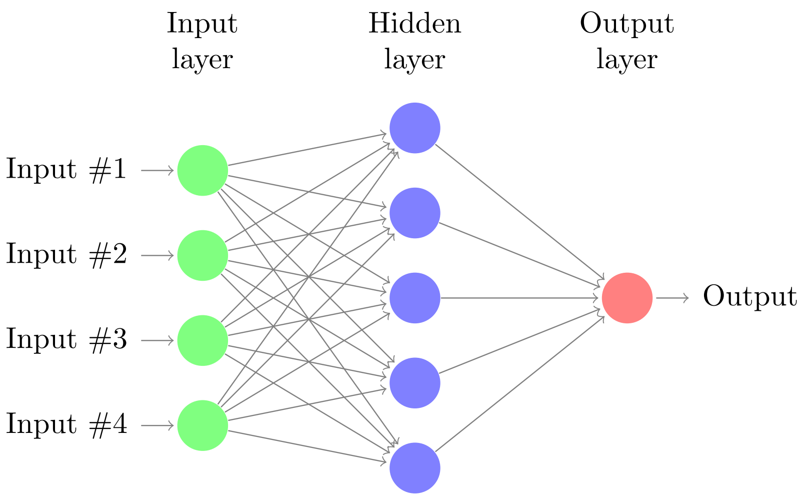


Figure 2.7: A Neural Network with one hidden layer

4 Source: Kjell Magne Fauske
`http://www.texample.net/tikz/examples/neural-network/`

ANNs are applied in many fields, such as handwritten character recognition [81]; system control [82], face detection [83], face recognition [84], traffic forecast [85], price forecast [86], but also applied in protein region detection [87], protein structure prediction [88], identifying lung cancer cells [89] and prognosing after breast cancer surgery [90].

ANNs appear often in online signature verificiation as well. McCabe et al. applied linear and Multilayer Perceptron (MLP) with one and two hidden layers [91]; Fahmy applied forward-backward propagation and hidden layer with 70 neurons using Discrete Wavelet Transformation (DWT) on 87 features [92]; Iranmanesh et al. applied MLP and features extracted using Pearson correlation coefficient on a Malaysian signature dataset [93]; Khalid et al. applied a three-layer fully connected MLP which was trained with back-propagation, used the outputs in fuzzy decision modules to decide about genuineness [39].

In a recent study Lai et al. applied Recurrent Neural Network (RNN) in online signature verification. The authors combined the SVC2004 and MCYT2003 to have enough training data and applied scale and rotation-invariant features using LNPS (length-normalized path signature) and GRU (Gated Recurrent Unit) with Adamax optimization as training. Equal Error Rate (EER) between 2.37 and 16.92% were reported using different number of training samples (6, 8, 10), sliding window size (7, 9, 11, 13, 15) and number of clients (0, 25, 50, 75, 100). Best EER (2.37%) was reached using combined training and testing on SVC2004 [94].

Support Vector Machine

Support Vector Machine (SVM) is a supervised classifier and it measures the similarity between instances using different kernel functions. The method tries to find the best separating hyperplane between two classes with the maximal margin based on the labeled training dataset. The original SVM was linear and was invented by Vapnik and Chervonenkis in 1964 [95]. Later it was extended to non-linear with kernel functions by Boser et al. in 1992 [96].

Given $\mathbf{x}_i \in \mathbb{R}^n$ instances $(i = 1, \ldots, L)$ and their $y_i \in \{1, -1\}$ labels (their classes), we have to solve the following optimization problem [97]:

$$\begin{aligned} \min_{\mathbf{w},b,\xi} \quad & \tfrac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{L} \xi_i \\ \text{subject to} \quad & y_i\left(\mathbf{w}^T\phi(\mathbf{x}_i) + b\right) \geqslant 1 - \xi_i, \quad \xi_i \geqslant 0 \end{aligned}$$

The training vectors $\mathbf{x}_i$ are mapped to a high dimension using the ϕ function and $C > 0$ is the penalty parameter of the error term. The $K(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)^T\phi(\mathbf{x}_j)$ is called the kernel function. The most commonly used kernel functions are

- linear: $K(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i^T\mathbf{x}_j$,
- polynomial: $K(\mathbf{x}_i, \mathbf{x}_j) = (\gamma\mathbf{x}_i^T\mathbf{x}_j + r)^d$,

- Radial Basis Function (RBF): $K(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\gamma \left\|\mathbf{x}_i - \mathbf{x}_j\right\|^2\right)$,
- sigmoid: $K(\mathbf{x}_i, \mathbf{x}_j) = \tanh(\gamma \mathbf{x}_i^T \mathbf{x}_j + r)$,

where $\gamma > 0, r$ and d are parameters.

Figure 2.8[5] represents the original instances and non-linear separator on the left, the right side represents the transformed instances with a linear separator.

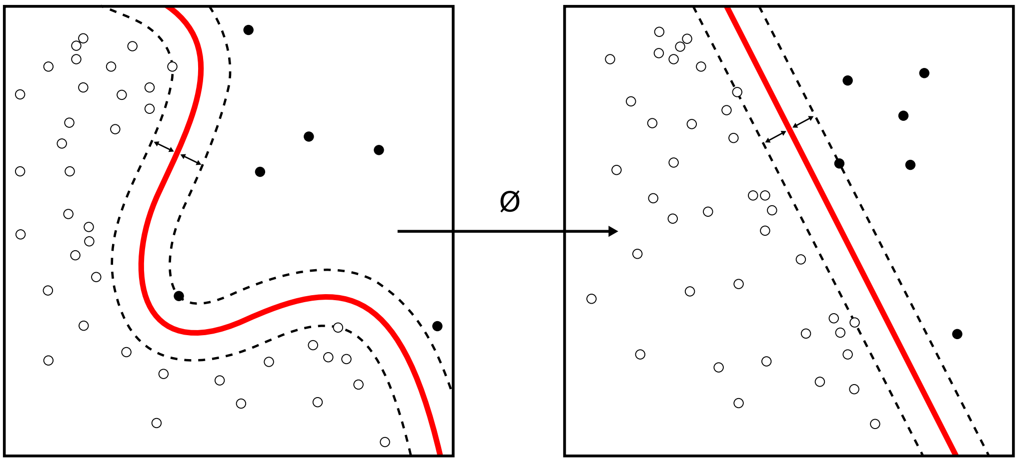


Figure 2.8: SVM applies a kernel function to determine a non-linear separator for the classes as a linear separator in a higher dimension

Kholmatov and Yanikoglu won the First Signature Verification Competition in 2004 with an online signature verifier combining three different classifier and DTW. One of the classifiers was SVM besides a Bayesian classifier and a PCA (Principal Component Analysis) linear classifier [19]. Gruber et al. applied SVM with longest common subsequences (LCSS) kernel [98]; Saeidi et al. cominbed extended regression and SVM to verify online signatures [99]; Huang and Gao applied genetic algorithms to optimize the parameters of SVM and the selection of feature subset [100]. Radhika and Gopika verified online and offline signatures and combined their results to perform verification using SVM classifier with projection, gradient and velocity features [101].

SVM was successfully applied in several other fields as well, such as spectroscopy [102], remote sensing [103], data mining [104], intrusion detection [105], handwritten digit recognition [106, 107], handwritten character recognition [108–110], face detection [111–113], image classification [114, 115] and classification and validation of cancer tissues, types and protein cells in bioinformatics [116–120].

5 Source: Wikipedia by user Alisneaky, svg version by Zirguezi
`https://en.wikipedia.org/wiki/File:Kernel_Machine.svg`

EVALUATION

During signature verification we test hypotheses where the null hypothesis H_0 corresponds to intra-source score (same author, genuine signature), alternative hypothesis H_1 corresponds to inter-source score (different author, the signature is forgery).

In hypothesis testing we define Type I and Type II error rates depending on the decision and the ground truth. Table 2.1 summarizes the types of errors which are generally used in the literature for hypothesis testing and with italic font style the terminology used in signature verification. Type I error represents the decision when we reject a genuine signature, Type II error occurs when a forged signature is accepted as genuine. Correct decision means the genuine signature was accepted or the forged signature was rejected.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Truth</td></tr>
<tr><td>H_0 true</td><td>H_0 false</td></tr>
<tr><td rowspan="2">Decision</td><td>Accept H_0</td><td>Correct
accepted genuine</td><td>Type II. error
accepted forgery</td></tr>
<tr><td>Reject H_0</td><td>Type I. error
rejected genuine</td><td>Correct
rejected forgery</td></tr>
</table>

Table 2.1: Error types

For the evaluation of different verification systems, Type I and Type II error rates are often called false rejection and false acceptance rates, respectively (False Rejection Rate (FRR) and False Acceptance Rate (FAR)). The traditional way to provide FRR and FAR depending on parameters, often a threshold. Using such a threshold one can examine that changing this threshold (e.g. a lower threshold is more restrictive towards forgeries) how FRR decreases and FAR increases. Besides this, sometimes accuracy is reported to measure the percentage of the correctly verified signatures.Accuracy can be often misleading especially when among questioned signature we have different number of genuine than forged. For example it is possible to reach 90% accuracy, but with this accuracy we accept half of the forgeries, and reject only a few genuine signatures. Moreover it is possible to have 0% FRR and 80% FAR the same time.

In order to avoid this bias, most often the equal error rate (EER) is reported for evaluation purposes[6]. This rate is often used to evaluate and compare biometric security systems. EER is the error rate when the FAR and FRR are equal. A lower EER means the verification system is more accurate towards genuine and forgeries as well.

Towards reducing the gap between verification systems created by computer scientist and forensic scientists, in ICDAR 2011 Signature Verification Competition (SigComp2011) a new measurement was suggested to measure the accuracy

6 also called as Crossover Error Rate (CER)

of a verification system, based on the Likelihood ratio (LR). Based on a similarity score produced by the verifier, LR is the probability of finding the similarity score given that the null hypothesis is true divided by the probability of finding the score when the alternative hypothesis is true [20]. Often the Log-likelihood ratio (LLR) is used which is the logarithm of LR and it is easier to optimize.

The costs of the LLR (denoted by $\hat{C}_{llr}$) and the minimal possible value of $\hat{C}_{llr}$ (denoted by $\hat{C}_{llr}^{min}$) was introduced by Brümmer and du Preez [121]. A smaller $\hat{C}_{llr}^{min}$ means better performance. Based on the results of the SigComp2011 the best FAR and FRR always had the best value of $\hat{C}_{llr}^{min}$.

DATABASES

The following section describes publicly and non-publicly available databases that have been used in the last one and half decades to evaluate online and offline signature verification methods. Table 2.3 provides an overview of these dataset, including the dataset size (number of writers, number of signatures per writer), types of forgeries, type of acquisition (offline/online), type of data (x_t, y_t coordinates, p_t pressure, azimuth, altitude, image), the nationality of the writers, acquisition devices and the related publications.

MCYT2003

The MCYT baseline corpus[7] is a bimodal biometric database containing two subcorpus: a fingerprint and a signature [122]. In both subcorpus 330 individuals provided fingerprints and signatures. In the case of the Signature subcorpus, 25 genuine signatures and 25 highly skilled forgeries (provided by 5 writers who could observe the static images of the signatures and practiced at least 10 times before recording) were obtained for each writer, alltogether this subcorpora contains $330 \times (25 + 25) = 16\ 500$ signatures samples. The WACOM Intuos A6 USB tablet was used as an acquisition device, which provides discrete-time dynamic sequences of the x_t, y_t positions, pressure p_t, azimuth and altitude angles γ_t, φ_t.

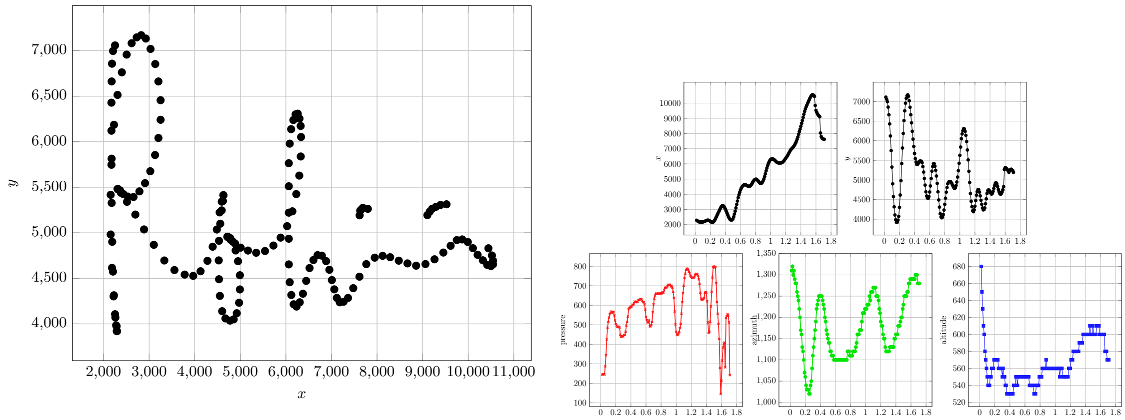
SVC2004

SVC is the abbreviation for the First International Signature Verification Competition[8] which consisted two tasks [18]. Each signature database created for the competitions involves 100 sets of signatures with 20 genuine (recorded in two sessions) and 20 skilled forgeries taken from at least 4 other contributors who were allowed to see the original signature and practise before recording forgeries. The database contains only coordinate information for Task 1, for Task 2

7 MCYT website `http://tinyurl.com/mcyt2003`
8 SVC website `http://tinyurl.com/svc2004`

pressure and pen orientation are also included. For both tasks the first 40 sets were provided as training data, while the remaining 60 sets were used during the evaluation of the competition. The training data are completely different for the two tasks, but the test subsets are the same except for the additional pen orientation and pressure data for the second task.

Figure 2.9.a is a sample signature taken from the database, while Figure 2.9.b shows the different features (x, y, p, azimuth and altitude values) of the corresponding sample.



(a) Signature

(b) x, y coordinates, pressure, azimuth and altitude time functions

Figure 2.9: Online signature sample taken from the SVC2004 database

SUSIG (2009)

The SUSIG database[9] consists of two subcorpora, namely the Visual subcorpus and the Blind subcorpus [123].

The Visual subcorpus contains genuine signatures from two sessions (approximately one week time passed between the recording sessions), from 100 writers, 10 signature per writer (in total 2000 samples). Skilled and highly skilled forgeries were provided by 100 writers, 5 samples per writer (in total 1000 samples). The validation set contains both genuine and forged signatures taken from 10 writers, 10 signature per writer (in total 200 samples).

The Blind subcorpus contains genuine signatures from one session, from 100 writers, 8/10 signature per writer (in total 940 samples), skilled forgeries[10] were provided by 100 writers, 10 samples per writer (in total 1000 samples). The validation set contains both genuine and forged signatures from 10 writers, 10 signature per writer (in total 200 samples).

9 SUSIG website `http://tinyurl.com/susig2009`

10 skilled forgeries could practice the signing based on the image, highly skilled forgeries was not clear how differ

Signatures in the Visual subcorpus were collected using an Interlink Electronics's ePad-ink tablet with a built-in LCD display such that people could see their signatures while signing. The device can store x, y coordinates, pressure at 128 levels and a timestamp. In contrast, no visual feedback was available for the Blind subcorporus which was collected with a Wacom's Graphire2 pressure sensitive tablet approximately 4 years before the Visual subcorpus. As a result, the people who donated the two subcorpora are largely different. However, the people who donated signatures are from the same demographics, resulting in similar signature complexity. The applied devices is capable of recording x, y coordinates of the trajectory, and pressure at 512 levels.

BSEC2009

The purpose of BSEC2009 competition[11] was to evaluate the algorithms depending on the quality of online signatures on the two BioSecure Data Sets DS2 and DS3 captured by a Wacom Intuos 3 A6 tablet and PDA (HP iPAQ hx2790), respectively [124].

The DS2 subcorpus contains x, y coordinates, pen pressure (at 1024 levels) and pen inclination angles (azimuth and altitude). The signatures were recorded in two sessions (the second session was 2 weeks after the first), during each session 15 genuine signatures and 10 skilled forgeries (of 2 other persons) were recorded from each writer.

The DS3 subcorpus contains x, y coordinates, and a timestamp. Signatures were also recorded in two sessions, the second session being around 5 weeks later. During each session 15 genuine signatures were recorded per writer and 10 forgeries (of two other persons' signatures). The forger was permitted to see the dynamics of the signature on the PDA screen before forging it in order to provide a highly skilled forgery.

Both datasets contains data taken from the same 382 people. A sample from each subcorpus is shown in Figure 2.10 and 2.11.

In 2009[12] Blankers et al. organized the first signature verification competition based on online and offline data, namely the SigComp2009 [125]. This name refers to both the dataset and the competition.

SigComp2009

Online and offline signatures were collected simultaneously at the Netherland Forensic Institute (NFI), the writers were employees of the Institute. For training the NISDCC signature collection was used, composed of offline signatures from 12 authentic writers (5 signatures per writer) and 31 forgers (5 forgeries per every

11 BSEC2009 website `http://tinyurl.com/bsec2009`
12 SigComp2009 website `http://tinyurl.com/sigcomp2009`

genuine signature), thus alltogether $12 \times 5 + 31 \times 12 \times 5 = 1920$. Due to an error of capturing the signatures, the online dataset contains only 1905 files.

The evaluation sets (NFI-online and NFI-offline) contains genuine signatures taken from 100 (new) writers, 12 per writer and forged signatures taken from 33 writers (6 forgeries per signature). Each authentic signature was forged by 4 writers. Altogether the evaluation dataset contains 1953 signatures both in online and offline format.

The training and evaluation online datasets were recorded using a Wacom Intuos2 A4-oversized tablet with a Wacom Intuos inking pen. The offline datasets were digitized originally in RGB24bit, with 600 dpi resolution, preprocessed images are also being provided in greyscale and binary format, both in 300 dpi and 600 dpi.

Figure 2.12.a shows a sample signature data taken from the online database, Figure 2.12.b shows the x, y coordinates, pressure, pen tilt and azimuth values of the corresponding signature.

SigComp2011

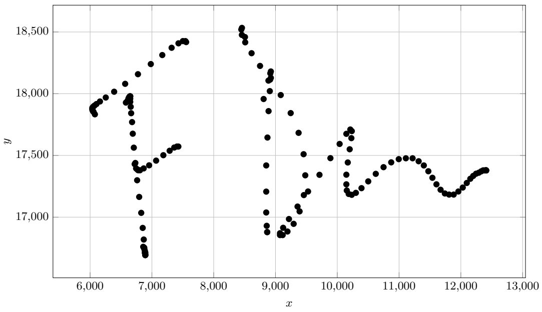
The primary aim of this competition[13] was to popularise the use of the likelihood ratio for decision which shows the strengh of the acceptance or rejection of the signatures [20]. It is very important to use this ratio, which supports the FHEs' evidence. In the competitions, the expected output for a questioned signature was a comparison score (degree of similarity) and an evidential value (i.e. likelihood ratio).

The collection contains offline and online Dutch and Chinese signatures acquired by a WACOM Intuos3 A3 wide USB pen tablet and software MovAlyzer. The Dutch training set contains genuine signatures taken from 10 writers and corresponding skilled forgeries, 449 online and 362 offline signatures. The Dutch test set contains genuine signatures taken from 54 writers and corresponding skilled forgeries, altogether giving a total of 1907 online and 1932 offline signatures. The Chinese training set contains genuine signatures taken from 10 writers and corresponding skilled forgeries, giving a total of 659 online and 575 offline signatures. The Chinese test set contains genuine signatures taken from 10 writers and corresponding skilled forgeries, altogether giving a total of 680 online and 602 offline signatures.

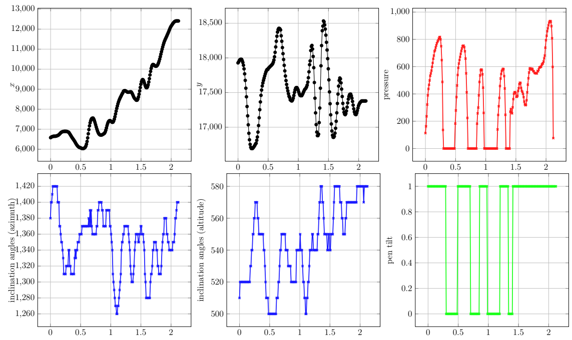
SigWiComp2013

The ICDAR 2013 Competitions on Signature Verification and Writer Identification for On- and Offline Skilled Forgeries (SigWiComp2013)[14] organized in 2013

13 SigComp2011 website `http://tinyurl.com/sigcomp2011`
14 SigWiComp2013 website `http://tinyurl.com/sigcomp2013`

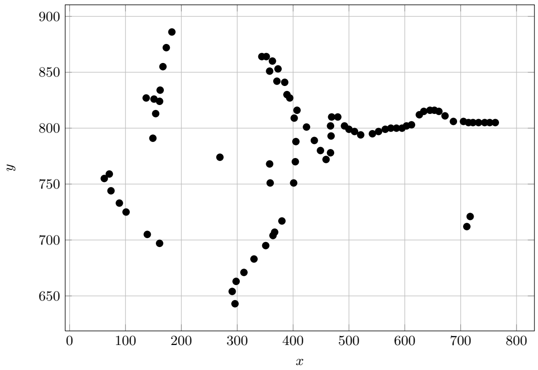


(a) Signature

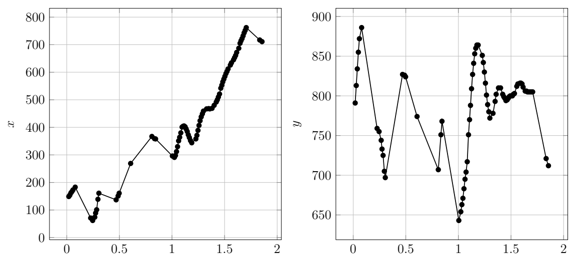


(b) x, y coordinates, pressure, inclination angles (altitude, azimuth) and pen tilt

Figure 2.10: Sample taken from BSEC'2009 DS2 database

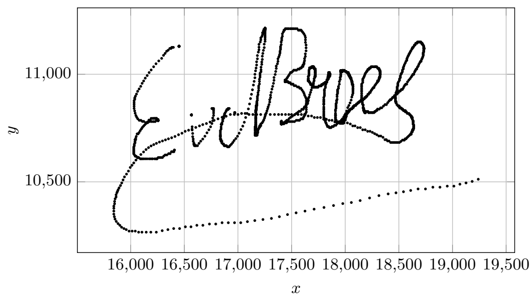


(a) Signature

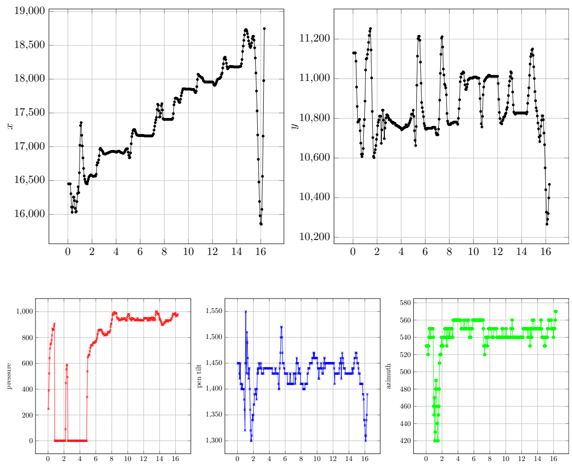


(b) x, y coordinates

Figure 2.11: Sample taken from the BSEC'2009 DS3 database



(a) Signature



(b) x, y coordinates, pressure, pen tilt and azimuth time functions

Figure 2.12: Online signature sample taken from the SigComp2009 database

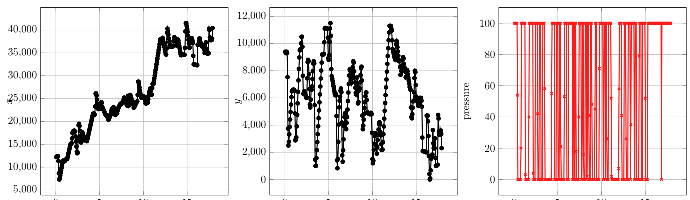
was the third in the SigComp series [126]. The database used during the competition contains Dutch offline, Japanese online and offline signatures and in addition to the previous competitions offline handwritten text samples (in English) for writer identification purposes.

