

**University of Szeged**  
**Department of Image Processing and Computer Graphics**

# **Discrete Tomographic and PACS Image Processing Systems**

Summary of the Doctoral Thesis

by

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# 1 Introduction

Image processing is one of the most dynamically developing part of computer science. Digital images are used on subsequent areas due to the evolution of the hardware. Images are archived, coded, transmitted, processed, and displayed by complex systems. The image processing systems have an important role in medicine, space research, meteorology, moreover we can say that it is useful in all disciplines. Besides generic image processing systems, special systems have been developed to solve particular tasks. This is the case when the crosssections of 3D objects are reconstructed from their projections or when medical images are collected and transmitted to a central image archive. These systems belong to the recent developments and the Thesis also deals with them.

The Thesis specifically deals with image processing systems related to Discrete Tomography, Emission Discrete Tomography, and Picture Archiving and Communication Systems.

## 2 Discrete Tomographic System

*Tomography* is an imaging procedure where the cross-sections of the 3D object being studied are determined from its projection images. The projection images can be created by some rays that are emitted from a source (like X-rays from an X-ray tube), transmitted through and partially absorbed by the object, and finally detected by some array (plane or line) of detectors. The pixels of the projection image represent the total absorption of the rays along the lines between the source and the corresponding detector elements. The pixels of the projection image can be measured in the following way (Beer law) in case of X-ray:

$$I = I_0 \cdot e^{-\int_0^d \mu(x) dx}, \quad (1)$$

where  $I_0$  is the source X-ray intensity,  $I$  is the detected X-ray intensity,  $\mu$  is the absorption coefficient of the object, and the  $d$  is the distance between the source and the detector. Pixel intensity of the projection image is given by an intensity of the ray which comes along the line between the source and the detector.

*Discrete tomography* (DT) is a special kind of tomography that can be applied if the object to be reconstructed consists of only a few known homogeneous materials (e.g., metal and wood). This information can be incorporated into the reconstruction process, giving one the opportunity of reconstructing simple objects from a much smaller number of projection values than is necessary for more complex objects. For this reason discrete tomography seems to be important in applications where the object is so simple and there is no opportunity or it is too costly to acquire lots of projections, like those in non-destructive testing, electron microscopy and medicine. For a summary of the theory and applications of DT, see [6].

There are basically two ways of acquiring the necessary projections. In the case of *parallel projections*, the rays parallel to a given direction are transmitted and measured in one phase of the acquisition process. By rotating the system other rays parallel to other directions can be created.

In the first part of the Thesis we discuss a special discrete tomography problem, namely the reconstruction of binary matrices from their fan-beam projections. Even the reconstruction from fan-beam (in 2D) or from cone-beam (in 3D) projections is well understood [7]. It is interesting that at the same time there are very few papers about DT using fan-beam/cone-beam projections. The main reason for this might be that from the mathematical point of view some

reconstruction results of parallel projections can be applied directly in the case of fan-beam projections. There are open questions in the last one (e.g. uniqueness and existence problems). There are several applications of tomography that make use of fan-beam projections and could be of interest in DT, e.g., non-destructive testing using X-rays or neutron beams.

We introduce a discrete tomographic system in the first part of the Thesis which is our development. We have also performed simulations based on this system. The results of the discrete tomographic system were published in [3, 9, 16] and article [8] will appear in a book chapter (accepted for publication).

## 2.1 The reconstruction problem for fan-beam projections

Let  $f$  be an integrable real function in the  $\mathbb{R}^2$  plane. Let  $S$  be a point called the *source point*, and  $v_\theta$  be a unit vector in the direction  $\theta \in [0, 2\pi)$  in the plane. Consider the integrals of  $f$  along the half-lines starting from  $S$  in direction  $v_\theta$

$$[\mathcal{R}f](S, \theta) = \int_0^\infty f(S + u \cdot v_\theta) du, \quad (2)$$

The transformation defined by (2) is called the *projection of  $f$  taken from the point  $S$  in the direction  $\theta$* , or the *fan-beam projection of  $f$  taken from the point  $S$* .

Given a set of the source points  $\mathcal{S}$ , the *reconstruction problem using fan-beam projections* can be stated as follows:

FB( $\mathcal{S}$ ) RECONSTRUCTION PROBLEM

Given: A function  $g : \mathcal{S} \times [0, 2\pi) \rightarrow \mathbb{R}$ .

Task: Construct a function  $f$  such that

$$[\mathcal{R}f](S, \theta) = g(S, \theta)$$

for all  $S \in \mathcal{S}$  for almost every  $\theta \in [0, 2\pi)$ .

There are accurate methods for solving the FB reconstruction problem due to the imprecise and inconsistency (acquired) data. Hence the solution of this problem is approximated.

We are interested in the reconstruction of special types of functions from fan-beam projections. Henceforth, let us suppose that the support of  $f$  can be covered by an  $n \times n$  regular lattice  $W$  such that  $f$  is constant on each  $1 \times 1$  square of the lattice, such that  $f$  can take a value of 0 or 1. That is,  $f$  can be represented by a binary-valued matrix or, equivalently, by a vector  $\mathbf{x} \in \{0, 1\}^J$  where  $x_j$  denotes the  $j$ th element of the matrix, say, in successive order, where  $j = 0, 1, \dots, J$  and  $J = n^2$ .

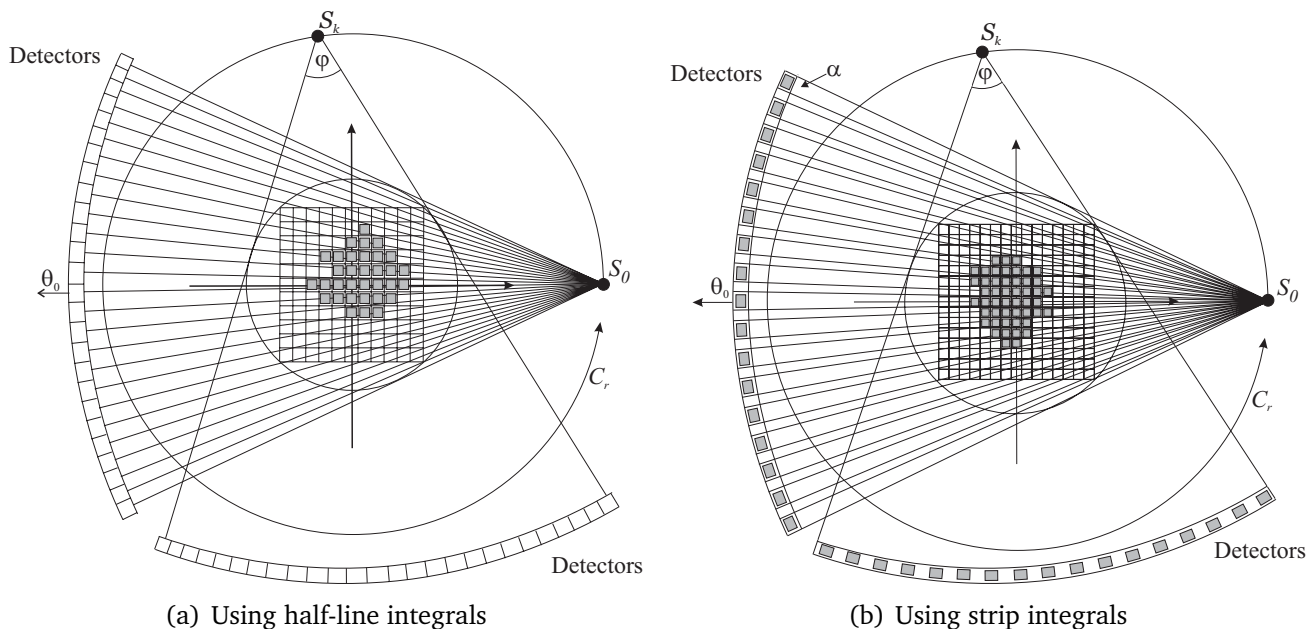
In the majority of applications the projections are acquired from only a finite number of points,  $S_k, k = 1, 2, \dots, K$ , along a finite ( $L$ ) number of half-lines from each point. In this case the  $i$ th projection,  $b_i$ , from the point  $S_k$  in direction  $v_\ell$  ( $i = (k - 1) \cdot K + \ell$ ) can be described by the linear equation

$$\sum_{j=0}^J a_{ij} x_j = b_i, \quad i = 1, 2, \dots, I, \quad (3)$$

where  $a_{ij}$  denotes the length of the intersection of the  $i$ th half-line with the  $j$ th unit square of  $W$  and  $I = K \cdot L$ . In the linear equation system (3) the projections are obtained (within

a certain error) by measurements. The elements of matrix  $A = (a_{ij})_{I \times J}$  can be computed knowing the positions of the squares in  $W$  and the half-lines starting from the source points. The special feature of (3) is that the unknown vector  $\mathbf{x}$  is binary here, i.e.,  $x_j \in \{0, 1\}$  for all  $j = 1, 2, \dots, J$ .

Each detector (Fig. 1) measures one projection value  $b_i > 0$ . Therefore, we can determine the effect of one pixel in case of half-line and strip integrals.