The training set for the Dutch offline competition comprised of signatures taken from the training and evaluation sets of SigComp2009 and SigComp2011. For evaluation new signatures from 27 authors were used, each author provided 10 genuine signatures captured over a five day period, in total 270 signatures. Here, the authors were allowed to use any writing instrument (pen, ball-point). The evaluation set contains 974 skilled forgeries, forged by 9 employees of the NFI, 36 skilled forgeries on average for each author. The dataset was scanned at 400 dpi resolution, RGB colour and saved as png images.

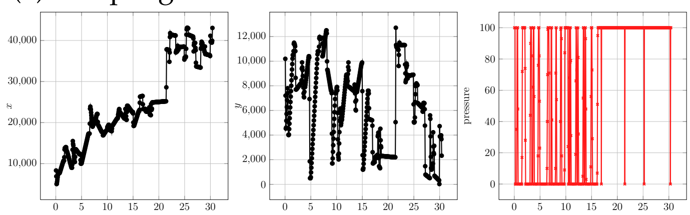
The Japanese offline and online dataset contains signatures from 31 individuals (11 in the training set and 20 in the evaluation set). The dataset was recorded with an HP EliteBook 2730p tablet PC and a software built with Microsoft INK SDK with a sample rate of 200 Hz. The online data consists (x_t, y_t) coordinates and z_t pen value with two values: pen down (100) and pen up (0). The offline database consists binary images generated based on the online data.

Each writer contributed 42 signatures to the database (15 signatures were captured on the first day, 9 signatures were captured in the following three days). Each signature was forged 36 times by 4 forgers (9 forgeries per forger). Altogether the database contains 1260 genuine signatures and 1080 skilled forgeries.

Two Japanese online signature samples are shown in Figure 2.13a and Figure 2.13b. One genuine time functions from the same author and one corresponding forgery are illustrated. Only time functions are depicted because according to the disclaimer of the database it is not allowed to display the images of the signatures in publications.



(a) Sample genuine time functions of the writer `000`



(b) A forgery sample time functions of the writer `000`

Figure 2.13: Samples taken from the SigWiComp2013 Japanese online database

SigWiComp2015

The ICDAR 2015 Competitions on Signature Verification and Writer Identification for On- and Offline Skilled Forgeries (SigWiComp2015)[15] consisted of 4 tasks: SigWestern, SigIndic, SigGerman and WI. The first three were signature competitions, SigWestern and SigIndic on offline modality, the SigGerman on online. WI stands for writer identification and retrieval, hence the WI dataset contains English handwritten text [127].

The SigWestern dataset contains Italian offline signatures collected over a period of time that is between 3 and 5 years depending on the author. The training dataset was provided by 50 author, 5-5 reference signatures in binarized format. The evaluation dataset consists of signatures taken from the same authors and corresponding forgeries, on average 10 questioned signatures per author, 229 genuine and 249 skilled forgeries.

The SigIndic dataset contains Bengali offline signatures, 240 genuine signatures from 10 authors (24 genuine per each) and 300 skilled forgeries taken from 10 individuals (30 signature per individual). The images were scanned at 300 dpi in 256 levels of greyscale. For training purposes the 240 genuine signatures were used, for testing the same genuine and the forgeries.

The SigGerman dataset contains online signatures that were collected using an Anoto pen (see Figure 2.4 earlier) taken from employees of different financial institutions and students of the University of Kaiserslautern (Germany). The dataset consists of x, y coordinates, p pressure with sampling rate 75 Hz and 85 dpi resolution. The training dataset was provided by 30 genuine authors, 10 genuine signatures per author. The evaluation dataset are taken from the same 30 authors with 10 reference and 15 questioned signatures per author, and (300 references, 150 genuines and 300 forgeries in total).

4NSigComp2010

The dataset of the ICFHR 2010 Signature Verification Competition (4NSigComp2010)[16] contains offline signatures collected under the supervision of Bryan Found and Doug Rogers in the years 2002 and 2006 and scanned for the purpose of the 4NSigComp2010 signature verification competition [128].

The competition consisted two scenarios. In scenario 1 the aim was to compare FHEs opinion on authorship and automatic systems The collection contains 209 signature images taken from 1 writer for training, 9 reference signatures and 200 questioned (76 genuine, 104 simulated forgeries and 20 disguised). The disguised signature is a signature taken from the authentic writer, which is altered somehow by the author in the purpose of denial. This is the first collection and

15 This dataset recently (May 2017) is not available, the training dataset was available at the time of the competition, but its website has been deleted

16 4NSigComp2010 website `http://tinyurl.com/4NSigComp2010`

competition where disguised signatures were present. The evaluation set contains 125 signatures taken from another writer: 25 reference signatures and 100 questioned signatures. The questioned signatures comprised 3 genuine, 90 simulated and 7 disguised signatures.

In scenario 2, the aim was to evaluate the performance of automatic verifiers in a security less critical environment. The reference set contains signatures from 400 writers, 4 genuine per person. The signatures were scanned at 300 dpi resolution and stored in black and white bmp format. The evaluation set contains 30 000 signatures, including original signatures, random signatures and simulated forgeries.

4NSigComp2012

The dataset of the ICFHR 2012 Signature Verification Competition (4NSigComp2012)[17] contains offline signatures which were collected under supervision of Bryan Found and Doug Rogers in the years 2001, 2002, 2004, 2005 and 2006 [129, 130].

The training set was the collection of the complete 4NSigComp2010 collection. The test set contains signatures from 3 writers: A1, A2 and A3, see Table 2.2 [130]. Signer A1 provided 3 normal genuine signatures and 6 disguised signatures everyday with a ball point pen for fifteen days and 6 genuine signatures with a pencil for three days. Signer A2 and A3 also provided normal and disguised signatures over a 10 and 15 day period respectively.

Skilled forgeries were provided as well. Each forger who forged signer A1 practiced 25 signatures and provided 5 forged signatures with a ball point pen and 5 forged signatures with a pencil every day over a 10 day period. For authors A2 and A3, forgers were provided 3 genuine samples of the corresponding specimen writer and were instructed to produce signature as forgers of signer A1. For author A2 31 forgers contributed signatures, for author A3 only 6 forgers provided forgeries.

No. of signatures	**A1**	**A2**	**A3**
Reference	20	16	15
Questioned	250	100	100
Disguised	47	8	9
Forged	160	42	71
Genuine	43	50	20

Table 2.2: Test data of 4NSigComp2012

17 4NSigComp2012 website `http://tinyurl.com/4NSigComp2012`

NAME	NUMBER OF WRITERS	TYPE OF FORGERY	NUMBER OF G/F SIGNATURES PER WRITER	NUMBER OF G/F SIGNATURES OVERALL	ONLINE OFFLINE	DATA	NATIONALITY	DEVICE	REFERENCE
MCYT2003	330	skilled	25/25	8250/8250	online	x, y, p, γ, φ		WACOM Intuos A6	[122]
SVC2004		skilled	20/20	2000/2000	online		English, Chinese	WACOM	[18]
Task1	100					x, y		Intuos tablet	
Task2	100					x, y, p, γ, φ			
BSEC 2009	382	skilled	30/20	11460/7640	online				[124]
DS2	-//-					x, y, p		Wacom	
						γ, φ		Intuos3 A6	
DS3	-//-					x, y		PDA	
			NUMBER OF TRAIN / TEST SIGNATURES [G+F(+D)] PER WRITER	OVERALL					
SigComp2009	12/100	skilled	5+5/12+6	1905/1953	online	x, y, p	Dutch	Wacom Intuos	[125]
	12/100	skilled	5+5/12+6	1920/1953	offline	image	Dutch	A4-oversized	
SigComp2011	10/54	skilled		362/1932	offline		Dutch	WACOM	[20]
	10/54	skilled		449/1907	online	x, y, p	Dutch	Intuos3 A3	
	10/10	skilled		575/602	offline		Chinese		
	10/10	skilled		659/680	online	x, y, p	Chinese		
SigWiComp2013	-/27	skilled	SC11/10+10	1356+270/3156+972	offline		Dutch	WACOM	[126]
	11/20	skilled	42/36	462+840/396+720	online	x, y, p	Japanese	Intuos3 A3	
	11/20	skilled	42/36	462+840/396+720	offline		Japanese		
SigWiComp2015	30/30	online	10/5+10	300/450	online	x, y, p	German	Anoto Pen	[127]
	50/50	offline	5/≈ 5+5	250/229+249	offline		Italian		
	10/10	offline	24/24+30	240/540	offline		Bengali		
			NUMBER OF SIGNATURE [GR+GQ+FQ+DQ] TRAIN	TEST					
4NSigComp2010	1/1	sim. forg.	9+76+104+20	25+3+90+7	offline				[128]
		disguised							
4NSigComp2012	2/3	disguised	previous		offline				[130]

Table 2.3: A comparison of publicly available signature databases. Here pressure is denoted by p, the azimuth angle by γ and the altitude angle by φ

WINNING METHODS

Here we give a brief overview of different winning systems for signature verification tasks tested on the previously described datasets. For details about the datasets, see Section 2.6. Table 2.4 shows overall results.

SVC2004

The winner team won in both tasks and achieved EER of 2.84% for Task 1 and an EER of 2.89% for Task 2 on skilled forgeries with a method based on DTW alignment and three different classifiers: a Bayes, SVM and a linear Principal Component Analysis (PCA) classifier, described in [19]. During the evaluation the systems were tested on random forgeries as well. The winning systems in these task achieved EER 2.04% and 1.70% although the description of these systems are missing from the summary article of the competition [18].

BSEC2009

In this competition there were 3 main tasks. The goal of Task 1 was to study the impact of mobility acquisition conditions on the verification methods' performance. The goal of the Task 2 was to examine what is the impact of the time variability on system's performance on DS2 dataset. The goal of Task 3 was to study the impact of information content in signature verifiers' performance. Eight participants submitted systems, 11 systems were submitted and all systems were tested on both tasks. Signatures were classified to High and Low Personal Entropy categories, based on the quality of the signatures in verification. These extreme categories means all the signatures from 60 and 161 writers among 382.

In Task 1 the lowest EER on DS2 dataset and skilled forgeries was achieved by a method based on DTW and local features (2.20%); on random forgeries by a DTW based system using 27 local features (0.51%). On DS3 dataset and skilled forgeries the winning system was based on DTW and local features as relative offsets of pen coordinates (4.97%); on random forgeries and DS2 dataset the same DTW based system won with 0.55% EER. In Task 2 the lowest EER was 1.71% without variability and 3.48% with variability. No further detail was provided. In Task 3 the lowest EERs was achieved on High Entropy category using local features and DTW distance (3.58%) and using fusion-based system and global features, on Low Entropy category an HMM and DTW based method won with 1.48% EER.

ESRA'2011

Here two tasks were announced: Task 1 was to assess the impact on the performance according to the quality of skilled forged signatures. The goal of Task 2 was to evaluate the systems depending on the representation of signatures. The competition used the BSEC2009 dataset to evaluate the systems.

In Task 1 only pen coordinates was given, during Task 2A besides the pen coordinates x, y the pen pressure p was also given and during Task 2B the data was extended with the inclination (azimuth and altitude) time functions. The database which was used to evaluate the systems is the same database used during the BSEC2009 competition (DS2 and DS3 datasets). In addition forged signatures were automatically classified as bad or good forgeries.

Different methods achieved the best performance based on the quality of the forged signatures. In Task 1 on bad quality forgeries 2.73% EER was achieved by a method based on HMM, on good quality forgeries 2.85% EER with a method based on HMM and local features. HMM obtained the best accuracy on the Task 2B with the inclination information (1.67% FRR and 2.43% FAR), but kernel based methods were better in Task 2A with only coordinates and pressure information (3.32% EER and 4.31% on good and bad forgeries respectively). DTW achieved the best accuracy in Task 1 on both good and bad forged signatures (6.05% FRR and 7.15% FAR).

SigComp2009

For each case only one reference signature was used and compared with the questioned signatures. The systems performed a similarity score (between 0 and 1, zero for non match and one for perfect match). During testing 6374 online matching, 9378 online non matching were executed.

For online task the system submitted by Parascript, LLC reached the lowest 2.85% EER value, for offline the system from Centre de Morphologie Mathématique won with EER value of 9.15%, details about the winning systems were not provided.

SigComp2011

Six systems were submitted to both Chinese and Dutch online competitions. The winning systems for both subtask was the same system based on statistical respectively empirical models. The winning system reached 93.17% accuracy on Chinese dataset ($\hat{C}_{llr} = 0.413413$ and $\hat{C}_{llr}^{min} = 0.217915$) and 96.27% accuracy on Dutch dataset ($\hat{C}_{llr} = 0.258932$ and $\hat{C}_{llr}^{min} = 0.122596$) .

SigWiComp2013

Only three systems were submitted to online japanese signature verification tasks, the winning system reached 72.55% accuracy with 27.56 FRR and 27.35% FAR ($\hat{C}_{llr} = 1.089183$ and $\hat{C}_{llr}^{min} = 0.744544$). The winning system is based on DTW described in [19].

SigWiComp2015

Several systems were submitted to the online signature verification competition: 7 of them based on DTW, one of them is on multiple time series representation, another one is on statistical data third is on differences between histogram and signal level. The winning system is based on huge number (70000) of movement characteristics had 90.27% accuracy with $\hat{C}_{llr} = 1.243891$ and $\hat{C}_{llr}^{min} = 0.290158$ developed by Cursor Insight[18].

The winner system for offline Italian signature applied HOG and LBP features and SVM classifier and achieved 99.16% accuracy, $\hat{C}_{llr} = 0.655109$ and $\hat{C}_{llr}^{min} = 0.021358$, details about the method can be found in [45]. The winner verifier on the Bengali offline signatures was based on run-length features and K-Nearest Neighbors (K-NN) and SVM , achieved 98.33% accuracy, $\hat{C}_{llr} = 0.923886$ and $\hat{C}_{llr}^{min} = 0.039721$, this method is described in [131]

18 www.cursorinsight.com

NAME	FORGERIES	TASK	BEST SYSTEM (%)	BASED ON
SVC2004	skilled	1	2.84	DTW
	skilled	2	2.89	DTW
	random	1	2.04	N/A
	random	2	1.70	N/A
		TASK	EER IN % DS2/DS3	
BSEC'2009	skilled	1	2.20/4.97	DTW
	random	1	0.51/0.55	DTW
	high entropy	3	3.58	DTW
	low entropy	3	1.48	fusion of 4 systems
		TASK	EER IN % WITHOUT / WITH VARIABILITY	
	skilled	2	1.71/3.48	N/A
	random	2	0.42/1.37	N/A
	QUALITY		EER (%)	
ESRA'2011	bad	1 DS2	2.73	HMM
	good	1 DS2	2.85	HMM
	good&bad	1 DS3	6.05 /7.15	DTW
	bad	2A DS2	3.32	kernel
	good	2A DS2	4.31	kernel
	good&bad	2B DS2	1.67/2.43	HMM
SigComp2009	skilled	online	2.85	N/A
	skilled	offline	9.15	N/A
			FRR/FAR (%)	
SigComp2011	sk. Dutch	online	3.70/3.76	statistics
	sk. Dutch	offline	2.47/2.19	edge-based distribution
	sk. Chinese	online	6.40/6.94	statistics
	sk. Chinese	offline	21.01/19.62	polar coordinates SVM
SigWiComp2013	sk. Dutch	offline	23.1/23.7	HOG, LBP
	sk. Chinese	offline	9.72/9.74	+SVM
	sk. Japanese	online	27.36/27.56	DTW
SigWiComp2015	sk. German	online	9.87/9.67	N/A
	sk. Italian	offline	0.87/0.80	HOG, LBP and SVM
	sk. Bengali	offline	1.67/1.67	run-length features and K-NN, SVM

Table 2.4: Performance of the winning systems on different online and offline signature verification competitions

Part II

ONLINE SIGNATURE VERIFICATION AND COMPARISON

3 DATASETS

Here we provide detailed description about the two datasets that we collected: the AccSigDb2011 (AccSigDb) and GyroSigDb2012 (GyroSigDb). Both datasets are online handwritten signature datasets, using a traditional ball-point pen with sensor(s) attached.

When we started collecting the dataset, only few public online signature databases were available. Since then some new databases were collected and published for research purposes, however still not many. Besides this none of the public dataset contains acceleration and angular momentum data. We wanted to examine the usability of the easily accessible accelerometer and gyroscope sensor and wanted to create public signature databases for further examination.

ACCSIGDB

Acquisition device

Here we used a ballpoint pen fitted with a three-axis accelerometer to follow the movements of handwriting sessions. Accelerometers can be placed at multiple positions of the pen, such as close to the bottom and/or close to the top of the pen [132, 133]. Sometimes grip pressure sensors are also included to get a comprehensive set of signals describing the movements of the pen, finger forces and gesture movements. In our study we focused on the signature-writing task, so we placed the accelerometer very close to the tip of the pen to track the movements as accurately as possible, see Figure 3.1.

The LIS352AX accelerometer chip was chosen in our design, because of its signal range, high accuracy, impressively low noise and ease-of-use. The accelerometer was soldered onto a very small printed circuit board (PCB) and this board was glued about 10 mm from the writing tip of the pen. Only the accelerometer, the decoupling and filtering chip capacitors were placed on the assembled PCB. A thin five-wire ribbon cable was used to power the circuit and carry the three acceleration signals from the accelerometer to the data acquisition unit. The cable was thin and long enough so as not to disturb the subject when s/he provided a handwriting sample. Our tiny general purpose three-channel data acquisition unit served as a sensor-to-USB interface.

The unit has three unipolar inputs with signal range of 0 to 3.3 V, and it also supplied the necessary 3.3 V to power it. The heart of the unit is a mixed-signal microcontroller C8051F530A that incorporates a precision multichannel 12-bit analogue-to-digital converter. The microcontroller runs a data logging program that allows easy communication with the host computer via an FT232RL-based USB-to-UART interface. The general purpose data acquisition program running on the PC was written in C#, and it allowed the real-time monitoring of signals. Both the hardware and software developments are fully open-source [134]. The block diagram of the measurement setup is shown in Figure 3.2. Note that originally the equipment was developed for educational purposes as described in [135] and our idea was to apply it in online signature verification.

The bandwidth of the signals was set to 10 Hz in order to remove unwanted high frequency components and prevent aliasing. Moreover, the sample rate was set to 1000 Hz. The signal range was closely matched to the input range of the data acquisition unit, hence a clean, low noise output was obtained. The acquired signals were then saved to a file for offline processing and analysis.

Subset 1 (AccSigDb1)

In order to make the signing process as natural as possible, there were no constraints on how the person should sign. This led to some problems in the analysis because during a signing session, the orientation of the pen can vary somewhat (e.g. a rotation with a small angle causes big differences for each axis), which influences the accelerometric signals along the three (x, y, z) axes. This is the reason why we transformed the 3-dimensional signals to 1 dimensional signals (we refer them later as reduced signals) and we compared the magnitudes of the acceleration vector data.

The signature samples were collected from 40 subjects between January and March of 2011 [136]. Each subject supplied 10 genuine signatures and 5 simple forgeries, so the dataset contains $40 \cdot 15 = 600$ signatures in total. The signature

Figure 3.1: The accelerometer is mounted close to the tip of the pen

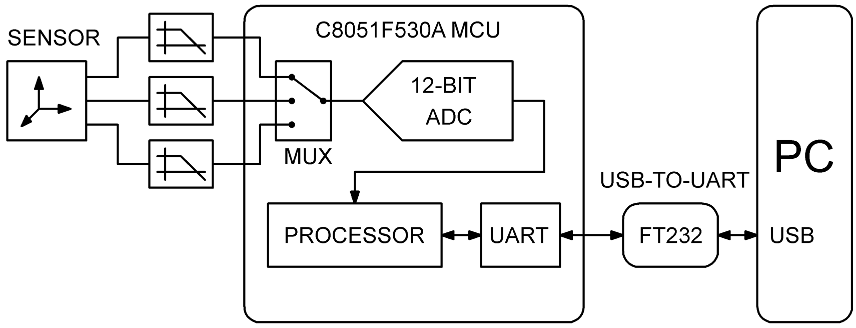


Figure 3.2: Block diagram of the data acquisition system

forgers were asked to produce 5 signatures of another person participating in the study. Each participant supplied forged samples and genuine samples as well.

Figure 3.3 shows the scanned images and the reduced acceleration signals of 2 genuine signatures and 1 forged signature. Subfigures a and b belong to the same author, and they appear quite similar. Subfigure c is a corresponding forged signature.

Subset 2 (AccSigDb2)

Afterwards the AccSigDb was extended [137], and the extended set contains 300 additional signatures. We asked 20 authors who contributed to the first version of the database and repeated the same process with them (10 genuine, 5 forged signatures per person) between April and May of 2011. This extension provided an opportunity to examine the similarities between signatures from the same author captured in two recording periods.

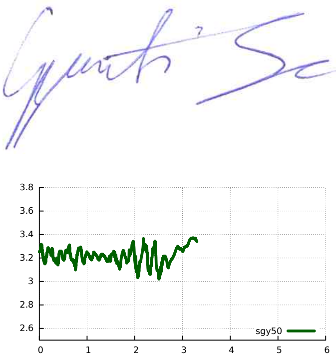
Figure 3.4 shows the scanned images and reduced acceleration signals of 2 genuine signatures and 2 forged signatures. Subfigures a and b show samples from the same author, and they appear quite similar. Subfigures c and d are the corresponding forged signatures.

Figure 3.6 shows the reduced signals of four signatures which belong to the same author. Subfigures a and b are the corresponding signals and signature from AccSigDb1, the subfigures c and d belong to two recorded signatures from the AccSigDb2 dataset.

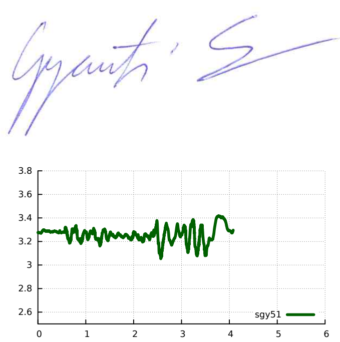
gyrosigdb

For comparison purposes we decided to try other sensor as well. Therefore we replaced the accelerometer with a 2-axis gyroscope to measure the angular momentum of the pen during the signing.

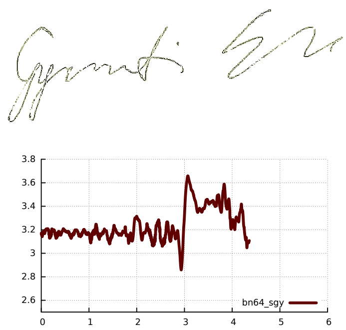
A low-power and dual-axis LPR530AL gyroscope sensor was applied to measure angular velocity along x and y axis. This sensor provides high resolution, has full scale of $\pm 300°/s$, and capable of detecting rates up to 140 Hz. Due to



(a) Genuine1

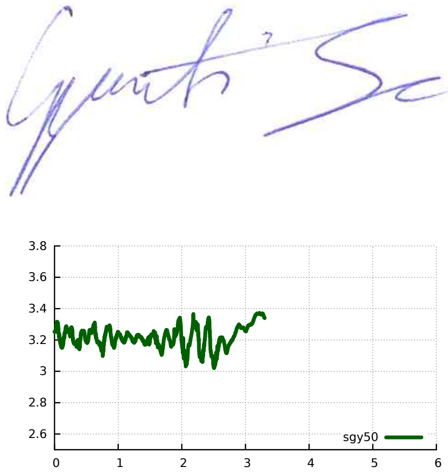


(b) Genuine2

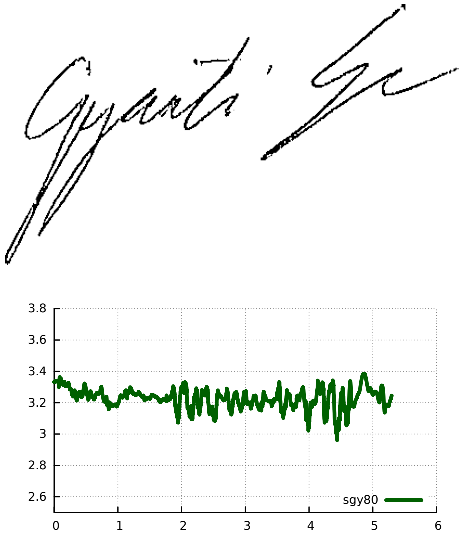


(c) Forgery

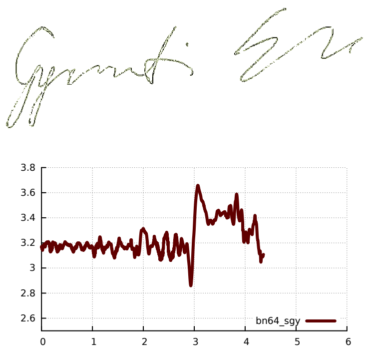
Figure 3.3: The images and acceleration signals of two genuine signatures and one forged signature from AccSigDb1



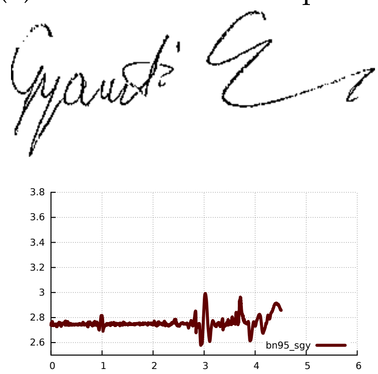
(a) Genuine - 1st time period



(b) Genuine - 2nd time period



(c) Forgery - 1st time period



(d) Forgery - 2nd time period

Figure 3.4: The images and corresponding acceleration signals of two genuine signatures and two forged signatures from AccSigDb1 and AccSigDb2

the size of this circuit board we could not attach the gyroscope to the tip of the pen (see Figure 3.5). We choose the middle part of the ball point pen so it is less inconvenient for the writer during writing.

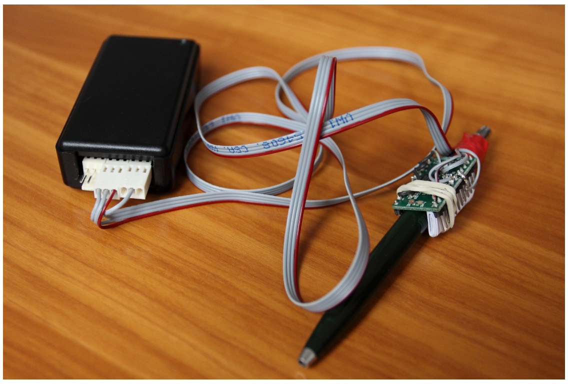


Figure 3.5: The gyroscope is mounted to the pen

Angular momentum

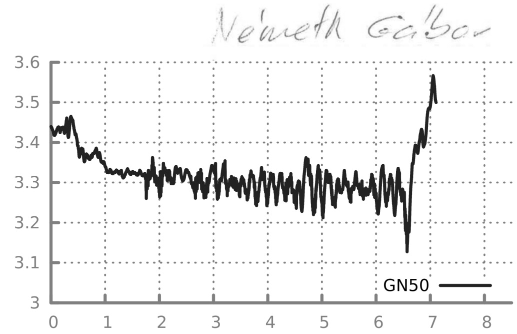
We used similar recording process to the one used during the recording of AccSigDb, however we used gyroscope attached to the ballpoint pen instead of the accelerometer. The sample rate during the signature recording was 100Hz. We refer to this database as GyroSigDb. 21 authors contributed to the GyroSigDb, each of them contributed ten signatures except one who gave 50 signature samples. Skilled forgeries were recorded from four of them as well.

Figure 3.7 shows two signatures and the corresponding signals from the same author as mentioned above. It shows the output voltage of the gyroscope directly. Each row belongs to one signature, the first column (left) shows the signal along the x-axis, the second (right) shows the signal along the y-axis.

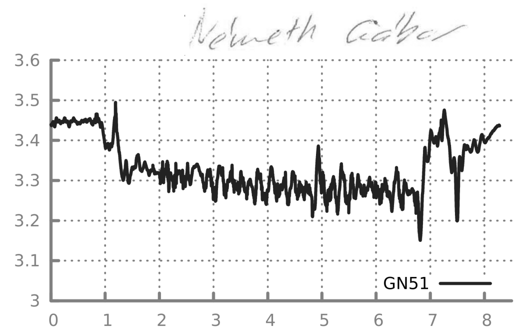
OVERLAP OF WRITERS

Some authors contributed to both AccSigDb and GyroSigDb.

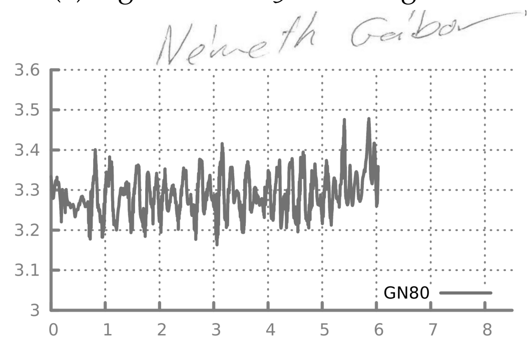
This overlap represents 10 authors and 300 genuine signatures: 20 genuine signatures per author from the AccSigDb (10-10 from the different recording periods) and 10 signatures per authors from the GyroSigDb. During the recording of GyroSigDb few authors were willing to contribute forged signatures, but due to the small number of forgeries, this dataset can be used principally to test the performance of writer identification methods.



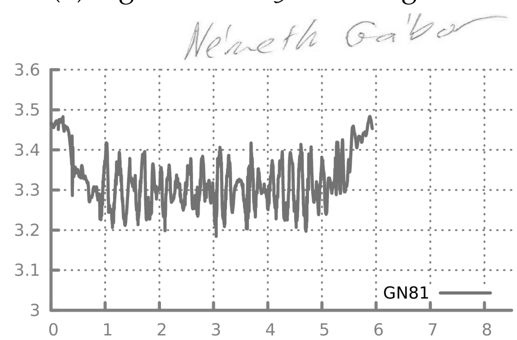
(a) Signature GN50 - AccSigDb1



(b) Signature GN51 - AccSigDb1



(c) Signature GN80 - AccSigDb2



(d) Signature GN81 - AccSigDb2

Figure 3.6: Four genuine signatures written by the same writer from dataset AccSigDb

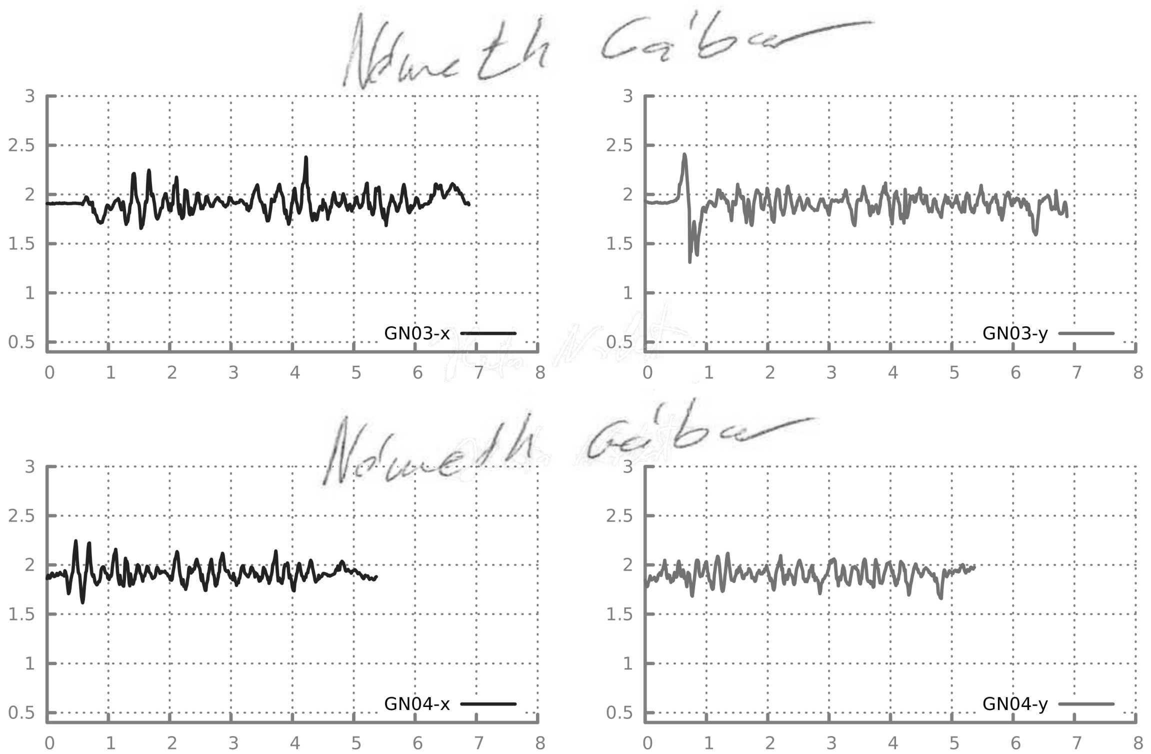


Figure 3.7: Two genuine signatures and the corresponding signals from writer NG (GyroSig, left/right: x/y axis)

4 ONLINE SIGNATURE VERIFICATION AND CLASSIFICATION

In this chapter we describe our online signature verification and classification methods that are based on the comparison of the triaxial acceleration and biaxial angular momentum of the handwriting process. We performed our experiments on the AccSigDb with genuine signatures and simple forgeries, and used the DTW method to measure the dissimilarities between signatures.

The novelty of our approach is a detailed investigation of the contribution of acceleration information in the signature verification process.

We also present a signature classification method applying Legendre approximation and SVM, performing analysis on the AccSigDb and GyroSigDb datasets [138, 139].

ACCELERATION BASED SIGNATURE VERIFICATION

Related work

In several studies, a special device (pen) was designed to measure the dynamic characteristics of the signing process. Accelerometric pens were first used by Herbst and Liu [140] (1977) with 200 Hz sampling rate and they collected 1332 signatures from 70 writers, 350 reference signatures.

Baron and Plamondon in 1989 considered the problem of measuring the acceleration produced by signing with a device fitted with 4 small embedded accelerometers and a pressure transducer [132]. It mainly focused on the technical background of signal recording. In [141], they described the mathematical background of motion recovery techniques for a special pen with an embedded accelerometer.

Rohlik et al. [142] and Mautner et al. [143] in 2001, 2002 employed a device to measure acceleration. This device is able to measure 2-axis accelerations and in [142] the authors aims were signature verification and author identification, while in [143] the aim was just signature verification. Both of them used neural networks.

Bashir and Kempf in 2008 used a Novel Pen Device and DTW for handwriting recognition and compared the acceleration, grip pressure, longitudinal and vertical axis of the pen. Their main purpose was to recognise characters and PIN words, not signatures [133].

Shastry et al. in 2011 presented an ordinary pen with sensors attached for signature verification that has a tri-axial accelerometer and two gyroscopes [144].

Examination on the AccSigDb1 dataset

Distance between time series

DTW elastic distance measure was applied to determine dissimilarities between the data. It has several versions, we apply the original version described in Subsection 2.4.3 (Page 15).

Since the order of the signal length (n and m) are around $10^3 - 10^4$, our implementation does not store the whole distance matrix C, whose size is about $n \times m \approx 10^6 - 10^8$. Instead, for each iteration, just the last two rows of the matrix were stored which is enough to determine the DTW distance.

For each writer who contributed k genuine signatures and had f corresponding forged signature we calculated $\binom{k+f}{2}$ DTW distance. One distance matrix belongs to writer AE (10 genuine and 5 forged signatures) is shown in Table 4.3 (on Page 51). The intersection of the first 10 columns and 10 rows shows the distance values between the genuine signatures. The intersection of the first 10 rows and the last 5 columns tells us the distances between genuine and the corresponding forged signatures. The rest (the intersection of the last 5 rows and last 5 columns) shows the distances between the forged signatures.

The distance between the genuine signatures varies from 60 to 308 (with average distance of 95), but between a genuine and a forged signature it varies from 157 to 950 (with average distance of 390).

The distance matrices belong to other writers are similar to that given above. In some cases the distance between genuine and forged signatures can be easily delimited, but in other cases we cannot define a strict line.

Results

We evaluated the above described method on the AccSigDb1 dataset. For each writer, 5 genuine signatures were chosen randomly as references. All the other signatures of this writer and skilled forgeries of their signature were used as questioned signatures: 5 genuine and 5 skilled forged signatures for each writer.

We first computed the average distance between the five references from ($D_{\mathbf{avg}}$). Then, for each questioned signature, the average distance between the questioned signature and the five reference signatures was found ($D_{\mathbf{dis}}$). If $D_{\mathbf{dis}} < c \cdot D_{\mathbf{avg}}$ then the questioned signature was accepted as a true signature (genuine), otherwise it was rejected (forgery).

Figures 4.1 shows the false reject and false accept rates depending on the constant multiplier (threshold) c of the minimum distance got from the training dataset. We can see that we get a zero FAR around $c = 7$. The curve decreases quite quickly while the increase of the FRR is less marked.

Besides the average distance we also used two other metrics, namely the maximum and minimum distances. These were calculated for the reference signatures:

$$D_{max}(R) = \max_{i,j=1,\dots,|R|, i \neq j} d_{DTW}(r_i, r_j)$$
$$D_{min}(R) = \min_{i,j=1,\dots,|R|, i \neq j} d_{DTW}(r_i, r_j),$$

where the set R contains the reference signatures, $|R|$ denotes the cardinality of R and r_i is the i^{th} reference signature.

We can use similar definitions to compute the distance between a questioned signature and a reference signatures:

$$D_{avg}(R, q) = \frac{\sum_{i=1}^{|R|} d_{DTW}(r_i, q)}{|R|}$$
$$D_{max}(R, q) = \max_{i=1,\dots,|R|} d_{DTW}(r_i, q)$$
$$D_{min}(R, q) = \min_{i=1,\dots,|R|} d_{DTW}(r_i, q),$$

where the set R contains the reference signatures, $|R|$ denotes the cardinality of R and r_i is the i^{th} reference signature and q is the questioned signature.

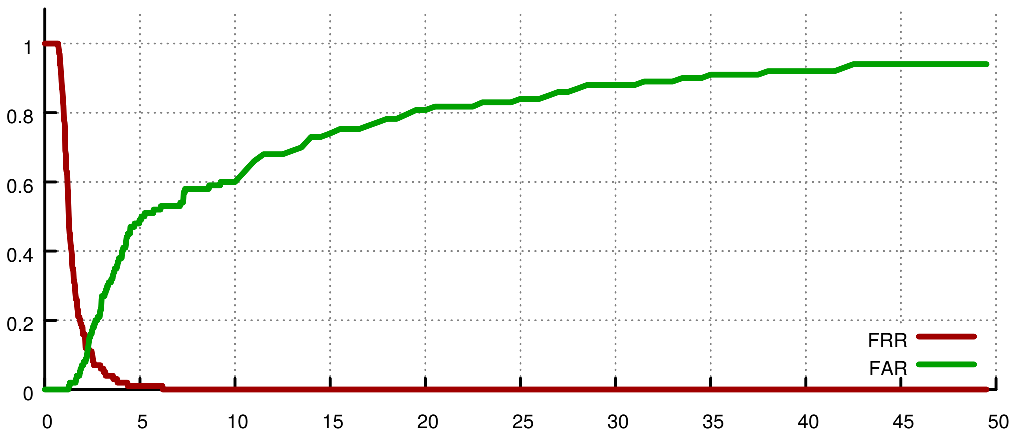


Figure 4.1: False rejection and false acceptance rates depending on the constant multiplier

Table 4.1 shows EER, the percentage where the false acceptance and the false rejection rates are equal. We see that we get the best results (the lowest EER), when we use minimum distance both for the reference and the questioned signatures.

		Test distance		
		average	**maximum**	**minimum**
Training	**average**	14.50% (1.36)	23.50% (0.56)	18.00% (3.34)
	maximum	17.25% (2.02)	29.50% (0.84)	23.25% (4.82)
	minimum	15.50% (0.98)	23.25% (0.38)	13.00% (2.28)

Table 4.1: EER depending on the chosen distance on the reference set and the chosen distance between references and the sample. The values in brackets are the corresponding c multipliers.

Summary

In this section an online signature verification method was applied to test the usability of accelerometric data for verifying human signatures. Here we found we had to limit the 3D acceleration vector data to 1D acceleration vector data so as to make the verification task more manageable. Using a time series approach and various metrics we were able to place signature samples into two classes, namely those that are genuine and those that are forged. The results were instructive and the method looks promising.

We attained the lowest 13.00% EER choosing the nearest neighbor approach (i.e. minimum distance on both test and training set) with $c = 2.16$ as a multiplier, and the highest 88.50% overall accuracy ratio choosing $c = 2.28$ as a multiplier. The method outlined in [143], used a similar device and neural networks to verify signatures, attained an overall accuracy ratio between 82.3% and 94.3%, depending on the author of the signatures (with an average of 87.88%). Thus our results compared to the above mentioned previous study is slightly better, despite the fact we used less data, as we did not have pressure information available.

There are several ways that the work described here could be extended. First, other metrics like DTW could be included and the results compared. Second, our method just uses the magnitude of the acceleration, not the direction. Thus our verification method could be improved by extracting more useful information from the 3-dimensional signals.

Third, we could compare other features (e.g. velocity, which can be computed from the acceleration data values) to learn which features are the most important in the signature verification process. A normalisation of the acceleration signals may be helpful too.

DIFFERENT REFERENCE SIGNATURE AND TIME PERIOD

Selection of reference signatures

In the previous section we examined the $40 \cdot 15 = 600$ signatures from the AccSigDb1. For each writer, 5 genuine signatures were chosen first randomly as references.

After we analyzed the results, we observed that the Type I (FRR) and II (FAR) errors depend on how we choose the reference signatures, so we checked all the possible choices of reference signatures and compared error rates. For each writer there were $\binom{10}{5} = 252$ possible ways of how to choose the 5 reference signatures from the 10 genuine signatures.

Based on our study described in Section 4.1.2, we set the multiplier c at 2.16 because we got the highest overall accuracy ratio (88.5%) with this value.

A typical distribution of Type I and Type II error rates are shown in Table 4.2a. The first two columns show the error rates, while the third one shows certain cases with the corresponding error rates. The last row shows the average error rate. According to this table, in 39 cases (out of 252) the Type I and Type II error rates are equal to 0. The average Type I error rate of 252 possibilities is 24.13%, while the average Type II error rate is 0. A much worse, but very rare case is shown in Table 4.2b.

For 27 authors (out of 40) and for each case, the FRRs were 0%. The average FAR was 14.34%, with a standard deviation of 13.62%; the average FRR was 12.89%, with a standard deviation of 24.33% on the AccSigDb1 .

False rejection/acceptance rates		
Type I	Type II	No of cases
0%	0%	39
20%	0%	135
40%	0%	68
60%	0%	7
80%	0%	3
Total		252
24.13%	0%	

(a) A typical distribution of error rates

False rejection/acceptance rates		
Type I	Type II	No of cases
0%	0%	13
0%	20%	52
0%	60%	45
20%	0%	8
20%	60%	58
20%	20%	45
40%	20%	8
40%	60%	22
60%	60%	1
Total		252
13.81%	38.33%	

(b) A different distribution of error rates

Table 4.2: Distribution of error rates for a given writer

Signatures from different time periods

Previously we investigated a procedure for signature verification which is based on acceleration signals. The necessary details about the method – applied in the earlier study and recent study – are explained in Section 4.1.2.

After we extended the AccSigDb with new recordings of the signatures from former signature suppliers, we were able to compare signature samples recorded in different time periods. In addition, we examined how the selection of reference signatures can affect the results of the verification process.

Table 4.3 and 4.4 (on page 51) are two distance matrices calculated for the same subject in the two time periods of signature recording (from AccSigDb1 and AccSigDb2, respectively).

The intersection of the first 10 columns and 10 rows shows the distance values between the genuine signatures (obtained from the same writer). The intersection of the first 10 rows and the last 5 columns tell us the distances between genuine and the corresponding forged signatures. The rest of the table (the intersection of the last 5 rows and last 5 columns) shows the distances between the corresponding forged signatures.

In Table 4.3 the distances between the genuine signatures varies from 60 to 317 with an average of 108 and a standard deviation 53, but between a genuine and a forged signature it varies from 158 to 977 with an average of 393 and a standard deviation of 211. In Table 4.4 the distances between the genuine signatures varies from 34 to 334 with an average value of 117 and a standard deviation 73, but between a genuine and a forged signature it varies from 165 to 770 with an average value of 382 and a standard deviation of 142. The distance matrices for other writers are similar to those given above.

In most cases there were no big differences between distance matrices calculated for different time periods (from the same author). Table 4.5 shows the DTW distance between all the genuine signatures taken from the same author for the different time periods. AE50-59 are from the first period, while AE80-89 are from the second. The average distance is 114, the minimum is 34, the maximum is 453 and the standard deviation of the distances is 70.3.

Figures 4.2.a and 4.2.b show the FRR and FAR as a function of the constant multiplier c of the minimum distance got from the training dataset.