**Figure 1:** The parameters of the fan-beam geometry.

The  $a_{ij}$  weights should be calculated in different ways depending on the geometry model according to (3):

- half-line integrals (Fig. 1(a)),  $a_{ij}$  is the length of the intersection of the  $i$ th half-line with the  $j$ th unit square of  $W$  and  $I = K \cdot L$ .
- strip integrals (Fig. 1(b)),  $a_{ij}$  is the common area between the  $i$ th fan — area between  $2i$  and  $2i + 1$  half-lines — and the  $j$ th unit square of  $W$  and  $I = K \cdot L$ .  $\alpha$  denotes the angle between the two half-lines.

In this way, the fan-beam geometry determined by  $r$ ,  $L$  (and  $\alpha$  in case of strip integrals) unambiguously. In real situations the projections are usually measured with a certain error. For this reason Gaussian noise can be generated and added to the exact (analytically computed) projections for creating noisy projection data.

## 2.2 Reconstruction as an optimization problem

As we saw earlier the solution of the  $FB(S)$  reconstruction problem in our fan-beam model is equivalent to finding a solution of the linear equation system

$$\mathbf{Ax} = \mathbf{b}, \quad \text{where } \mathbf{x} \text{ is a binary-valued vector.} \quad (4)$$

The reconstruction methods like the Algebraic Reconstruction Techniques (ART) [5] do not necessarily provide a binary-valued  $\mathbf{x}$  that satisfies (3). It cannot be applied here because a non-binary solution might be quite different from a binary one.

Since any half-line in our model at most intersects  $\mathcal{O}(n)$  squares of  $W$ , the matrix  $\mathbf{A} = (a_{ij})_{I \times J}$  is sparse (it contains only a few non-zero elements). Another important property of this matrix equation in DT applications is that the number of equations (i.e. the number of projections) is usually much less than the number of unknowns, hence  $I \ll J$ . It means that it can have several solutions, even binary-valued ones.

Furthermore, due to measurement errors it is also possible that (4) has no exact solution, so it is better to try to find a binary-valued  $\mathbf{x}$  which satisfies (4), at least approximately. Actually, a possible way of solving (4) at least approximately is to reformulate it as an optimization problem. Formally, we should find the minimum of the following objective function

$$C(\mathbf{x}) = \|\mathbf{A}\mathbf{x} - \mathbf{b}\| + \Psi(\mathbf{x}), \quad (5)$$

where  $\mathbf{x} \in D$  and  $\Psi(\mathbf{x}) = \gamma \cdot \Phi(\mathbf{x})$ . The first term on the rhs ensures that we have an  $\mathbf{x}$  satisfying (4) at least approximately. The second term allows us to include *a priori* knowledge about  $\mathbf{x}$  into the optimization. The second term in (5) provides that for the  $\mathbf{x}$  which gives low value for the first term,  $C(\mathbf{x})$  will be also low with  $\Psi(\mathbf{x})$ , i.e. grants the required property as well. The regularization coefficient  $\gamma$  is needed to weight the two terms in  $C$ .

Since we are looking for a binary-valued  $\mathbf{x}$  in the optimization of (5), the usual numerical optimization methods seem unsuitable here. The combinatorial optimization methods looked more promising and turned out to be useful. Among them we selected the *simulated annealing* (SA) optimization procedure [15].

We have used a special  $\Psi(\mathbf{x})$  function in our experiments, namely

$$\Psi(\mathbf{x}) = \Psi_{\text{poz}}(\mathbf{x}) = \gamma_{\text{poz}} \cdot \Phi_{\text{poz}}(\mathbf{x}) = \gamma_{\text{poz}} \cdot \sum_{j=0}^{J-1} \text{poz}(f_j - f_j^{(\text{proto})}), \quad (6)$$

where *poz* denotes the positive part of  $y$ . Formally,

$$\text{poz}(y) = \begin{cases} y, & \text{if } y > 0, \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

and  $f_j^{(\text{proto})}$  is a so-called *prototype function*. The regularization parameter and penalty term bias the algorithm according to the mask function. Optimizing the objective function we get a result like this, which will lie inside the given mask.

We can extend the  $\Psi(\mathbf{x})$  regularization term with additional terms. For example

$$\Psi(\mathbf{x}) = \Psi_{\text{mix}}(\mathbf{x}) = \Psi_{\text{poz}}(\mathbf{x}) + \Psi_{\text{sm}}(\mathbf{x}), \quad (8)$$

where

$$\Psi_{\text{sm}}(\mathbf{x}) = \gamma_{\text{sm}} \cdot \Phi_{\text{sm}}(\mathbf{x}), \quad (9)$$

and  $\Phi_{\text{sm}}(\mathbf{x})$  can be defined in the following way:

$$\Phi_{\text{sm}}(\mathbf{x}) = \sum_{j=0}^{J-1} \sum_{\ell \in Q_j^m} g_{\ell,j} \cdot |x_j - x_\ell|, \quad (10)$$

where  $Q_j^m$  is the set of indices of the  $m \times m$  adjacent pixels of the  $i$ th lattice pixel. The  $g_{\ell,j}$  is the corresponding element of the  $m \times m$  Gaussian matrix. The  $g_{\ell,j}$  scalar weights the differences according to the distance of the  $\ell$ th and  $j$ th lattice points. Using  $\Psi_{\text{mix}}(\mathbf{x})$  regularization term we can reconstruct objects with big homogeneously connected regions.