We can see that in both time intervals we get a zero FAR when $c = 7$. The curves decrease quite quickly, while the increase of the FRR is less marked. The main difference between the two time intervals and the FRR curves is that in the first time interval it increases faster than in the second one. The reason is probably that in the second time interval the acceleration signals were quite similar (see Tables 4.3 and 4.4).

DTW	AE50	AE51	AE52	AE53	AE54	AE55	AE56	AE57	AE58	AE59	ME60	ME61	ME62	ME63	ME64
AE50	0														
AE51	63	0													
AE52	98	64	0												
AE53	125	71	105	0											
AE54	116	65	67	101	0										
AE55	63	113	136	167	157	0									
AE56	114	80	76	127	67	155	0								
AE57	104	68	76	115	73	147	63	0							
AE58	74	66	63	111	59	105	37	49	0						
AE59	233	173	86	177	82	317	165	152	122	0					
ME60	344	239	254	281	386	532	333	202	234	372	0				
ME61	274	232	252	285	441	450	402	239	246	501	135	0			
ME62	237	177	175	231	255	350	222	179	158	316	70	107	0		
ME63	318	259	260	304	410	494	334	221	227	372	50	83	67	0	
ME64	710	677	697	716	875	854	796	670	684	977	260	198	395	269	0

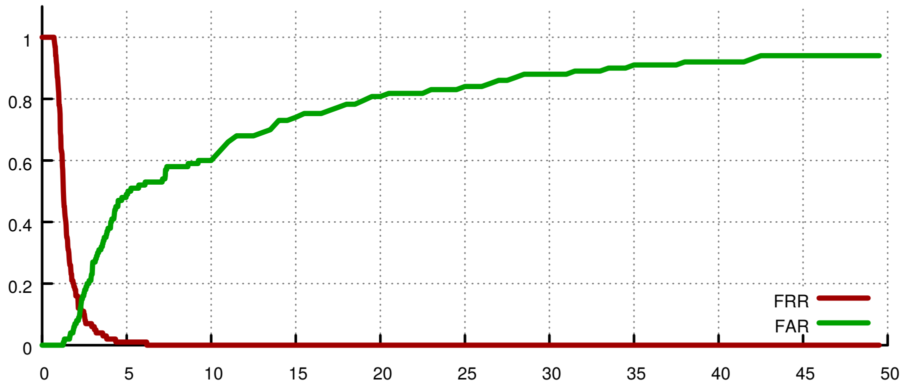
Table 4.3: Sample distance matrix of signer AE from AccSigDb1

DTW2	AE80	AE81	AE82	AE83	AE84	AE85	AE86	AE87	AE88	AE89	ME90	ME91	ME92	ME93	ME94
AE80	0														
AE81	34	0													
AE82	34	41	0												
AE83	50	63	47	0											
AE84	52	58	43	49	0										
AE85	217	213	179	227	206	0									
AE86	139	130	152	150	145	325	0								
AE87	117	103	144	154	147	339	81	0							
AE88	55	52	52	91	82	140	154	121	0						
AE89	65	63	60	71	65	233	105	125	92	0					
ME90	293	245	270	355	310	236	336	302	228	328	0				
ME91	227	198	208	295	252	245	275	262	165	259	54	0			
ME92	339	298	322	419	387	288	393	348	273	413	45	106	0		
ME93	617	625	569	617	699	473	518	415	473	770	202	260	117	0	
ME94	388	425	492	540	582	293	469	376	395	582	67	150	40	100	0

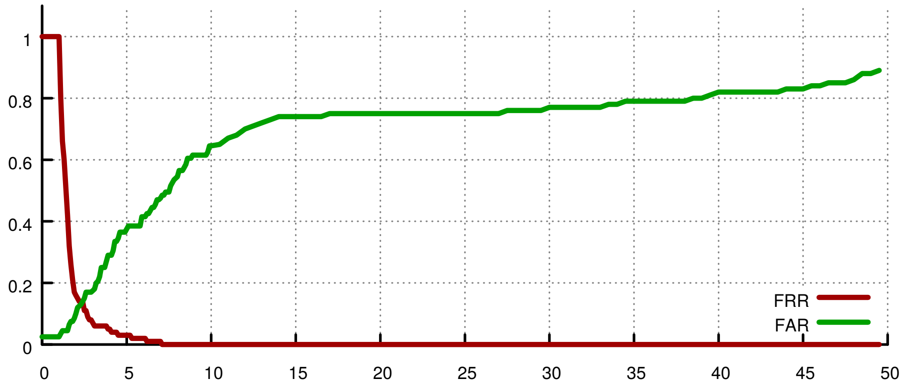
Table 4.4: Sample distance matrix using AccSigDb2

DTW	**AE50**	**AE51**	**AE52**	**AE53**	**AE54**	**AE55**	**AE56**	**AE57**	**AE58**	**AE59**	**AE80**	**AE81**	**AE82**	**AE83**	**AE84**	**AE85**	**AE86**	**AE87**	**AE88**	**AE89**
AE50	0																			
AE51	63	0																		
AE52	98	64	0																	
AE53	125	71	105	0																
AE54	116	65	67	101	0															
AE55	63	113	136	167	157	0														
AE56	114	80	76	127	67	155	0													
AE57	104	68	76	115	73	147	63	0												
AE58	74	66	63	111	59	105	37	49	0											
AE59	233	173	86	177	82	317	165	152	122	0										
AE80	74	51	47	95	75	112	65	67	50	168	0									
AE81	75	51	50	102	69	119	64	59	47	179	34	0								
AE82	67	40	48	96	54	104	74	66	57	179	34	41	0							
AE83	94	63	58	94	58	121	78	75	68	129	50	63	47	0						
AE84	90	54	57	87	44	120	65	53	49	124	52	58	43	49	0					
AE85	84	238	265	259	251	147	352	303	268	453	217	213	179	227	206	0				
AE86	223	145	111	192	141	306	128	145	110	92	139	130	152	150	145	325	0			
AE87	179	126	126	190	170	252	84	108	96	203	117	103	144	154	147	339	81	0		
AE88	45	63	77	132	105	82	87	83	64	217	55	52	52	91	82	140	154	121	0	
AE89	133	70	55	120	52	185	67	77	65	109	65	63	60	71	65	233	105	125	92	0

Table 4.5: Distances between genuine signatures from both time periods (AccSigDb1 and AccSigDb2)



(a) 1st time period



(b) 2nd time period

Figure 4.2: False acceptance and false rejection rates

Conclusion

In this study we examined how the reference set selection influences the accuracy and whether different acquisition time can influence the false acceptance and false rejection rates. The dataset used in the experiment contained $600 + 300 = 900$ signatures, where 600 signatures were genuine and 300 were forged.

Using the $c = 2.16$ threshold and calculating the error rates for 27 authors (out of 40) and for each possible choice of the 5 reference signatures, the FRRs were 0%, thus the Type I error rate did not depend on the choice of reference signatures in 67.5% of the cases. The average FAR was 14.34%, with a standard deviation of 13.62%; the average FRR was 12.89%, with standard deviation of 24.33% on the whole AccSigDb1. The much higher standard deviation of FRR together with result that FRR was 0% for 27 author, shows that FRR much more depends on the author and has much greater variability.

We found that the performance of a verifier depends largely on the reference set and writer's signature may not vary much over a period of weeks or months, but it could vary more over longer periods.

LEGENDRE APPROXIMATION IN ONLINE SIGNATURE CLASSIFICATION

The motivation for this study was to compare the expedience of two easy to use and cheap devices for online signature verification and to use a state-of-the-art feature extraction.

Different types of approximations have been successfully used in several different research field [145–147]. Due to the fact that each online signature data is large (depending on the number of dynamic features and the sample rate), an efficient feature extraction can reduce the amount of data and decrease the differences between signatures from the same author, thus hopefully improve the verification performance as well. Parodi et al. showed in [42], that the Legendre approximation combined with SVM and Random Forest (RF) classifiers could outperform earlier methods on the SigComp2011 database, hence we want to apply the proposed method on our databases and use it for comparison .

Here we show how this earlier published Legendre approximation and the SVM classifier performs on AccSigDb and GyroSigDb datasets to examine what accuracy can be achieved on different types of dynamical data.

Legendre approximation for feature extraction

Legendre approximation is based on Legendre polynomials which have the sum formula

$$p_n(x) = 2^n \sum_{k=0}^{n} x^k \binom{n}{k} \binom{\frac{n+k-1}{2}}{n}.$$

or the equivalent recursive form, where $p_0(x) = 1$, $p_1(x) = x$ and $(n+1)p_{n+1}(x) = (2n+1)p_n(x) - np_{n-1}(x)$.

We can approximate a function using linear combination of these polynomials in the following form:

$$P(x) = c_0 + c_1 p_1(x) + \cdots + c_N p_N(x), \tag{4.1}$$

where N is the order of the polynomial.

We varied the order of the Legendre polynomial from $N = 1$ to 40 in order to examine which order performs the best. Every signature in the AccSigDb can be represented with the $(c_0, c_1, \ldots, c_{N-1})$ feature vector which is constructed from the coefficients of the Legendre polynomial (see Equation 4.1 above) with length N. Signatures in the GyroSigDb were represented similar way as in [42]. Thus we calculated for each signature the Legendre polynomial with order N for axes x and y, respectively, and concatenated the coefficient vectors of the polynomials, so the feature vectors in these case have length of 2N.

SVM classifier for classification

For classification the SVM classifier was used with RBF. The parameters of the classifier were tuned automatically during the training phase by the Library for Support Vector Machines (libsvm), see [97]. Scaling was used before the testing and training as well. Half of the signatures were used for training purposes and we tested the classifier on the other half of the overlapping part of the databases.

We tested two types of classification. First we considered every possible pairing of the authors and used two-class SVM classifier with RBF. The accuracy of the classification can be seen in Figure 4.3a.

Second we used SVM classifier with every signature of the 10 authors mentioned above to see how SVM can separate the signatures of the 10 authors from each other into ten classes. The accuracy of the classification is presented in Figure 4.3b.

Results

Figure 4.3a and 4.3b show the accuracies depending on the order of the Legendre polynomials. We varied the order N from 1 to 40.

Considering the binary classification of the signatures, the average accuracy is around 85%-89% in AccSigDb and weakly dependent on the order of the polynomials. Contrarily the accuracy of the method on the GyroSigDb is always less than on the AccSigDb. The accuracy is increasing with the order of the polynomials until the order is less than 15 on AccSigDb and it is decreasing if larger than 15 (see Figure 4.3a). This growth lasth until order 30 on the GyroSigDb.

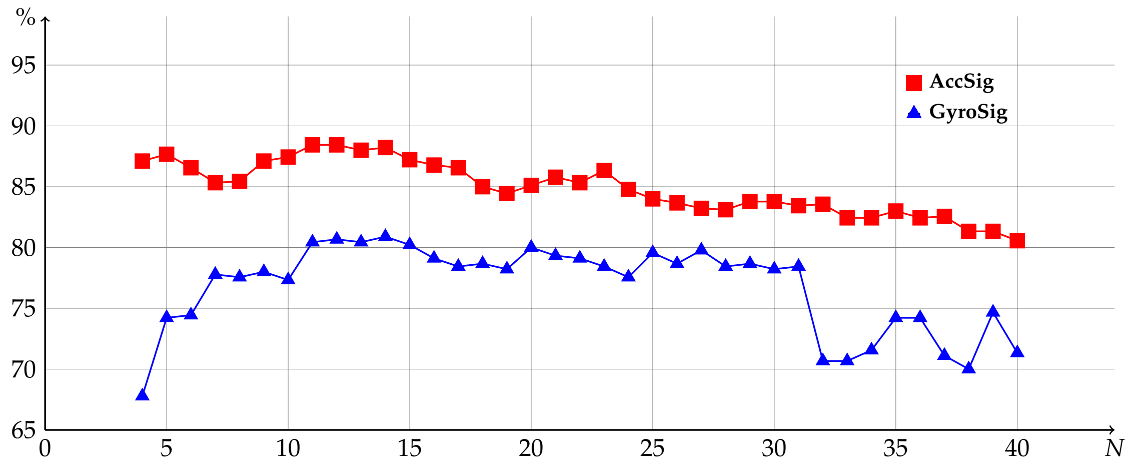
While using multi-class classification the accuracy is much lower than in previous case since we have more classes. In this case the proposed method results higher accuracy for GyroSigDb compared to AccSigDb. It is around 50% if the order of the Legendre polynomials order is less than 20, while it is just around 35-40% on the AccSigDb (see Figure 4.3b).

Conclusion

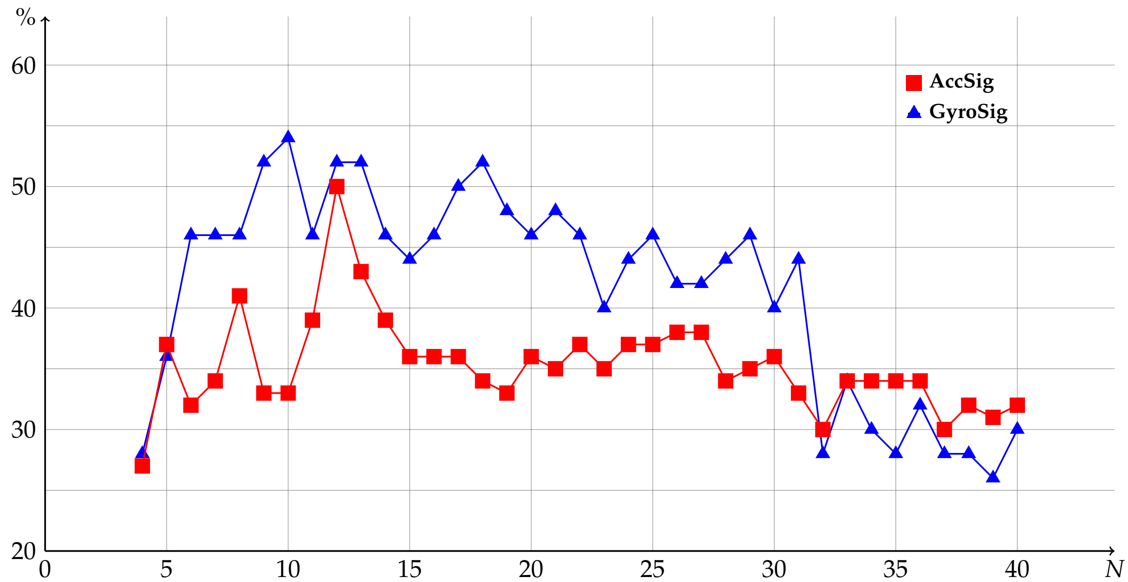
In this study we examined 300 signatures of 10 authors: 200 signatures from AccSigDb (from two different time periods) and 100 signatures from GyroSigDb was written by the same 10 authors (20+10 signature per author).

We used Legendre approximation as a feature extraction method and SVM as a classifier to compare the different features from two aspects. Above order of 20, higher order does not yield better accuracy, thus it is not necessary to increase the order of the polynomials from a certain point.

In order to get more insights in the analysis of the behavior of these two types of signals, we plan to extend the databases both in number of the contributors and number of the forged signatures. Furthermore, we want to apply other approximation and classification methods as well.



(a) pairwise binary



(b) multiclass classification

Figure 4.3: Accuracy of the classification

5 KOLMOGOROV-SMIRNOV METRIC IN SIGNATURE VERIFICATION

Kolmogorov-Smirnov statistics measures the distance between two distribution functions. It is applied in the non-parametric Kolmogorov-Smirnov test (KS test) to examine equality of one-dimensional, continuous probability distributions [148]. Besides the statistical examinations [149–152] it is applied in several fields such as image segmentation [153], image comparison [154] and to improve SVM by better feature selection [155].

Only few related research can be found concerning the topic of this dissertation, most of them applies KS test to compare distribution of features, e.g. Zois and Anastassopoulos in writer identification, Kővári and Charaf in offline signature verification [157]. An offline signature verification method of Srinivasan et al. applied the Kolmogorov-Smirnov statistics in the classification step [158].

In this chapter we present an online signature verification approach based on the Kolmogorov-Smirnov distribution metric. Different feature distributions are compared in order to perform classification and distinguish between forged and genuine signatures [159].

We have not found any online signature verification analysis or method in the literature which is based on statistical metric, hence we decided to carry out studies in this direction. We considered the relevance of the Kolmogorov-Smirnov statistics in online signature verification and here we evaluate a method based on the distribution distance determined by applying the Kolmogorov-Smirnov statistic.

THE PROPOSED ALGORITHM

Our idea was to keep the simple, given features without excessive transformations. The SigComp2011 database [20] was used for the evaluation of our method, which contains Dutch and Chinese online and offline signatures as well.

We performed the verification on the Dutch online data. The signatures were collected by a WACOM Intuos3 A3 Wide USB Pen Tablet with sampling rate of 200Hz. The (x_t, y_t, p_t) values are captured as x and y coordinates of the signature, and pressure p at a given time t.

Preprocessing

The WACOM tablet is a large tablet with an active area of 488×305 mm and it is possible for two signatures from the same person to be written on any part of the active area. Therefore we shifted the x, y coordinate values to the origin, so for each signatures, the minimal x and y values were substracted from the original coordinates. Thus, we got

$$\begin{aligned} x'_t &= x_t - \min_k (x_k) \quad (t = 1, \dots, T) \\ y'_t &= y_t - \min_k (y_k) \quad (t = 1, \dots, T) \end{aligned}$$

shifted coordinates, where T is the number of sampled values of the signature.

Feature extraction

In addition to the above-mentioned features (i.e., shifted x, y), we calculated the absolute velocity values based on the difference between the coordinates and time.

The velocity values are calculated in the following way:

$$v_k = \sqrt{\left(\frac{\Delta x_k}{\Delta t_k}\right)^2 + \left(\frac{\Delta y_k}{\Delta t_k}\right)^2} = \sqrt{\left(\frac{x_{k+1} - x_k}{t_{k+1} - t_k}\right)^2 + \left(\frac{y_{k+1} - y_k}{t_{k+1} - t_k}\right)^2}.$$

Classification

The classification is based on the distribution of the different features, especially the pressure and velocity values of the signatures.

In order to compare two distribution functions, the Kolmogorov-Smirnov distance (KS distance) was used which calculates the maximal difference between the cumulative distribution functions. If $F_x = f(X < x)$ and $G_x = g(X < x)$ are the two distributions, then the KS distance between the two distribution functions is

$$D(f, g) = \sup_x |F_x - G_x|.$$

Figure 5.1 shows two empirical cumulative distribution functions and the arrow indicates the KS distance of these distributions.

Figures 5.2a and 5.2b show empirical distribution functions of different features for the same writer from the SigComp2011 dataset. The rows represent the distribution of x and y coordinates, pressure p and velocity v for three different signatures in three columns. Figure 5.2a depicts the distribution of the examined four features (x, y, p, v) of three reference signatures, while the distribution functions of three questioned forgeries can be seen in Figure 5.2b.

During the verification process the KS distance was calculated pairwise between each reference signature for the same author and the questioned signature and the references for all the features. Dutch evaluation data set usually contains 12 reference signatures and 24 questioned ones for each author. All together there are 36 signatures per person. The KS distance is always in the range $[0, 1]$ interval so we can represent the distance matrices as a grayscale image.

We present some KS distances in Tables 5.1a-5.1e and Figure 5.3. Table 5.1a shows the KS distance of pressure distributions of writer 022 between the reference signatures. Here the diagonal is 0 because same empirical distribution leads to 0 distance. Tables 5.1b and 5.1c show the KS distance of pressure distributions of the same writer between the genuine-reference and forged-reference signatures, respectively. Table 5.1d shows the KS distance of velocity distributions of the same writer between the questioned genuine and reference. Table 5.1e shows the KS distance of velocity distributions of the same writer between questioned forged and reference signatures.

Figure 5.3 visualizes the distances belonging to the author denoted by id 022. Each column represents different features (i.e., x, y, p, v) and contains three figures in a row. The first row shows the distances between the reference signatures. The diagonal of these subfigures are black (this colour is assigned to zero), because there is no difference between identical distributions, thus the KS distance between identical signatures is zero. Lighter gray represents larger KS distance. In this coloured representation white colour would represent the largest possible KS distance, which is 1. The second row shows the distance values between the reference and the questioned forged signatures. Each row in the picture contains 12 pixels which represent the 12 distances between the corresponding forged, questioned signature and the reference signatures. The third row shows the distances between questioned genuine and reference signatures in the same way as the forged ones: each row in the image represents the distances between the corresponding questioned genuine signature and the reference signatures.

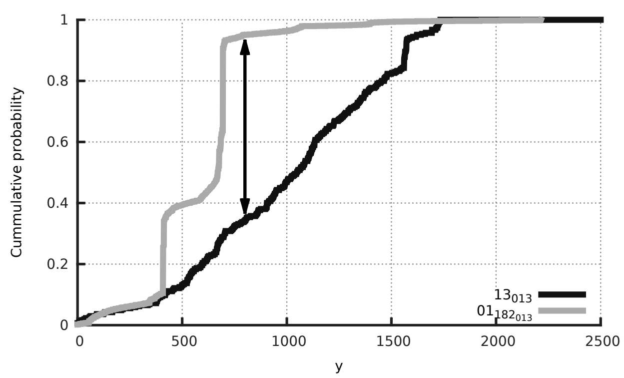
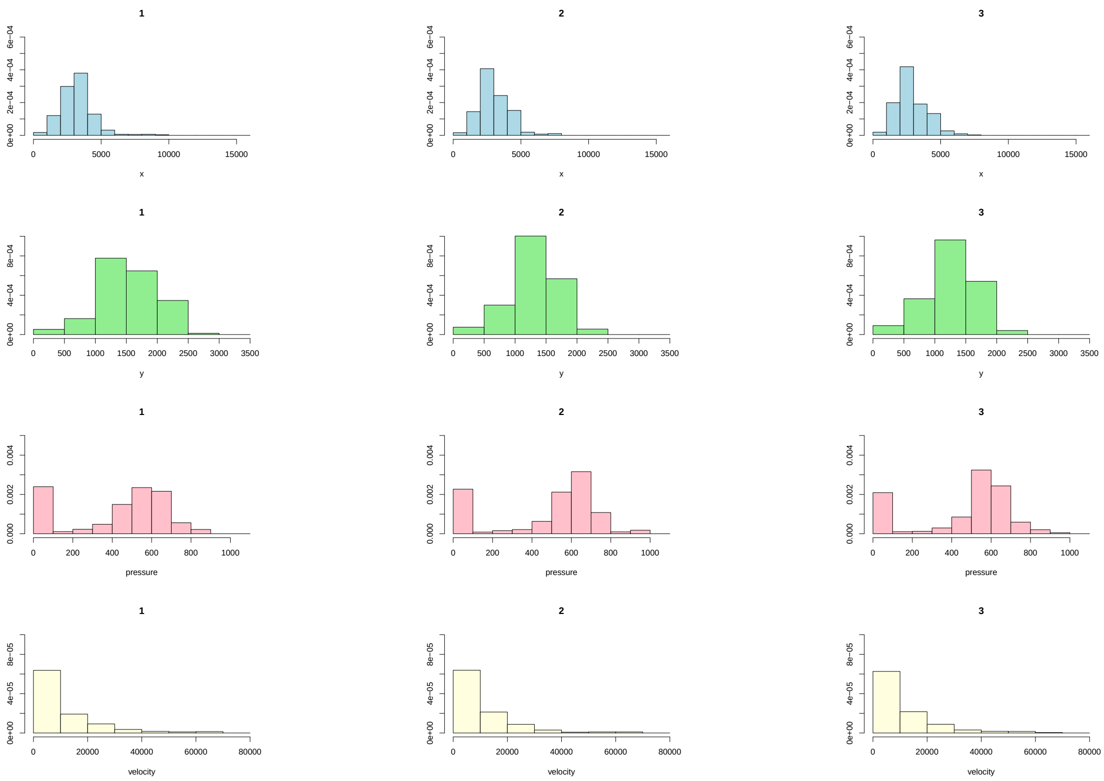
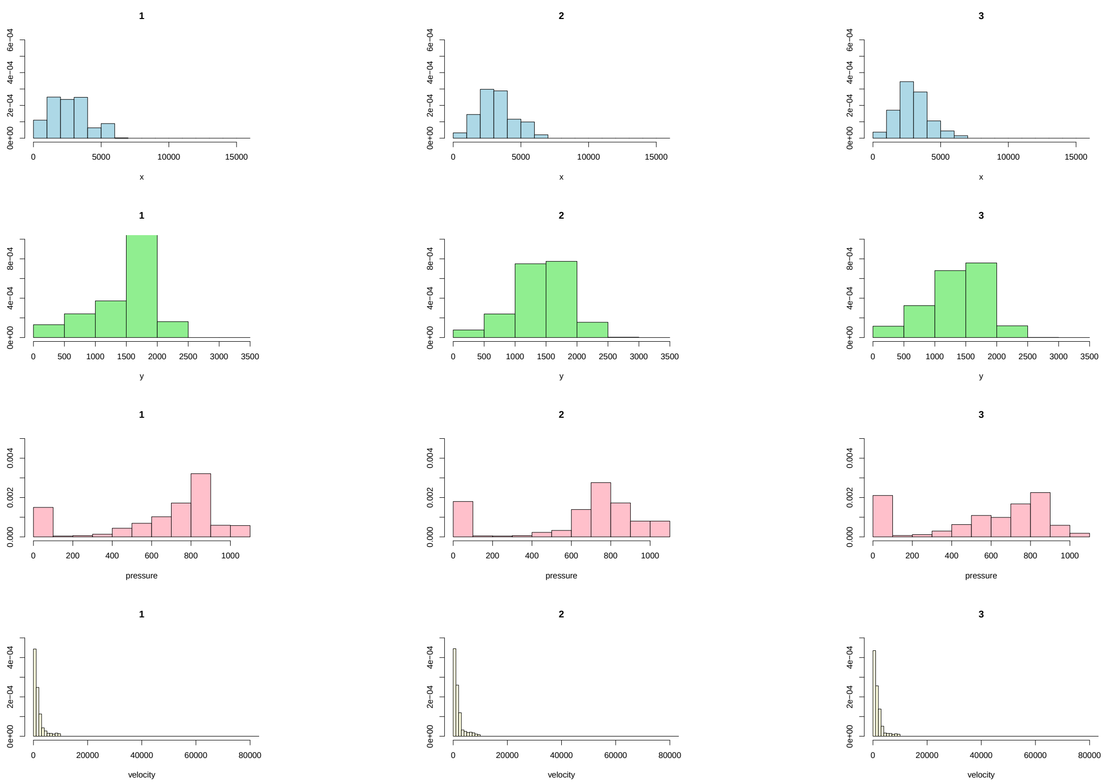


Figure 5.1: Plots of two empirical cumulative distribution functions, the black arrow shows the maximal difference between the functions, thus the two-sample KS distance



(a) Empirical distribution of reference signatures



(b) Empirical distribution of questioned forged signatures

Figure 5.2: Empirical distributions of signatures of writer 022

	Reference signatures											
Reference signatures	0.0000	0.1762	0.1363	0.1620	0.1372	0.0509	0.0738	0.0754	0.0935	0.1275	0.0672	0.1178
	0.1762	0.0000	0.1517	0.0857	0.0959	0.1424	0.2212	0.1575	0.1523	0.2436	0.1707	0.2490
	0.1363	0.1517	0.0000	0.0786	0.0579	0.1054	0.1938	0.1266	0.1102	0.2171	0.1466	0.2067
	0.1620	0.0857	0.0786	0.0000	0.0526	0.1346	0.2207	0.1545	0.1397	0.2511	0.1799	0.2274
	0.1372	0.0959	0.0579	0.0526	0.0000	0.1080	0.1836	0.1302	0.1171	0.2177	0.1465	0.2094
	0.0509	0.1424	0.1054	0.1346	0.1080	0.0000	0.0959	0.0568	0.1069	0.1193	0.0612	0.1192
	0.0738	0.2212	0.1938	0.2207	0.1836	0.0959	0.0000	0.1156	0.1275	0.1111	0.0805	0.1199
	0.0754	0.1575	0.1266	0.1545	0.1302	0.0568	0.1156	0.0000	0.1050	0.1250	0.0632	0.1437
	0.0935	0.1523	0.1102	0.1397	0.1171	0.1069	0.1275	0.1050	0.0000	0.2031	0.1452	0.1989
	0.1275	0.2436	0.2171	0.2511	0.2177	0.1193	0.1111	0.1250	0.2031	0.0000	0.0776	0.0373
	0.0672	0.1707	0.1466	0.1799	0.1465	0.0612	0.0805	0.0632	0.1452	0.0776	0.0000	0.0890
	0.1178	0.2490	0.2067	0.2274	0.2094	0.1192	0.1199	0.1437	0.1989	0.0373	0.0890	0.0000

(a) between pressure p values of reference signatures

	Reference signatures											
Questioned genuine signatures	0.1103	0.0901	0.0911	0.1188	0.0833	0.0584	0.1343	0.0703	0.0804	0.1604	0.0896	0.1591
	0.1881	0.3443	0.2229	0.1934	0.2308	0.3140	0.4030	0.3139	0.3277	0.4200	0.3520	0.4176
	0.1381	0.2920	0.1719	0.1396	0.1827	0.2615	0.3531	0.2761	0.2771	0.3708	0.3019	0.3581
	0.1742	0.0667	0.1543	0.1869	0.1497	0.0719	0.0786	0.0772	0.0906	0.1581	0.1187	0.1728
	0.1347	0.1369	0.0725	0.0972	0.0689	0.1351	0.1909	0.1467	0.0932	0.2247	0.1836	0.2382
	0.0944	0.1211	0.0594	0.0595	0.0322	0.1195	0.1847	0.1229	0.1028	0.2098	0.1488	0.2049
	0.1102	0.1667	0.1474	0.1088	0.1260	0.1346	0.2100	0.1566	0.1436	0.2405	0.1683	0.2374
	0.1404	0.0598	0.1127	0.1422	0.1030	0.0820	0.0944	0.0857	0.0576	0.1743	0.1230	0.1682
	0.2700	0.1546	0.2446	0.2776	0.2443	0.1463	0.1119	0.1275	0.2195	0.0321	0.1035	0.0598
	0.1939	0.0470	0.1653	0.1951	0.1554	0.0708	0.0679	0.0831	0.0979	0.1346	0.1029	0.1480
	0.1561	0.0600	0.1353	0.1601	0.1247	0.0677	0.0959	0.0769	0.0574	0.1569	0.1170	0.1672
	0.1974	0.0451	0.1681	0.1995	0.1584	0.0759	0.0574	0.0882	0.1009	0.1109	0.0729	0.1173

(b) between pressure p values of questioned genuine and reference signatures

	Reference signatures											
Questioned forged signatures	0.5549	0.5125	0.5456	0.5402	0.5303	0.5162	0.5177	0.4994	0.5518	0.5234	0.4978	0.4886
	0.4339	0.5817	0.5364	0.4841	0.5111	0.5586	0.6309	0.5338	0.5717	0.6216	0.5657	0.6517
	0.6766	0.7209	0.7340	0.6947	0.7014	0.7103	0.7405	0.7225	0.7108	0.7160	0.6982	0.7334
	0.5448	0.4756	0.5272	0.5402	0.5377	0.4862	0.4876	0.4540	0.5514	0.4365	0.4342	0.4277
	0.4926	0.6273	0.6253	0.5618	0.5824	0.6318	0.6677	0.5970	0.6294	0.6558	0.6032	0.6711
	0.6732	0.7193	0.7325	0.6925	0.6983	0.7083	0.7390	0.7211	0.7092	0.7133	0.6967	0.7308
	0.4076	0.3603	0.4013	0.3979	0.3888	0.3698	0.3691	0.3360	0.4067	0.3617	0.3362	0.3265
	0.5250	0.6278	0.6344	0.5771	0.5787	0.6221	0.6447	0.6029	0.6276	0.6329	0.6011	0.6435
	0.6686	0.7163	0.7289	0.6888	0.6897	0.7065	0.7352	0.7182	0.7070	0.7095	0.6962	0.7289
	0.4442	0.3928	0.4369	0.4358	0.4260	0.4036	0.3998	0.3534	0.4460	0.3695	0.3595	0.3582
	0.3998	0.5350	0.5239	0.4525	0.4818	0.5307	0.5765	0.5012	0.5372	0.5653	0.5063	0.5816
	0.6469	0.7015	0.7142	0.6754	0.6736	0.6915	0.7187	0.7026	0.6920	0.6924	0.6801	0.7118

(c) between pressure p values of questioned forgeries and reference signatures

	Reference signatures											
Questioned genuine signatures	0.0389	0.0207	0.0662	0.0758	0.0694	0.0855	0.0986	0.1201	0.0543	0.0641	0.0642	0.0839
	0.0827	0.0645	0.0884	0.0917	0.1070	0.1233	0.1383	0.1602	0.0961	0.1061	0.0966	0.1289
	0.0517	0.0379	0.0623	0.0675	0.0697	0.0797	0.0926	0.1129	0.0549	0.0657	0.0634	0.0766
	0.0390	0.0269	0.0564	0.0535	0.0629	0.0710	0.0974	0.1132	0.0554	0.0621	0.0555	0.0831
	0.0360	0.0225	0.0676	0.0862	0.0777	0.0873	0.0950	0.1206	0.0626	0.0734	0.0756	0.0831
	0.0501	0.0311	0.0685	0.0846	0.0774	0.0798	0.1042	0.1124	0.0639	0.0662	0.0760	0.0897
	0.0719	0.0508	0.1123	0.1111	0.1173	0.1272	0.1336	0.1602	0.1009	0.1117	0.1074	0.1161
	0.0505	0.0675	0.0302	0.0576	0.0472	0.0398	0.0608	0.0677	0.0418	0.0358	0.0389	0.0322
	0.0414	0.0577	0.0328	0.0617	0.0517	0.0431	0.0624	0.0746	0.0400	0.0363	0.0428	0.0423
	0.0403	0.0400	0.0642	0.0895	0.0794	0.0673	0.0906	0.0995	0.0552	0.0656	0.0661	0.0670
	0.0336	0.0246	0.0573	0.0764	0.0664	0.0713	0.0834	0.1043	0.0506	0.0573	0.0578	0.0671
	0.0598	0.0688	0.0290	0.0337	0.0334	0.0556	0.0689	0.0853	0.0326	0.0330	0.0347	0.0547

(d) between velocity v values of questioned genuine and reference signatures

Table 5.1: KS distance matrices of writer 022

	Reference signatures											
Questioned forged signatures	0.4902	0.4964	0.5396	0.5709	0.5609	0.5472	0.5710	0.5715	0.5366	0.5423	0.5481	0.5389
	0.2964	0.2934	0.3482	0.3638	0.3575	0.3621	0.3674	0.3953	0.3378	0.3484	0.3537	0.3573
	0.3132	0.3036	0.3741	0.3703	0.3776	0.3733	0.3864	0.4101	0.3574	0.3677	0.3758	0.3698
	0.5063	0.5140	0.5575	0.5884	0.5783	0.5635	0.5883	0.5893	0.5541	0.5601	0.5643	0.5522
	0.2678	0.2576	0.3277	0.3252	0.3360	0.3340	0.3382	0.3688	0.3092	0.3195	0.3266	0.3342
	0.4569	0.4432	0.4822	0.5093	0.4958	0.4853	0.5124	0.5188	0.4898	0.4879	0.4903	0.4794
	0.5164	0.5208	0.5666	0.5954	0.5854	0.5741	0.5931	0.6009	0.5612	0.5711	0.5763	0.5652
	0.2271	0.2205	0.2936	0.2793	0.2969	0.2919	0.2992	0.3274	0.2702	0.2828	0.2906	0.2920
	0.3895	0.3825	0.4396	0.4516	0.4506	0.4409	0.4570	0.4762	0.4241	0.4398	0.4488	0.4369
	0.4916	0.4888	0.5380	0.5610	0.5527	0.5456	0.5650	0.5761	0.5313	0.5425	0.5487	0.5370
	0.2280	0.2252	0.2827	0.2961	0.2932	0.2838	0.3000	0.3175	0.2649	0.2790	0.2919	0.2821
	0.2355	0.2201	0.2761	0.2852	0.2876	0.2968	0.3031	0.3345	0.2715	0.2814	0.2785	0.2871

(e) KS distance matrix between velocity v values of questioned forgeries and reference signatures

Table 5.1: KS distance matrices of writer 022

	Duration of signatures (in seconds)												AVG $\pm$ SD
Reference	4.6	4.7	4.4	4.4	4.6	4.5	4.3	4.1	4.4	4.5	5.0	4.4	4.5$\pm$ 0.2
Questioned genuine	5.1	5.0	4.5	4.6	4.6	4.6	5.3	4.6	4.5	4.6	4.8	4.6	4.7$\pm$0.3
Questioned forged	24.4	11.3	8.3	27.7	11.2	13.6	26.9	11.2	15.1	22.6	10.9	8.1	15.9$\pm$7.3

Table 5.2: Time duration of writer 022

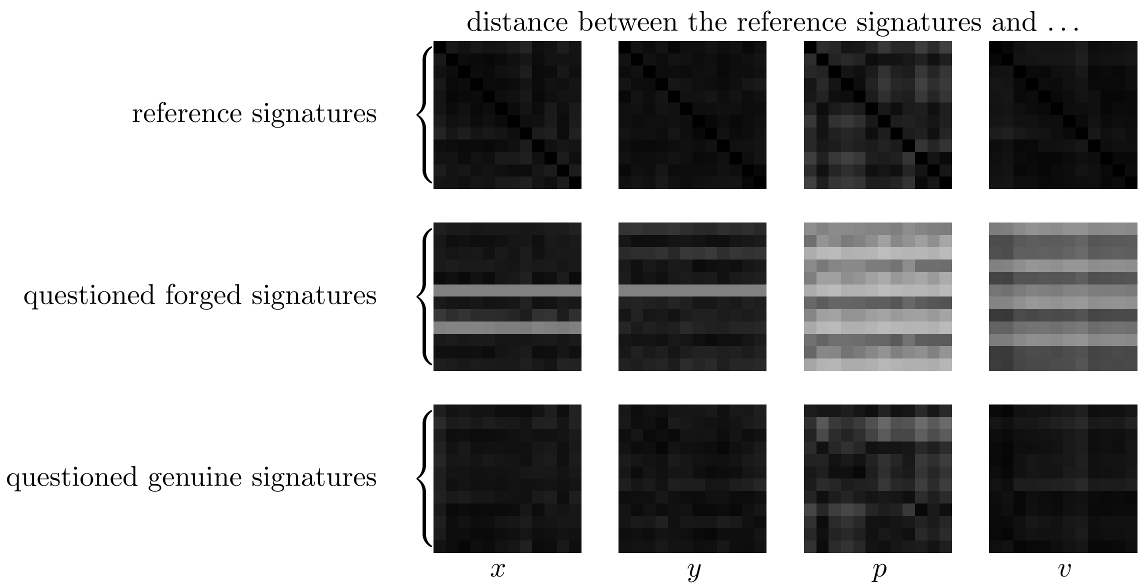


Figure 5.3: KS distance matrix of the coordinates, pressure and velocity features of writer 022 from the evaluation set of the SigComp2011 dataset

Algorithm

Based on the reference signatures of a particular author, the reference distance was determined for each signature of every authors. First, the KS distances between each pair of reference signatures were calculated for an arbitrary feature and the reference distance was determined by taking the average, maximum or minimum of these distances. During the evaluation, this reference distance was used for comparison purposes.

For each questioned signature the KS distance from each corresponding reference signature was calculated and the KS distance of the given questioned signature was defined as the average, maximum or minimum of these KS distances.

Algorithm 5.1: Verification based on Kolmogorov-Smirnov distance

```
Input  : feature, references and questioned signature, threshold c
Output: genuine or forgery (decision)
k = 1
for i=1 to NoReferences do
    for j=(i+1) to NoReferences do
        distRef_k = KS(reference_i, reference_j, feature)
        k = k + 1
    end
end
for i=1 to NoReferences do
    dist_i = KS(reference_i, questioned, feature)
end
if duration(questioned) > 1.5 max(duration(references)) then
    return forgery
else
    if max / min /avg(dist) < c · max / min /avg(distRef) then
        return genuine
    else
        return forgery
    end
end
```

Algorithm 5.1 shows the verification steps. First the distances for the reference set are calculated, while as next step the distances between the corresponding questioned signature and the reference signatures are calculated. The last part of the algorithm takes into account the constraint for the (minimal, maximal or average) distance of the questioned signature from the references and based on their comparison classifies the signature as genuine or forgery.

We compared two main scenarios and several combination of different features. The verification performance was examined with and without the constraint of the time duration of the signing process, see the 11th line of the Algorithm 5.1. If the questioned signature duration was greater than 1.5 times the maximal duration of the reference signatures, and the same hold for the pen down duration (when $p_i > 0$), then the questioned signature was rejected. Table 5.2 shows the duration of the reference, questioned genuine and questioned forged signatures for writer 022.

The constraints of the different features were taken into account separately (for x, y, p and v and for maximum, minimum and average distance it means $4 \cdot 3 = 12$ different cases, if the time duration constraint is taken into account it makes $2 \cdot 12 = 24$ cases). For example, if the minimal KS distance of the pressure values fulfilled the constraint $\min(\text{dist}, p) < c \cdot \min(\text{distRef}, p)$ for the questioned signature, it was accepted as a genuine signature, otherwise it was rejected. The threshold c was varied from 0 to 7 by 0.01.

In the next step these constraints were combined in several ways: the constraints regarded to the x and y coordinates were combined with logical `AND` ($\wedge$) and logical `OR` ($\vee$), the pressure p and velocity v the same way. The pressure was combined with the x, y coordinates as well, as shown below.

RESULTS

Table 5.3 lists the false acceptance and false rejection rates (FAR and FRR) on the Dutch dataset of SigComp2011 based on the proposed method. The table reports those FAR and FRR which were closest to each other to a given threshold c, thus the provided FAR and FRR values can be considered as approximation of the EER.

The rows show which feature (x, y coordinates, pressure p or velocity v) and which distance was used (average, maximum, minimum). The columns show two scenarios. In scenario I the duration constraint was applied (line 11 in Algorithm 5.1), while in scenario II this constraint was not applied. In addition we tested two scenarios (besides the time restriction): in those we excluded the reference signature which differed the most from the other reference signatures, so it was not used during the decision process here. These experiments that negligibly worsened both the accuracy and equal error rates (by approximately $0.5 - 1.0\%$) produced results which were not included here.

For comparison purposes, the DTW distance for pressure and velocity features is included in Table 5.3. Moreover, Table 5.4 shows the results of the competing systems submitted to the SigComp2011. In Table 5.3 values in bold indicate the lowest FAR/FRR rates in that section of the table. Different combination of constraints are compared when the same type of statistic is used for comparison (maximum, minimum and average distances are compared both on the reference signatures and between the references and each questioned). Values in italics indicate the lowest FAR/FRR rates when only single features are considered.

		With time constraint	Without time constraint
x	average	12.27%/12.19%	29.62%/29.01%
y		11.13%/11.27%	30.28%/30.56%
p		*9.66%/9.57%*	19.97%/19.91%
v		11.13%/11.11%	27.66%/27.78%
$x \wedge y$		10.47%/10.34%	26.19%/25.15%
$x \vee y$		12.44%/12.50%	29.79%/29.01%
$p \wedge v$		**7.86%/8.02%**	15.55%/15.74%
$p \vee v$		10.31%/10.19%	26.35%/26.54%
$(x \wedge y) \wedge p$		8.02%/7.72%	16.04%/16.05%
$(x \wedge y) \vee p$		10.80%/10.96%	21.93%/22.22%
x	max	11.95%/12.04%	31.75%/33.95%
y		11.29%/11.42%	32.90%/30.71%
p		*9.66%/9.88%*	23.73%/23.61%
v		10.97%/10.49%	26.35%/27.31%
$x \wedge y$		10.64%/10.34%	30.93%/31.02%
$x \vee y$		10.31%/11.73%	32.24%/33.64%
$p \wedge v$		**7.86%/8.02%**	16.69%/16.67%
$p \vee v$		11.46%/11.73%	27.00%/26.85%
$(x \wedge y) \wedge p$		8.84%/9.26%	19.97%/20.06%
$(x \wedge y) \vee p$		12.44%/11.73%	25.37%/26.23%
x	min	11.78%/11.88%	27.82%/27.62%
y		10.97%/10.96%	27.99%/28.55%
p		*9.00%/8.95%*	20.13%/20.22%
v		4.42%/50.46%	21.11%/50.46%
$x \wedge y$		9.82%/9.88%	22.91%/23.30%
$x \vee y$		11.29%/11.27%	26.35%/25.93%
$p \wedge v$		2.13%/52.01%	10.64%/52.01%
$p \vee v$		9.49%/9.41%	19.31%/19.44%
$(x \wedge y) \wedge p$		**8.67%/8.64%**	17.02%/17.13%
$(x \wedge y) \vee p$		10.47%/10.49%	20.13%/19.75%
		DTW distance	
p	average	13.75%/13.89%	38.13%/38.89%
v		14.08%/14.04%	33.06%/33.18%
p	max	13.91%/14.04%	30.44%/30.86%
v		14.57%/14.66%	29.95%/30.09%
p	min	8.67%/62.19%	49.75%/62.04%
v		6.87%/74.54%	25.86%/74.54%

Table 5.3: FAR/FRR values using KS distance

It is evident that the classification scheme based on the KS distance perform better than the DTW and it's performance is comparable to the systems submitted to the SigComp2011 competition. We observe if the time constraint is applied (column with label "with time constraint"), the equal error rates are roughly less than 13%. If the time constraint is not applied, the error rates increase by $10-20\%$.

For almost all constraints the best results were achieved with a minimum distance. In this case the reference distance was the minimum distance between the reference signatures and it was compared with the minimum distance of the questioned signature and the reference signatures. If we compare the performance of the different single features, we see the smallest error rates appear in the rows which belongs to the pressure feature. The minimum KS distance gives good results when the coordinate constraints and pressure are used together ($x \wedge y \wedge p$) otherwise (in the case of average and maximum) the constraint for pressure and velocity together ($p \wedge v$) gives the lowest equal error rates. Besides the minimum distance, the time constraint improved the results significantly.

CONCLUSION

An online signature verification method based on simple statististical distance and time constraints were proposed and evaluated. The x, y coordinates, pressure, and velocity features were tested both separately and combined. The performance was evaluated on a public Dutch dataset.

A further development direction can be the examination of other features, such as acceleration, curvature, polar coordinates. Another approach could be the generation of one reference signature model per author (or reference distribution of the features) based on all references and compare the questioned signatures using this generated distribution. We intend to evaluate the method on different databases and investigate the stability of other features.

ID	FAR	FRR
4	3.76	3.70
5	3.44	3.86
7	6.87	7.25
1	7.69	7.56
KS	**7.86**	**8.02**
6	8.02	8.33
9	11.27	11.11

Table 5.4: FAR and FRR values of SigComp2011 Dutch online competitors and the proposed KS distance

Part III

ONLINE HANDWRITING CLASSIFICATION

6 HANDEDNESS DETECTION

FHEs have to expand their work field and elaborate supplementary methods because of spreading of online signatures. Special devices, as digitizing tablets, enable us to analyze additional information of online writings. These data are suitable for verification of expert's observations and may help in identification and classification of groups of writers.

This chapter describes an automatized method of handedness detection on online handwriting samples based on approach of FHEs. With the guidance of the Hungarian Institute for Forensic Science (NIFS) I examined online handwriting samples and classified them by handedness based on forensic-methodology. We attempted to ensure the efficiency of the investigation by using only significant (and avoiding unnecessary) features to classify the handwriting samples. Besides this, instead of a microscopic examination applied by FHEs, we evaluated absolutely objective data of online handwriting samples [160].

The novelty of this study is the automatic measurement of the most reliable feature in handedness detection, the automatic classification of handwriting samples based on this feature and the contribution to the scientific background of examination of handedness in Forensic Handwriting Examination.

RELATED WORK

Differentiation of handedness plays an important role in Forensic Handwriting Identification, particularly in studying disguises, sometimes forgeries, and in each case when taking specimens. If an expert examines lot of handwriting samples it might be necessary to distinguish subgroups of right-handed and left-handed writers. The recognition of handedness in forensic handwriting examination is mainly associated with the characteristics of left-hand writing or the unaccustomed hand [161].

According to the classical handwriting literature, in left-hand writing samples we often observe the lowest level of execution, slower speed, tendency to vertical slope or backhand, along with irregularity in spacing between letters and words. Apart from these general features there are some characteristics like heavier pressure on upstrokes than downstrokes, right-to-left horizontal strokes and dots, clockwise circles, upwards or to left terminal strokes.

Huber and Headrick state in their book [162] that in practice the above general features did not prove to be consistent. The authors pointed to the examination of the direction of horizontal strokes applied over a long time. The latest studies also provide convincing data concerning this feature [163, 164].

Saran et al. in [164] showed that right-handed people write horizontal lines from left to right most of the time (99.51%–100%), while left-handers write 68.75–82.69% of these strokes from right to left.

Algorithmic approaches have been introduced in the last decade as well. Bandi and Srihari [165] performed demographic classification on offline data, determining binary classes (e.g. gender, age, handedness) based on bagging and boosting, which make use of a combination of neural network classifiers using up to 11 features.

Liwicki et al. [166] automatically detected gender and handedness from online handwriting and conducted the experiments on the IAM-OnDB [167] (which we employ in our experiment as well). For classification, 29 features and both an SVM classifier with different kernel functions (linear, radial bases, sigmoid and polynomial) and GMM were applied.

METHODOLOGY

Our efforts for automatization are based on the detection of the principal characteristic of handedness, namely the direction of isolated horizontal strokes. By stroke we mean all points between two pen lifts as the term is used in the IAM-OnDB dataset.

In our approach isolated horizontal strokes/lines are the following:

1. crossings of the letters **A, E, F, H, I, T, f, t**. (There are fewer strokes in the cursive writings.)

2. other punctuation marks such as dashes – , hyphens or negative signs -, and underlines, deletion lines.

The direction of strokes is observed through a stereomicroscopic examination by forensic handwriting experts. In contrast, we conducted our experiments on the IAM-OnDB (using the x, y coordinates and timestamp), which is available on the website of Universität Bern[1] after registration for research purposes.

The database consists of 1568 handwriting samples taken from 196 authors, 177 of which are right-handed and 19 are left-handed, whose ratio is similar to that for the general population. Handwritten English texts were acquired on a whiteboard in online and offline format. It can be used to train and test handwritten text recognizers and to perform writer identification and verification experiments [167].

1 IAM-OnDB website `http://tinyurl.com/iam-online-hwr`

variable	description
avg_width	average width of strokes width of a stroke is $\max(x_i) - \min(x_i)$
avg_height	average height of strokes height of a stroke is $\max(y_i) - \min(y_i)$
avg_diag	average diagonal length of the bounding boxes diagonal length of a stroke is $\sqrt{\text{width}^2 + \text{height}^2}$
avg_length	average length of strokes the average number of points within a stroke
dev_length	deviation of the length of strokes the deviation of the number of points within a stroke
total_length	number of points within the sample
no_strokes	number of strokes within the whole sample

Table 6.1: Overall statistics for the whole sample

variable	description
act_avg_slope	average slope for each consecutive point pairs
act_dev_slope	deviation of the slope for each consecutive pairs
act_diag	the diagonal length of the bounding box

Table 6.2: Statistics for a stroke

ALGORITHM

Here we present a detailed description of our handedness detection system. The method first calculates some overall statistics about the whole sample. Each handwriting sample consists of several strokes, where each stroke consists points between two pen lifts.

Table 6.1 contains the variable names and short description of these statistics. For each sample global features are calculated, such as `avg_width`, `avg_height` and `avg_diag` as the average width, height and average diagonal of the strokes. We calculate also statistics related to the length of the sample, where length means the number of points within stroke. The `avg_length` and `dev_length` are the average and the standard deviation of the number of points, `total_length` and `no_strokes` denotes the total number of points and the number of strokes for the whole sample.

After the statistical values above have been determined, the method analyzes the data stroke-by-stroke and for each stroke calculates stroke statistics, given in Table 6.2.

The slope is calculated for each consecutive point pairs within a stroke using the

$$m_i = \frac{y_i - y_{i-1}}{x_i - x_{i-1}}$$

formula omitting the points where there is no change in the x coordinate, i.e. $x_i - x_{i-1}$ equal to zero. In average 5.3% of the points were omitted with standard deviation 1.5%.

The average slope and deviation of the slope (`act_avg_slope` and `act_dev_slope`) are compared with thresholds to determine whether a stroke can be considered horizontal or not. In handwritings we can hardly find stroke which lies perfectly on a line, so we found it reasonable to allow of a small deviation of slope values within stroke. The diagonal length of the boundig box (`act_diag`) was applied to determine whether the stroke is short.

Using these statistical values the right-to-left and left-to-right horizontal strokes are detected for each sample based on the following conditions.

If a particular stroke diagonal length (variable name is `act_diag`) lies between a quarter of the average diagonal length for the entire sample (`avg_diag`) and the average diagonal length, then the stroke is classified as *short* (but not as short as dots). If it is *short* it is examined further to decide whether it is a horizontal stroke or not.

If a stroke fulfills the conditions

- `act_dev_slope < DL` and
- `|act_avg_slope| < HL`

it is classified as *horizontal stroke*. Here, `DL` refers to the slope deviance limit and `HL` refers to the slope horizontal limit, which are two parameters of the procedure. The thresholds for horizontal stroke and short stroke were determined intuitively and experimentally, for the applied `DL, HL` values see the *Experiments and results* section at Page 78.

The conditions

1. `act_dev_slope<DL` is a constraint for the deviation of the slope, which ensures that the particular stroke almost lies on a straight line.
2. `|act_avg_slope|<HL` is a constraint for the absolute value of the average slope, which ensures that the particular stroke is nearly horizontal.

After a horizontal stroke is detected, based on the starting and end points (in the figures, Figure 6.1 for instance is marked by ○ and • respectively), the direction of the stroke is determined to decide whether it is from left-to-right or right-to-left. Based on the online data, the determination of the direction of the stroke is straightforward and objective.

A basic assumption is that a left-to-right horizontal stroke is an indication of a right-handed person, while a right-to-left horizontal stroke indicates a left-handed person. For the sample we count the number of left-to-right and right-to-left horizontal strokes (variable `no_right_cross` and `no_left_cross` where right/left indicates the handedness type).

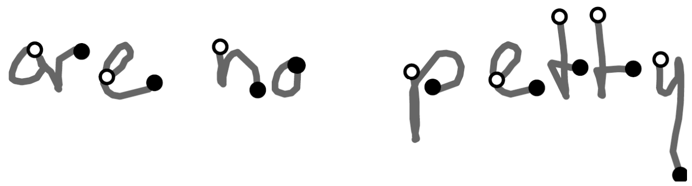


Figure 6.1: Part of a sample with marked starting and end points

Figure 6.2.a represents a right-handed sample with 16 detected left-to-right horizontal strokes (thick lines marked in red). Figure 6.2.b is a left-handed handwriting sample with 13 detected right-to-left horizontal strokes (thick lines marked in blue).

The notation above the handwriting samples are the following. After each label the related value can be found between rectangular brackets [and]. Labels are:

F filename with path (in the original dataset)

W writer identification number

H handedness (left, right)

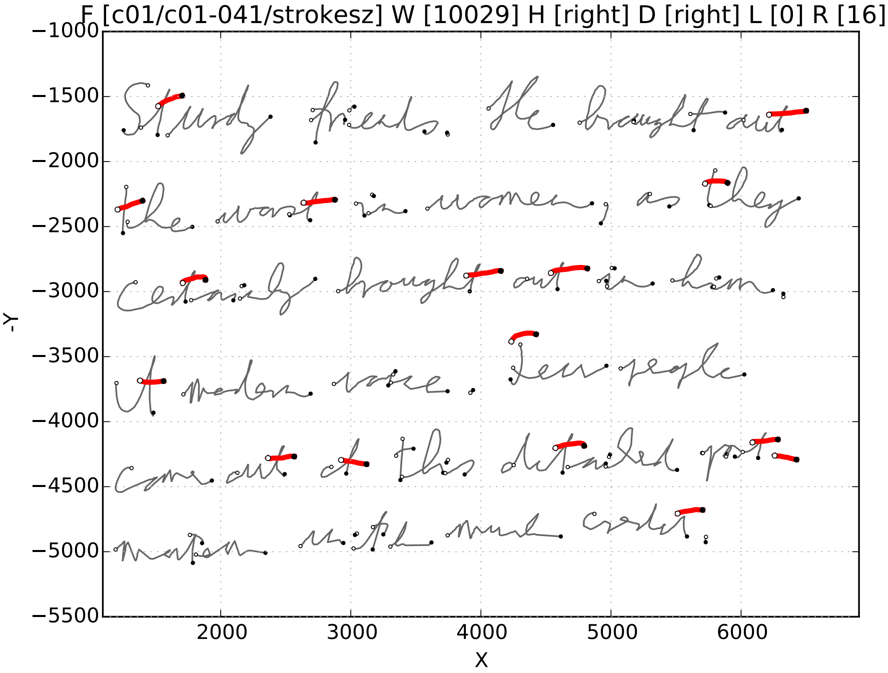
D decision (left, right, undecided)

L number of right-to-left short horizontal strokes (indicator of left-handedness)

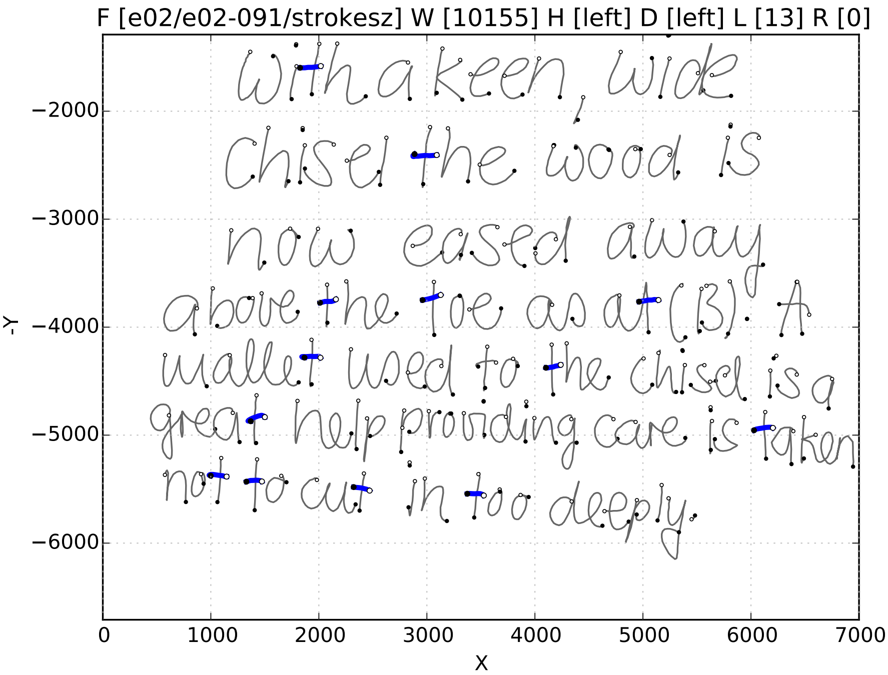
R number of left-to-right short horizontal strokes (indicator of right -handedness)

Majority voting (MV)

If only one horizontal strokes were detected or the number of the two different horizontal stroke types are equal, the decision is inconclusive; otherwise the decision is based on majority voting. The method described above is sketched out in Algorithm 6.1 using pseudo code.



(a) Right-handed writer sample



(b) Left-handed writer sample

Figure 6.2: Detection of horizontal strokes (sufficient number of strokes)

Algorithm 6.1: Detection of handedness with majority voting

```
INPUT:       SAMPLE // online handwriting sample
PARAMETERS:  DL, HL // limits
OUTPUT:      RIGHT/LEFT/INCONCLUSIVE // decision
Determine avg_height, avg_width, avg_diag
Determine avg_length, dev_length
Determine total_length, no_strokes
no_left_cross ← no_right_cross ← 0
for each stroke in the SAMPLE do
  is_crossing ← FALSE
  Determine act_avg_slope, act_dev_slope
  Determine act_diag
  if avg_diag/4 < act_diag < avg_diag then
    if act_dev_slope<DL ∧||act_avg_slope|| <HL then
      is_crossing ← TRUE
    end if
  end if
  if is_crossing then
    if the crossing is left-to-right then
      no_right_cross += 1
    else
      no_left_cross += 1
    end if
  end if
end for
no_crossing ← no_right_cross + no_left_cross
if no_crossing < 2 then
  decision ← INCONCLUSIVE
else
  if no_right_cross > no_left_cross then
    decision ← RIGHT
  else
    if no_left_cross > no_right_cross then
      decision ← LEFT
    else
      decision ← INCONCLUSIVE
    end if
  end if
end if
return decision
```

Left-condition (LC_k)

Based on latest studies [163, 164], we decided to apply another constraint. If a sample contains right-to-left horizontal stroke it most likely belongs to a left-handed writer, regardless of the number of left-to-right horizontal strokes. However, as the automatic horizontal stroke detection that we applied is not perfect, sometimes there is false horizontal stroke detection (e.g. do to retouches) or the automatic detection did not recognise horizontal stroke (e.g. due to the large variation of the slope), we can not apply such a strict constraint that if the sample contains one right-to-left horizontal stroke, it is originated from a left-handed writer.

Thus we introduce another classification (in addition to majority voting), were we only change the classification condition in Algorithm 6.1 and replaced it with the following: if more or equal than k right-to-left horizontal strokes are present in the sample, the writer is classified as left-handed, as described in Algorithm 6.2.

As regards samples taken from the same person, the classifier should give consistent results, see Figure 6.3, where RH and LH indicate right-handed and left-handed samples, respectively. Here we can see that more inconsistency was present for left-handed writers when MV method was applied, it gave different decision for the same writer in 1 out of 5 the cases for left-handed writers. The inconsistency is more biased if LC_2 was applied.

If the recognition of handwriting details attains a sufficient level in the current application, the basis of the evaluation could be the fact of whether the sample being examined contains separate horizontal lines in the left direction.

Figure 6.4.a and Figure 6.4.b represent two samples with only one horizontal stroke, which leads to inconclusive decisions. Figure 6.4.c represents another sample which contain misdetected horizontal strokes and includes left-to-right and right-to-left directions as well (referred to as mixed direction of strokes).

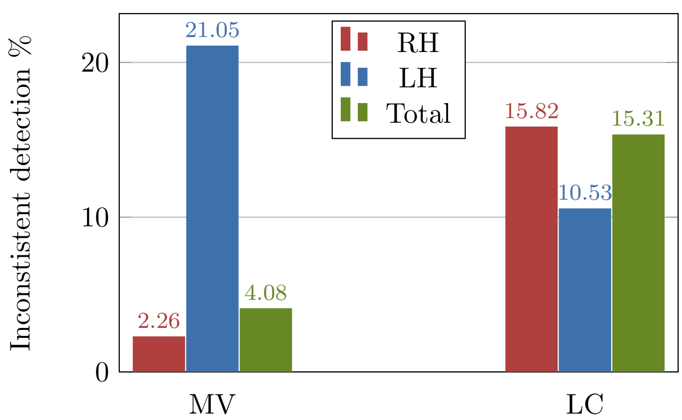
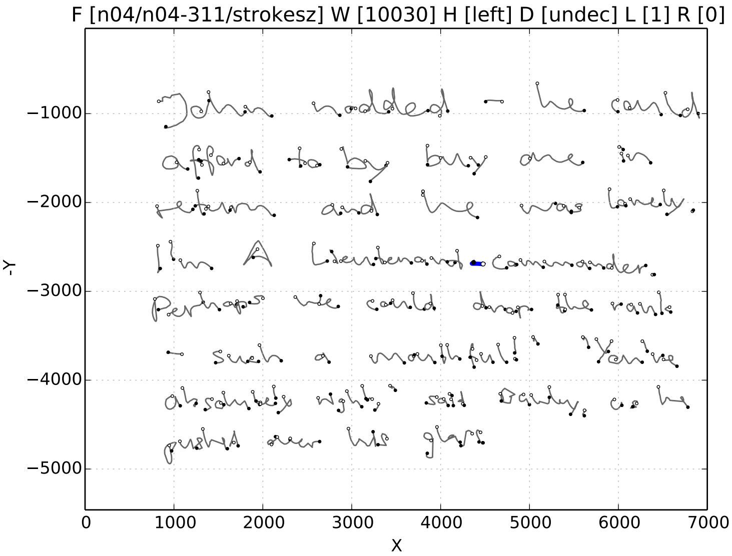
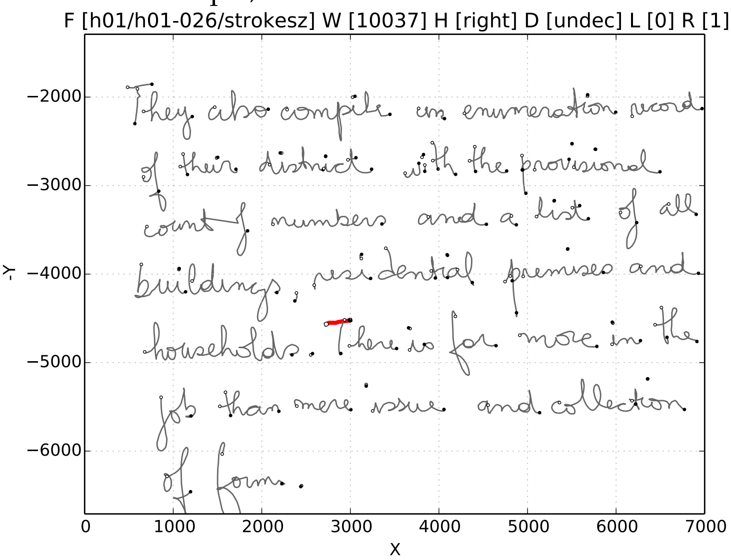


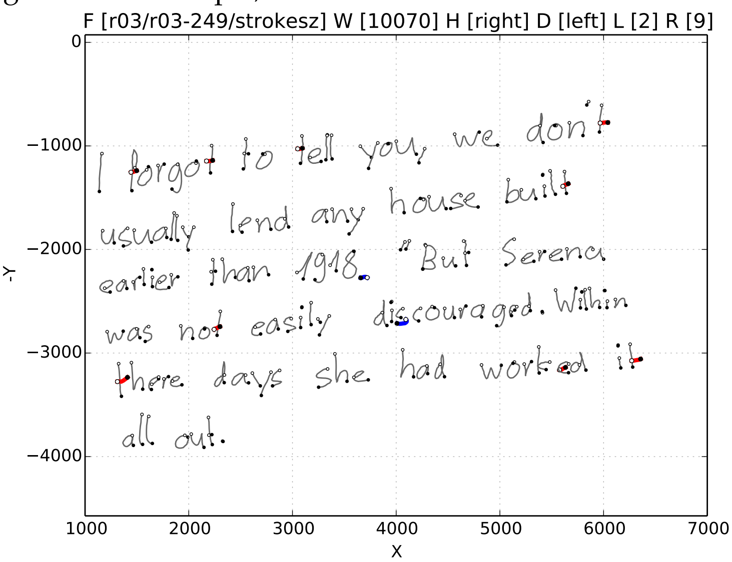
Figure 6.3: Different handedness detection of the samples taken from the same person in % terms, Here we compare MV with LC_2



(a) Left-handed sample, insufficient number of horizontal stroke



(b) Right-handed sample, insufficient number of horizontal strokes



(c) Right-handed sample, mixed directional detected strokes

Figure 6.4: Detection of horizontal strokes
F - Filename, W - writer id, H - handedness (real), D - decision about handedness, L/R - number of right-to-left/left-to right short horizontal strokes

EXPERIMENTS AND RESULTS

The dataset contains online information about handwriting samples in xml format. Each stroke is stored separately in a `<Stroke>` node. Each stroke contains the list of points between two pen lifts in `<Points>` nodes in temporal order. Each `<Points>` node has three attributes: the x, y coordinates and the related timestamp. Altogether the dataset consists of 1568 handwriting samples which contain the writer identification number, so for these samples the handedness of the writer is known.

Since the method does not require training as supervised learning techniques (e.g. SVM, HMM), we evaluated the classifier on the entire dataset. As mentioned previously, our classifier can distinguish three classes, namely right-handed, left-handed and if insufficient information is available about the sample (e.g. not enough horizontal lines), the classifier produces an *inconclusive* result. The two parameters of the horizontal stroke detection method (DL and HL) were tuned experimentally and were each set to 0.6. The parameter of the LC method was tested for $k = 1, 2, 3$, so respectively more or equal than 1, 2 or 3 right-to-left horizontal stroke resulted in immediate left-handed decision.

The summary tables have 3 rows for each class, 2 columns for right-handed and left-handed samples and the last column and row denotes the total. RH and LH indicate right-handed and left-handed samples, respectively. In the row of correctly classified cases and incorrectly classified cases the sum represents the result. The first term of the sum is the number of samples including horizontal strokes in only one direction (for right-handed writer samples it is left-to-right), the second term represents the number of samples that include both left-to-right

Algorithm 6.2: Decision about handedness with left-condition

```
PARAMETER: k
if no_crossing < 2 then
  decision ← INCONCLUSIVE
else
  if no_left_cross > no_right_cross ∨ no_left_cross >= k then
    decision ← LEFT
  else
    if no_right_cross > no_left_cross ∧ no_left_cross < k then
      decision ← RIGHT
    else
      decision ← INCONCLUSIVE
    end if
  end if
end if
return decision
```

and right-to-left directional horizontal strokes. Here percentages in brackets are calculated according to the whole dataset, e.g. 80.23% of the samples were correctly classified right-handed sample when MV was applied.

Table 6.3 lists the classification result when majority voting was used, Table 6.4, 6.5 and 6.6 show the accuracy of the LC_k method which changes the decision rule of MV with the following: if a sample contains at least $k = 1, 2$ or 3 right-to-left horizontal strokes; then the writer is considered left-handed. The highest accuracy 85.97% on the whole dataset was achieved using MV. The LC_k method gave the highest overall accuracy 84.63% with $k = 3$, in contrast to $k = 2$ which achieved 83.55%. Higher k value increases accuracy in the right-handed, however decreases accuracy in the left-handed class.

Figure 6.5.a, Figure 6.5.b, Figure 6.5.c and Figure 6.5.d are barcharts that show the percentage values of each class separately for right-handed and left-handed writers for MV, LC_1, LC_2 and LC_3. While Figure 6.6.a and Figure 6.6.b show only the conclusive cases for MV and LC_2. E.g. if inconclusive cases are included, 88.84% of the right-handed samples were correctly classified in contrast to 59.21% of the correctly classified left-handed samples, if MV was applied. If only conclusive cases are taken into account, 99.29% of the right-handed samples and 80.36% of the left-handed samples were correctly classified with MV. Applying the LC_2 the accuracy for right-handed class decreases to 95.58%, however for left-handed increases to 86.10%.

In addition to the automatical detection of the horizontal strokes, further difficulties arose from the fact that the samples contained only a few sentences with different content. For writing samples with an insufficient number of horizontal strokes (not found or only with other writing formations) the result was inconclusive.

In a detailed analysis of the reasons why we sometimes incorrectly classified writings, samples revealed that some letters, especially the letter **s** and sometimes the letters **a, o, d** were written with pen lift and hence it contained an extra horizontal line (see Figure 6.4.c). For example when a right-hander writes the finishing line of the letter **s**, the direction of the movement is right-to-left according to the copy-book style. These extra elements therefore cannot be considered indicators of handedness.

Some wrong decisions originated from the exceptions to the handwriting motion rule we applied in this experiment (for example in MV 8 cases among 31 incorrect decisions).

Using the above-mentioned approaches, Table 6.7 summarizes the best performance scores for earlier applied and our methods. The approach Tomai et al. in [168] is not comparable with our method because their way of handedness detection is based on character discriminatory capability. However in their study the most discriminative characters for handedness detection were *N*, *C*, *R*, *n*, *9* and *t* with 58-61% performance.

Class	RH	LH	Total
correct	1085+173 (80.23%)	41+49 (5.74%)	1348 (85.97%)
incorrect	2+7 (0.57%)	10+12 (1.40%)	31 (1.98%)
inconclusive	149 (9.50%)	40 (2.55%)	189 (12.05%)
Total	1416 (90.31%)	152 (9.69%)	1568 (100.0%)

Table 6.3: Summary of results – MV

	RH	LH	Total
correct	1085+0 (69.20%)	41+65 (6.76%)	1191 (75.96%)
incorrect	2+188 (12.12%)	10+0 (0.64%)	200 (12.76%)
inconclusive	141 (8.99%)	36 (2.30%)	177 (11.29%)
Total	1416 (90.31%)	152 (9.69%)	1568 (100.00%)

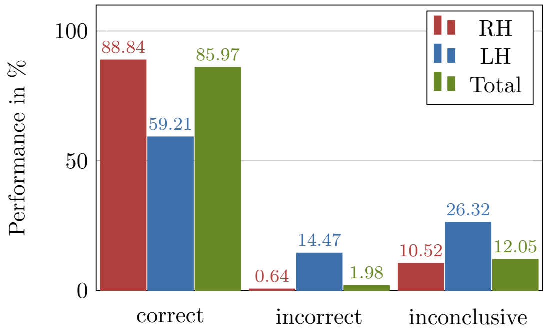
Table 6.4: Summary of results – LC_1

	RH	LH	Total
correct	1085+126 (77.23%)	41+58 (6.31%)	1310 (83.55%)
incorrect	2+54 (3.57%)	10+6 (1.02%)	72 (4.59%)
inconclusive	149 (9.50%)	37 (2.36%)	186 (11.86%)
Total	1416 (90.31%)	152 (9.69%)	1568 (100.00%)

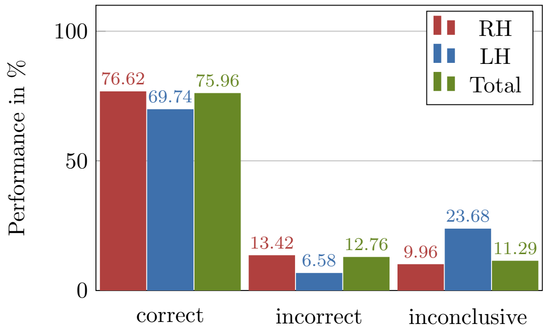
Table 6.5: Summary of results – LC_2

	RH	LH	Total
correct	1085+150 (78.76%)	41+51 (5.87%)	1327 (84.63%)
incorrect	2+30 (2.04%)	10+11 (1.34%)	53 (3.38%)
inconclusive	149 (9.50%)	39 (2.49%)	188 (11.99%)
Total	1416 (90.31%)	152 (9.69%)	1568 (100.00%)

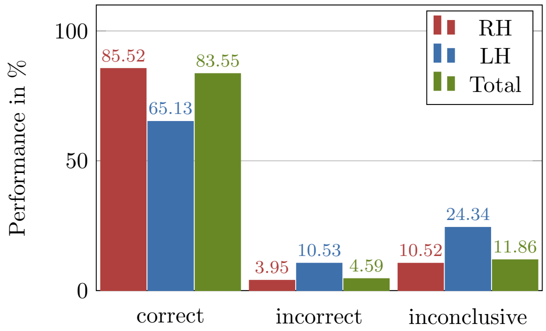
Table 6.6: Summary of results – LC_3



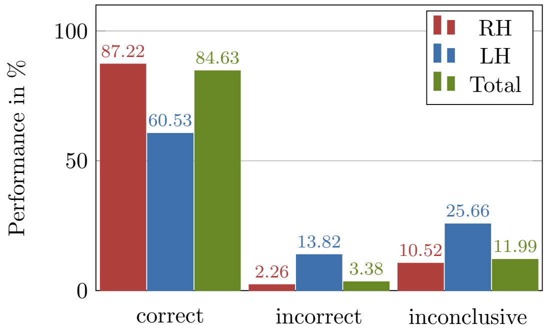
(a) MV



(b) LC_1

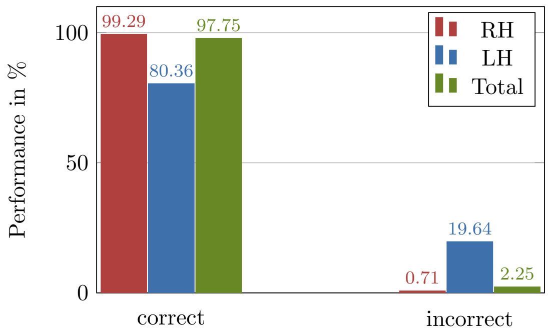


(c) LC_2

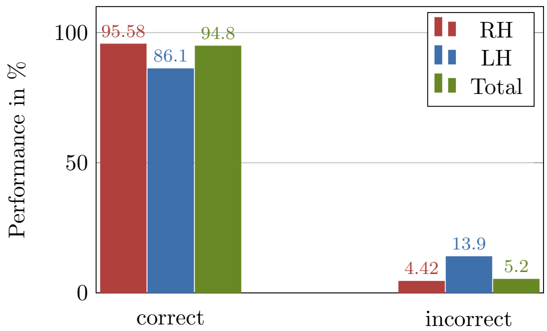


(d) LC_3

Figure 6.5: Performance in % terms with inconclusive cases



(a) MV



(b) LC_2

Figure 6.6: Performance in % terms with only conclusive cases

Method	**Performance**
SVM-linear [166]	54.69%
SVM-RBF [166]	62.57%
SVM-sigmoid [166]	54.24%
SVM-polynomial [166]	61.46%
GMM [166]	84.66%
Single Neural Network [165]	68.5%
Bagging [165]	70.1%
Boosting [165]	74.4%
Proposed method LC_1	**75.96%**
Proposed method LC_2	**83.55%**
Proposed method LC_3	**84.63%**
Proposed method MV	**85.97%**

Table 6.7: Comparison with other approaches

CONCLUSION AND FUTURE PLANS

I developed an automatic method for handedness detection based on forensic handwriting knowledge. The results are promising and competitive with most of the methods described in the literature [165, 166, 168].

With the application every starting point and end point in the pictures of online writing samples becomes clear. This facilitates further objective expert observations and the verification of the handwriting motion hypothesis after taking into account those handwriting samples that did not yield a result with the automated method in this phase of our research.

As a future plan we would like to develop our methodology, by improving the recognition of horizontal lines, examining smaller parts of the strokes and adding other significant features to reduce the number of inconclusive cases (especially for the left-handed samples). This improvement might lead to apply new parameters and fine-tuning them by training.

Our long-term plan is to create a representative online Hungarian dataset that considers not only handedness but also gender, age, and education level; and also to develop a useful module for forensic experts. We already collected handwritten text and signature samples from 20 person in online and offline format using a template. The samples include handwritten consecutive numbers, sentences and signatures as well.

7 GENDER DETECTION

In several criminal cases it is necessary to narrow the focus of the investigation, the demographic scope of suspects. For this task, in the case of handwritten anonymous letters, the investigating authority may delegate of a forensic handwriting expert. One of the professional questions is whether the expert can give any evidence of the gender of the writer.

In this chapter we examine whether there is significant difference between the male and female handwriting based on the characteristics applied in forensic examination. We do not introduce new features, rather we focus on the features mainly applied in forensic examination. The following study analyses online handwriting samples, thus we can also evaluate dynamical, time-dependent feature as well.

The novelty of this study is the comprehensive statistical examination of several features according to the biological gender and automatic measurement of some of the features. All together 25 features were revealed manually by the examination of the Hungarian Institute for Forensic Science and 5 features were measured automatically [169], which were attempt to give a deep analysis of these features in order to move even more towards a statistically assisted daily work of Forensic Handwriting Experts.

INTRODUCTION

In addition to the examination of specific biological material residue, other research studies are conducted in connection with performing biological gender classification based on evaluation of different characteristics of the body parts (e.g. face, shape of hand, fingerprint, iris) and human behavioural characteristics (such as walking, speaking, play behaviour etc.). Our recent study is based on handwriting, learned and conditioned behaviour as well, which is affected by several internal and external factors, among others, the conditions and circumstances of the writing, the writing routine, the age and the gender of the writer. The gender of the writer is affected psychologically, biologically, by social stereotypes, but primarily hormonally. Several disciplines examine the relationship of the biological sex and handwriting, such as psychology, sociology, philosophy, pedagogy and computer science. Lots of psychological studies examine how reliably non-professionals can determine the gender of the writer. These studies clearly show that the judgement is better than random chance, Hayes showed that this accuracy is 75% [170], moreover according to Burr the gender recognition can be improved [171]. Surprisingly the recognition accuracy of gender does not have direct relationship between the woman cultural role, e.g. participants of a test could not distinguish better the gender of Pakistani handwritings than the English. Average accuracies were 67.8% and 70% respectively in the experiment of Hamid and Loewenthal [172].

One of the most interesting investigation of Beech and Mackintosh showed that the proportion of the index and ring finger length is related to the characteristics of feminine and masculine handwritings. This study was based on the hypothesis that those who had strong testosterone effect have larger ring fingers (especially on the right hand) than index fingers. Woman showed masculine performance in 23.3%, man got feminine evaluation in 55% of the cases, based on evaluators scores. Overall, this study found that handwriting of woman reflects more the sexual nature than man [173].

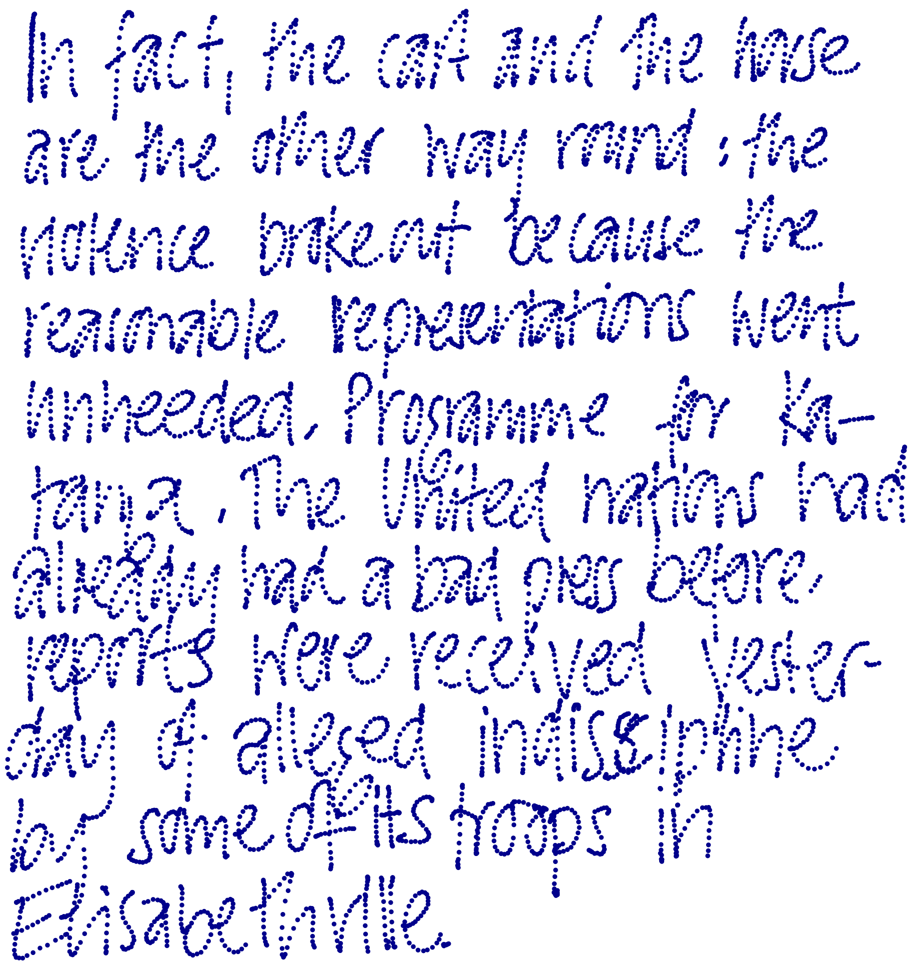
RELATED WORK

Few automatic systems were developed to detect gender. Bandi and Srihari applied micro and macro features and combined neural networks using bagging and boosting to perform binary classification on offline handwritings considering handedness, gender and age category (below 24 and above 45) and achieved 73.2% accuracy with a single and 74.7% (bagging) and 77.5% (boosting) with combination of neural networks in gender detection [165]. Hassaïne et al. also examined offline handwriting based on geometrical features such as directions, curvatures, tortuosity, chain code features and edge based directional features using χ^2 distance for comparison to perform writer identification and with combination of the best ten features could achieve 86-89% accuracy with different classifiers [174].

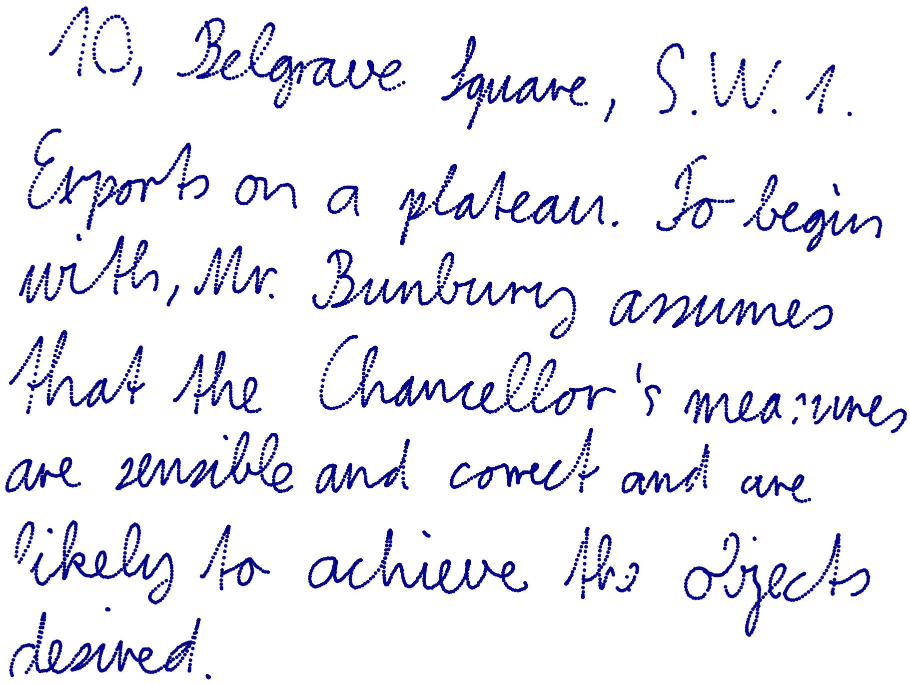
Liwicki et al. applied GMM and SVM to classify online handwriting to distinguish the handedness and gender of the writer [166]. Liwicki et al. also applied GMM to detect genderness using online and offline information [175]. Both systems were tested on the IAM-OnDB dataset, described in [167]. These methods achieved 62% and 67-68% classification accuracies with SVM and GMM, respectively.

The gender-based classification has a long history in Forensic Handwriting Examination, the automated methods only appeared in the last one and half decades besides the identifying attempts of other demographic classes. In Forensic Handwriting Examination a big change can be observed in this kind of classifications: the actual feasibility to identify different groups of writers is decreasing. For example with the marginalization of handwriting and the changes in writing habits, the educational level is decreasingly reflected by the quality of the handwriting.

Due to the conditions of examinations have changed, age can be determined only in exceptional cases and with large time interval. In the 70s examining



(a) Female (10191): Standard letters and even placement of the handwriting more frequently occur in female handwriting



(b) Male (10218): Narrow letters, the bigger size of the upper zone, larger capital letters more frequently occur in male handwriting

Figure 7.1: Handwriting samples

the young, middle-aged and old people's handwriting expert could reveal different copy-book style which was taught in Hungary. Conclusion in connection with the writers age based on this knowledge. Similarly English and Canadian experts could conclude the female writers on the fact that girls' schools taught different standard handwritings, see [176]. In addition the experts analysed large number of handwriting and observed different proportions of handwriting characteristics by genders. According to the evolved hypotheses based on these observations the literature differentiates such kind of features like female handwriting is more regular, more bounded, has more starting and ending lines and more rounded; the male handwriting has stronger pressure, their handwritings contains more narrow middle-zone letters, have larger upper zone according to Huber and Headrick [176] and especially their signatures are more illegible according to Mohammed et al. [177]. Several studies have dealt with the difference between girls and boys' coordination (see [178]) and relative speed (see [179]) of their writings from elementary to high school.

Currently, most of the Forensic Handwriting Experts (FHEs) do not make any conclusion related to the gender. Over and above there is an American handwriting examination standard (ASTM 2007) which states that the handwriting examinations cannot determine the sex (and some other traits either). The approach of Australian experts are more aware, Haines et al. conducted research about gender, and recognized some results in connection with demographic features extraction however in their view the techniques cannot be applied to casework[1] [180] .

In summary, there is only a small number of gender research studies reported by computer scientists and FHEs. The most recent study of [181] based upon mainly the examination and statistics of several special features of the letters and their elements (counters, loops, strokes, accents etc.). In contrast to this our experiment focuses on other feature groups, detailed in the examination section.

FORENSIC EXAMINATION AND RESULTS

We used the publicly available IAM-OnDB online handwriting dataset with writer genderness information and filtered the dataset on the basis of criteria of representative samples, considering gender (50-50), age (mean $\pm$ SD is 26.57 ± 7.37), nationality, handedness (81 right and 19 left-handed) and profession for examining the specimens on broader demographic scope. According to these aspects 100 samples were selected, from which 50 were written by female, and 50 by male. Both examinations were performed on the same samples, the computer based was completed on the online data directly, while the manual examination was based on the visualized online data[2].

1 `http://www.signatureforensic.com.au/content/view/36/68/`

2 Chosen writers id endings. Each writer id starts with 10. 027–033, 036–037, 040–041, 045, 047–048, 052, 057, 062–066, 068, 069, 071, 078, 081, 084–085, 088–089, 092, 093, 095–097, 101, 104–106, 108,

In the handwriting examination we analysed several features, the placement of handwritings, the general features and some special features that had largest discriminative power in earlier studies (see [176, 181]).

PLACEMENT OF HANDWRITINGS direction of the left margin, form of the right margin, direction of the lines, form of the lines, evennes of the placement of writing

GENERAL FEATURES extent of the writing, type of handwriting, legibility, structure, coordination, writing level, slant, form of the letters, proportions of capital letters, largest zone

SPECIAL FEATURES accents, fluorish at start, upward fluorish at end, hooks at starting and ending lines, cross bar of the letter *t*, ending line of the letter *y*, arcaded starting lines, without ending lines

As a result of our examination these features occurred more frequently in female handwritings: even placement, lines with convex form and up direction, better than medium coordination and writing level, simplified and cursive letters, larger scale of lower zones, equal proportion of capital letters and upper zones, consistent cross bar of the letter "t", arcade and fluorish at start, rounded ending lines of the letter "y" and upward fluorish at the end of the letters.

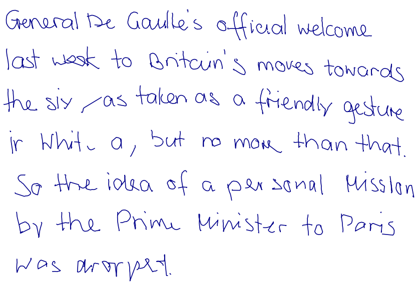
Contrarily the following features were more frequent in male writings: wavy form of the lines, medium coordination, simple structure, narrow letters, uneven slant, larger scale of upper zone and comparing with upper zone the larger proportion of capital letters, hooks and straight ending lines.

There was not distinct difference between female and male writings in accents, characteristics of the margins, form of the letters and loops, extent of the writings and legibility.

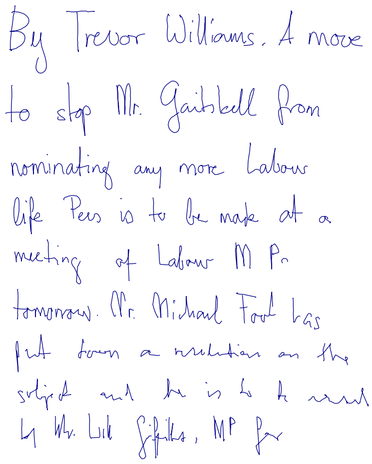
Fisher's exact tests were conducted to examine independence between gender and the revealed features. The null hypothesis of this test is that the proportions of the two examined variables (e.g. gender vs. slant) are equal, the variables are independent. The p-value of the conducted test is lower than 5%, indicates significant association between the examined variables. For the purpose of an error analysis we have monitored the results which is done similarly in daily routine work. There is a possibility performing indepedent investigations with storing feature dataset.

The main significant differences were in evenness of the placement of handwritings, form of lines, coordination, zone proportions, proportions of capital letters, structure of the letters, direction and form of starting and ending lines, consistency of the cross bar. Other special significant features were extracted, such as arcade in starting lines which did not occur in male, but occurred in 16%

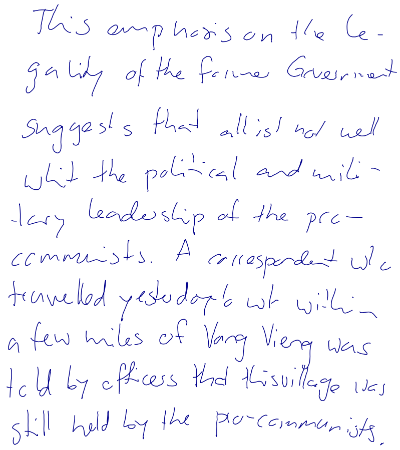
110–111, 115–116, 118, 120, 125, 127, 131–133, 135, 137, 142, 149–152, 155–161, 164–165, 168–169, 172–176, 178, 180–181, 185–186, 188–191, 194, 198–201, 204–208, 210, 212, 215–219, 221



(a) even



(b) mostly even



(c) mixed

Figure 7.2: Samples of different handwriting evenness

(a) straight

(b) rounded

(c) mixed

Figure 7.3: Samples of different ending lines of the letter *y*

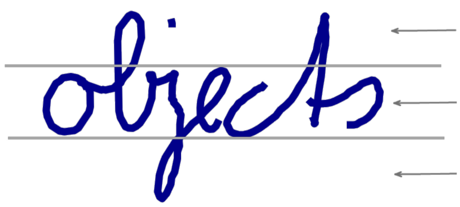
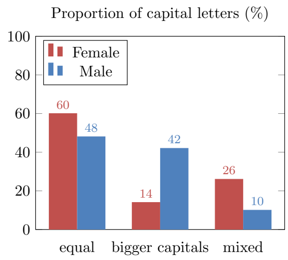
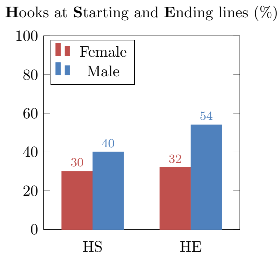


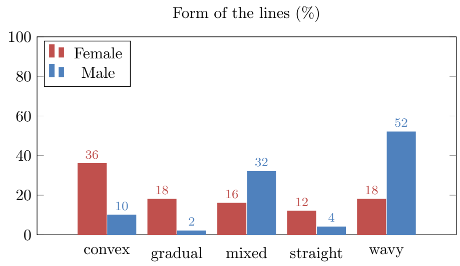
Figure 7.4: Zones in handwriting – lower, middle, upper



(a) Proportion of capital letters

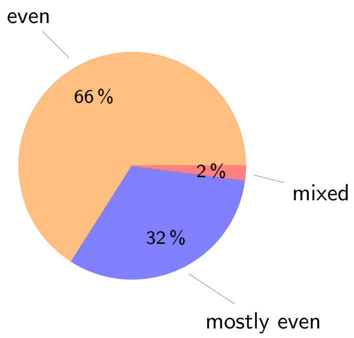


(b) Hooks

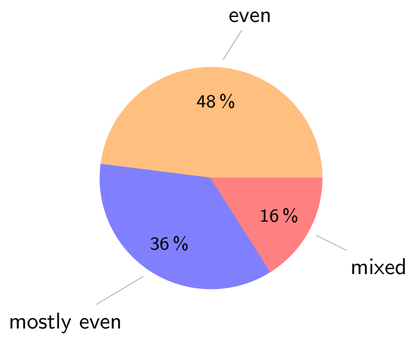


(c) Form of the lines

Figure 7.5: Barplots from each feature group

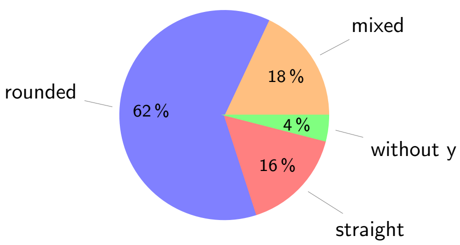


(a) Female

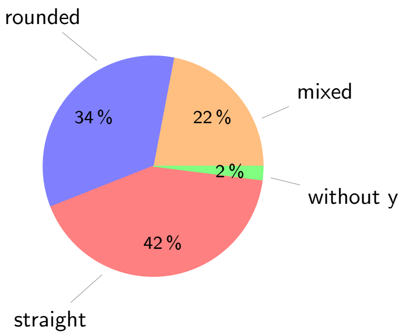


(b) Male

Figure 7.6: Distribution – Handwriting evenness

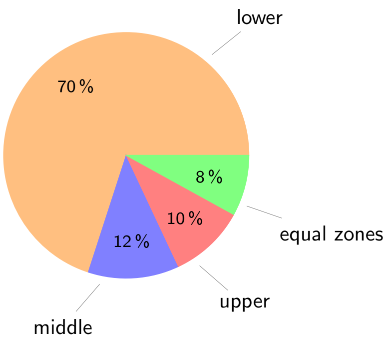


(a) Female

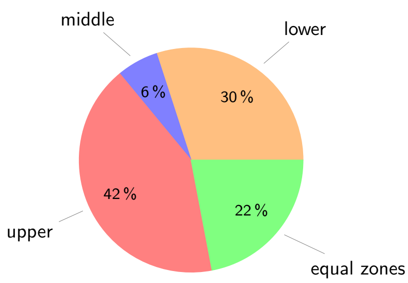


(b) Male

Figure 7.7: Distribution – Ending line of the letter *y*



(a) Female



(b) Male

Figure 7.8: Distribution – Largest zones

of the female handwriting. At the same time there was no female handwriting without ending line nevertheless its proportion in male writing was 8%.

Related to the limitations of this study we have to mention the under-representativity of the dataset in terms of educational level. It may caused the small number of low-coordinated and low-level handwritings. The condition of the sampling process could also affect the proportion of different handwriting types, inasmuch as under normal circumstances cursive handwriting is more common.

Table 7.1 summarizes each analysed feature, the corresponding p-values and the proportion of different characteristics amongst female and male samples.

Feature	**F**	**M**	p-**value**
Form of the lines			$\mathbf{9.999 \cdot 10^{-6}}$
convex	0.36	0.10	
gradual	0.18	0.02	
mixed	0.16	0.32	
straight	0.12	0.04	
wavy	0.18	0.52	
Evenness			$\mathbf{2.775 \cdot 10^{-2}}$
even	0.66	0.48	
mostly even	0.32	0.36	
mixed	0.02	0.16	
Coordination			$\mathbf{8.309 \cdot 10^{-5}}$
weaker than medium	0.00	0.02	
medium	0.20	0.58	
better than medium	0.80	0.40	
Largest zone			
lower	0.70	0.30	$\mathbf{1.207 \cdot 10^{-4}}$
middle	0.12	0.06	$4.870 \cdot 10^{-1}$
upper	0.10	0.42	$\mathbf{4.728 \cdot 10^{-4}}$
Proportion of capital letters			$\mathbf{3.046 \cdot 10^{-3}}$
equal	0.60	0.48	
capitals are bigger	0.14	0.42	
mixed	0.26	0.10	
Structure			$\mathbf{2.538 \cdot 10^{-2}}$
simple	0.18	0.44	
simplified	0.34	0.20	
complex	0.18	0.08	
mixed	0.30	0.28	
Hooks			
starting lines	0.30	0.40	$4.019 \cdot 10^{-1}$

	ending lines	0.32	0.54	$\mathbf{4.282 \cdot 10^{-2}}$
Arcaded	starting lines	0.16	0.00	$\mathbf{5.77 \cdot 10^{-3}}$
Without	ending lines	0.00	0.08	$1.175 \cdot 10^{-1}$
Fluorish at start		0.38	0.16	$\mathbf{2.328 \cdot 10^{-2}}$
Upward fluorish at end		0.58	0.30	$\mathbf{8.471 \cdot 10^{-3}}$
Cross bar of the letter "t"				$\mathbf{6.013 \cdot 10^{-4}}$
	consistent	0.68	0.32	
Ending line of the letter "y"				$\mathbf{9.329 \cdot 10^{-3}}$
	mixed	0.18	0.22	
	rounded	0.62	0.34	
	straight	0.16	0.42	
	without y	0.040	0.02	
Dot accents		0.98	0.94	0.6173
Acute accents		0.64	0.72	0.5205
Direction of the lines				0.1566
	down	0.10	0.24	
	horizontal	0.20	0.26	
	mixed	0.34	0.28	
	up	0.36	0.22	
Direction of the left margin				0.2948
	left	0.10	0.02	
	right	0.44	0.38	
	upright	0.38	0.50	
	mixed	0.08	0.10	
Form of the right margin				0.3921
	almost linear	0.38	0.24	
	linear	0.06	0.02	
	curved	0.06	0.10	
	gradual	0.42	0.50	
	mixed	0.08	0.14	
Form of the loops				0.4076
	angular	0.10	0.04	
	mixed	0.24	0.22	
	rounded	0.40	0.34	
	without	0.26	0.40	
Form of the letters				1.0000
	angular	0.24	0.24	
	rounded	0.40	0.42	
	mixed	0.36	0.34	

Type of handwriting			0.2029
block of letters	0.30	0.44	
cursive	0.54	0.36	
mixed	0.16	0.20	
Legibility			0.6820
legible	0.64	0.58	
mixed	0.36	0.42	
Writing level			0.2463
weaker than medium	0.00	0.05	
medium	0.48	0.50	
better than medium	0.52	0.44	
Slant			0.1679
backhand	0.12	0.04	
forehand	0.24	0.16	
vertical	0.30	0.26	
mixed	0.34	0.54	
Extent of the writing			0.3687
standard	0.28	0.42	
compressed	0.42	0.34	
mixed	0.30	0.24	
Letter width			0.2430
mixed	0.26	0.18	
narrow	0.22	0.38	
standard	0.46	0.34	
wide	0.06	0.10	

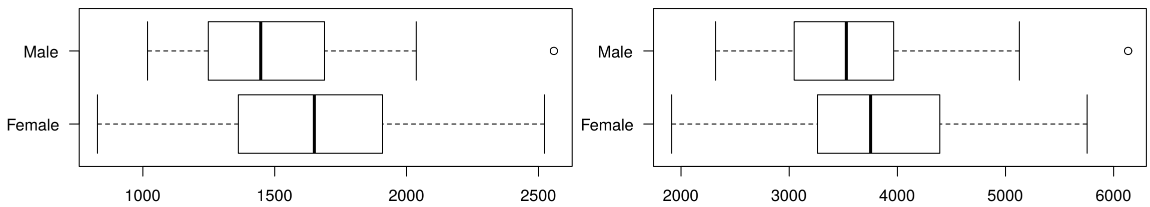
Table 7.1: p-values of Fisher's exact test for independence testing

AUTOMATIC EXAMINATION AND RESULTS

We examined the first line indent, the left margin and different velocity features for each sample with a software developed for this experiment. After removing noise, the left margin was measured as the average horizontal distance from origin to the minimal x coordinate for each line except the first. The first line margin was measured as the horizontal distance from origin to the minimal x coordinate, and the first line indent was measured as the difference of the first line margin and the left margin.

Velocity was measured using the x_t, y_t coordinates and t timestamp of the dataset. For each sample we calculated $v_k = \sqrt{(\Delta x_k/\Delta t_k)^2 + (\Delta y_k/\Delta t_k)^2}$ absolute velocity for each stroke pointwise, for comparison the maximum, the median and the mean was calculated as features. We noticed that due to the noise, extremely large maximum velocities occurred, therefore we examined 95% percentiles instead of the maximum values.

These automatically evaluated features were quantitative, thus Welch t-test was conducted to compare the mean of these features with respect to males and females. There were no significant differences between the means of first line indent, the left margin and the 95% percentile of velocities. However significant differences were found in velocity means and velocity medians ($p = 0.0339$ and $p = 0.01116$ respectively). The measurement of velocity complements the results of the previous literature [179] that a difference can be experienced not only in the writing speed of the student girls and boys, but also in the average speed of adult male and female writings. Table 7.2 shows the corresponding p-values, Figure 7.9 shows the boxplot for velocity means and 95% percentiles.



(a) mean (b) 95% percentile

Figure 7.9: Boxplots of velocity mean and velocity 95% percentile

Feature	p**-value**	**95% CI**
First line indent	0.5722	$[-185, 103]$
Left margin	0.9887	$[-121, 120]$
Velocity		
95% percentile	0.14310	$[-84, 569]$
median	$\mathbf{1.116 \cdot 10^{-2}}$	$[36, 276]$
mean	$\mathbf{3.39 \cdot 10^{-2}}$	$[12, 284]$

Table 7.2: p-values and confidence intervals of Welch's t-tests

CONCLUSION

The aim of this study was to assess whether pre-filtering by gender has relevance. Based on this experiment and the revealed significant differences, further research can be conducted towards this direction through deeper analysis on a large and representative dataset. This kind of statistical data might be a potential base for appreciating features of the questioned handwritings and performing a quantitative report about gender of the writer.

Part IV

SUMMARY

SUMMARY IN ENGLISH

MOTIVATION

In the new digital era paperless office is not an utopistic idea anymore. With the spread of easily available smart devices and sensors, more and more companies introduce their systems to change from paper-based contracts to digital. Thus there is a need to develop better devices, algorithms and softwares to analyse online handwriting and verify signatures and to support the work of Forensic Handwriting Examiners.

CONTRIBUTIONS

My PhD dissertation includes several studies in online handwriting analysis. The first thesis group consists studies in online signature verification and classification, while the second thesis group focuses on online handwriting classification and analysis based on features applied by FHEs.

My main contributions to these studies are the followings:

I. THESIS GROUP - SIGNATURE VERIFICATION AND CLASSIFICATION

1. I recorded two online signature datasets based on data measured by 3D accelerometer and 2D gyroscope. I evaluated the applicability of the databases in verification using baseline verifiers and classification and examined how training data selection effects the results (Chapter 4)

   Based on the results and an online signature verification method implementation which is based on the often used DTW, the accelerometric is applicable in online signature verification and analysis, however the accuracy does not reach the accuracy of the recently available systems on the market which are more expensive, but due to the more accurate measurement with advanced hardware, more sophisticated methods can be applied in the verifier able to detect forgeries.

   During the analysis of the database recorded with the accelerometer, I found that the selection of the test dataset greatly influences the verification results.

   Signature classification was performed on the signature samples recorded by accelerometer and gyroscope. Features extracted using Legendre approximation and classification carried out with SVM showed that approximation with higher than 20 order, the accuracy did not increase. Multi-class classification with 10 classes provided 43%/52% accuracy (on accelerometric/gyroscopic data, with order 13), binary classification provided 88%/80.44%. In binary classification accelerometric data clearly outperformed gyroscopic in terms of accuracy, however in multi-class classification gyroscopic data provided slightly higher accuracy.

2. I introduced an online signature verification method based on Kolmogorov-Smirnov statistical distance (Chapter 5)

   The method was tested on the SigComp2011 Dutch online dataset, thus x, y coordinates and pressure data were available. I added velocity feature and examined which single feature and feature combinations were most suitable for signature verification purposes. The developed method is competitive to the competing methods of the SigComp2011 competition and I found that the primarily pressure, secondarily the velocity features proved to be the most suitable features in online signature verification.

II. Thesis group - Handwriting examination

3. With the guidance of the Hungarian Institute for Forensic Science (NIFS) I examined online handwriting samples and classified them by handedness based on forensic-methodology (Chapter 6)

   Previous results of forensic handwriting examinations showed that horizontal lines (e.g. crossings) were the most reliable feature to distinguish left-handed and right handed handwriting, I automatically detected horizontal strokes and by taken to account their directions, made conclusion about the handedness of the handwriting samples. The results obtained are competitive to the available methods. The actual error rates compared to the results of other methods was lower, with the newly introduced inconclusive class, which does not give conclusion about handedness if insufficient information is available to make decision. Thus, based on the majority voting (MV), we could achieve a 1.98% error rate for 1568 handwritings with 12.05% inconclusive cases.

4. I examined handwriting differences of several discrete and continuous features comparing the writers' biological gender using statistical tests (Chapter 7)

   Based on the examination of the Hungarian Institute for Forensic Science, which revealed 2500 features and automatic measurement of velocity-related features carried out by my software, I analyzed these extracted features using statistical tests (Fisher's exact test for discrete variables and Welch t-test for continuous), examining which features might be suitable to distinguish male and female handwriting.

Publications by the author, related to theses

Publication	Thesis point			
	I. group		II. group	
	1	2	3	4
[136]	•			
[137]	•			
[138]	•			
[139]	•			
[159]		•		
[160]			•	
[169]				•

ÖSSZEGZÉS MAGYARUL

MOTIVÁCIÓ

A mai digitális korban a papírmentes iroda már többé nem egy utópisztikus elképzelés. A manapság könnyen elérhető okos eszközök és szenzorok elterjedésével egyre több cég vezet be digitális rendszereket, és próbálja dokumentumait, szerződéseit digitális formában tárolni. Emiatt továbbra is nagy szükség van olyan eszközök, algoritmusok illetve szoftverek fejlesztésére, melyek online kézírások elemzésére, aláírások hitelesítésére alkalmasak és mindemellett az igazságügyi írásszakértők mindennapi munkáját támogatják.

HOZZÁJÁRULÁS

Eredményeim két téziscsoportban, téziscsoportonként két-két tézispontban foglalhatóak össze. Az első téziscsoport tézispontjai online aláíráshitelesítéssel és aláírások osztályozásával, a második tézicsoport tézispontjai online kézírások osztályzásával illetve elemzésével kapcsolatosak.

I. TÉZISCSOPORT - ALÁÍRÁSOK HITELESÍTÉSE, OSZTÁLYZÁSA

1. Rögzítettem két online aláírás adatbázist 3-irányú gyorsulásmérő illetve 2-irányú giroszkóp használatával. Aláíráshitelesítő módszereket és osztályzókat értékeltem ki az adatokon, vizsgálva az adatok alkalmazhatóságát.

   Az eredmények alapján elmondható, hogy az aláíráshitelesítésben korábban is alkalmazott DTW a gyorsulás adatokon alkalmazható, az elért pontosság viszont nem közelíti meg a piacon manapság elérhető, pontosabb mérést lehetővé tevő (és szofisztikáltabb osztályzót alkalmazó) eszközöket.

   Megállapítottam a gyorsulásmérővel rögzített adatbázis elemzése során, hogy a tanítóhalmaz kiválasztása nagy mértékben befolyásolja a hitelesítési eredményeket.

   Aláírás-osztályzást végeztem a gyorsulásmérővel és giroszkóppal mért adatokon ugyanazon aláírók aláírásain. Legendre approximáció és SVM osztályzó használatával megmutattam, hogy 20-ad rendűnél magasabb approximáció esetén csökken az osztályzó pontossága, és 10 aláíró osztály esetén többosztályos osztályzónál nem magasabb, mint 43% / 52% (gyorsulásmérő/giroszkóp adatokon, 13-ad rendű approximáció), bináris osztályzás esetén a pontosság 88% / 80.44%.

Bináris osztályzás esetén egyértelműen a gyorsulásadatok adtak pontosabb osztályzást, többosztályos osztályzás esetén viszont a giroszkóppal rögzített adatokkal tudtunk kevesebb hibával osztályozni, bár utóbbi esetben a különbség nem számottevő a kétféle adat által elérhető pontosság(ok) között.

2. Egy publikusan elérhető adatbázison vizsgáltam egy általam fejlesztett Kolmogorov-Smirnov távolságon alapuló hitelesítő módszer eredményességét.

   A módszer a SigComp2011 holland adatbázison lett tesztelve, melynél a koordináta és nyomás jellemzők adottak és emellett vizsgáltam az abszolút sebességet is. Megvizsgáltam, hogy mely egyszeri és mely kombinált jellemzők a legalkalmasak aláíráshitelesítésben, Kolmogorov-Smirnov távolsággal mérve a tapasztalati eloszlásfüggvények közötti különbségeket. A módszerem versenyképes a kapcsolódó SigComp2011 versenyen résztvevők módszereivel. Megállapítottam, hogy a vizsgált jellemzők közül elsősorban nyomás, másodsorban a sebességérték bizonyultak a legalkalmasabb jellemzőknek az aláíráshitelesítésben.

II. Téziscsoport - Kézírások vizsgálata

3. A Nemzeti Szakértői és Kutató Központ írásszakértői iránymutatása szerint osztályoztam kézírásmintákat az alapján, hogy azok jobb vagy bal kézzel készültek . Az írásszakértők által legmegbízhatóbbnak értékelt, az online területen objektíven megismerhető sajátosságot, az áthúzó vonalak irányát automatikusan detektáltam és ezek balról jobbra vagy jobbról balra irányuló mozdulata alapján hoztam döntést az író személy kezességéről. Az elért eredmények versenyképesnek bizonyultak a szakirodalomban megtalálható, jóval több jellemzőt alkalmazó, hasonló módszerekkel. A tényleges hibaszázalék az ismert eredményekkel összehasonlítva kisebb volt azáltal, hogy ahol nem voltak megfelelőek a feltételek, a szakértői következtetési skálához hasonló, döntés nélküli (ún. inkonklúzív) következtetést is bevezettünk. Ezáltal a többségi döntés (MV) alapján 1568 db kézírásra 1,98%-os hibaarányt tudtunk elérni, 12,05%-os inkonklúzív eset mellett.

4. A Nemzeti Szakértői és Kutatóközpont IAM-OnDB adatbázison végzett írásszakértői vizsgálatából kiindulva elemeztem, hogy a feltárt 2500 sajátosság a férfi és női kézírásokban milyen arányban fordul elő. A manuálisan vizsgált jellemzőket és néhány általam automatikusan mért jellemzőt statisztikai próbáknak vetettem alá, tanulmányozva annak lehetőségét, hogy a kizárólag szakirodalmi adatokon és írásszakértői tapasztalatokon nyugvó következtetéseket meg lehet-e támogatni egzaktabb módszerekkel. 100 személy minta anyagának jellemzőit értékelve több szignifikáns eltérést is találtam. Az eredmények abba az irányba mutatnak, hogy egy jelentős szá-

mú, reprezentatív adatbázis segítségével az írásszakértők a jelenlegi módszertant statisztikai következtetésekkel egészíthetik ki.

A DOLGOZATHOZ KAPCSOLÓDÓ PUBLIKÁCIÓIM

Publikáció	**Tézispont**			
	I. csoport		II. csoport	
	1	2	3	4
[136]	•			
[137]	•			
[138]	•			
[139]	•			
[159]		•		
[160]			•	
[169]				•

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