## 2.3 Simulation experiments

We have performed simulation experiments to show the effects of parameter changing to the fan-beam and Simulated Annealing. Our aim was to present information about the efficiency of the discrete tomography system according to the different parameter sets. Our experiments was used in the applications and in the planning of the tomographic hardware [3, 9].

The simulation experiments were performed with phantom images each having a size  $200 \times 200$ . The projections of the phantom images were computed based on (3) for each parameter setting. The images were then reconstructed from the projections using the SA algorithm. In order to get quantitative results, the original phantom images were compared pixel-by-pixel according to the relative mean error (11), that is,

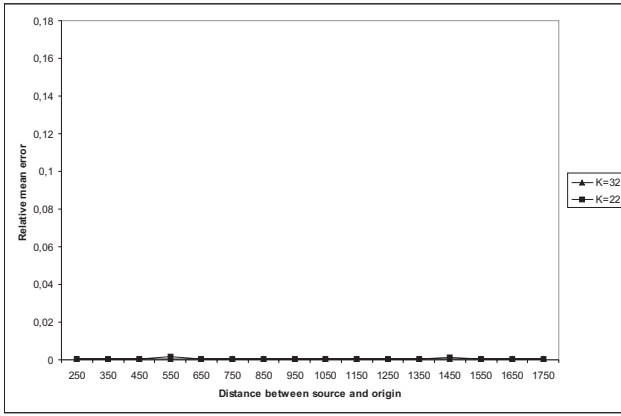
$$M_e = \frac{\sum_{j=1}^J |x_j - \hat{x}_j|}{\sum_{j=1}^J \hat{x}_j}, \quad (11)$$

where  $\hat{\mathbf{x}} = \{\hat{x}_j\}_{j=1}^J$  denotes the vector of the original image. Clearly,  $M_e \geq 0$  and the smaller value indicates better comparison result. Furthermore,  $M_e = 0$  if and only if  $\mathbf{x} = \hat{\mathbf{x}}$ . The relative mean error is essentially the pixel ratio between the deviation between the original and the reconstructed image and the number of 1s in the original image.

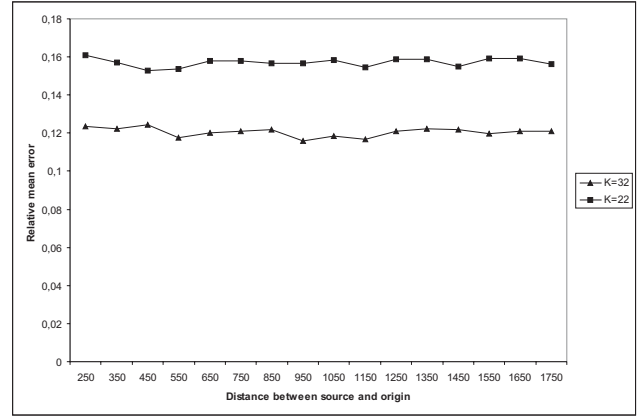
Since we had an optimization process based on random-search, we repeated each test 100 times with the same parameter setting. The mean of the 100 values of the  $M_e$  was computed and presented later as the result for each test with the given parameter setting. The average image of the 100 binary images was given as the result of the reconstruction for one parameter setting.

Several parameter settings were tested. One of them, the so-called *baseline parameter setting*, played a special role. Here only one of the parameters was allowed to change at a time, the others having the same values as in the baseline parameter setting case. In order to see the effect of the parameters on the quality of the reconstruction, a sequence of tests was performed for each parameter. During a test sequence only the value of the selected parameter was changed and the other parameters always had the same values as in the baseline parameter setting.

We have made experiments by varying of the parameters of the cone-beam data acquisition geometry in case of using line and strip integrals. According to the results of the experiments there is no substantive difference between the cone-beam and the parallel beam data acquisition geometry using the given DT reconstruction method on the studied phantoms (Figs. 2 and 3). It was an important result that further quality improvement can be achieved by using prototype ( $\Psi_{\text{poz}}$ ) and regularization terms preferring large coherent areas ( $\Psi_{\text{sm}}$ ) in the case of noisy projections of the given phantoms. We found that there is no major difference when line and strip integrals were used during the reconstruction in the case of the same phantoms.

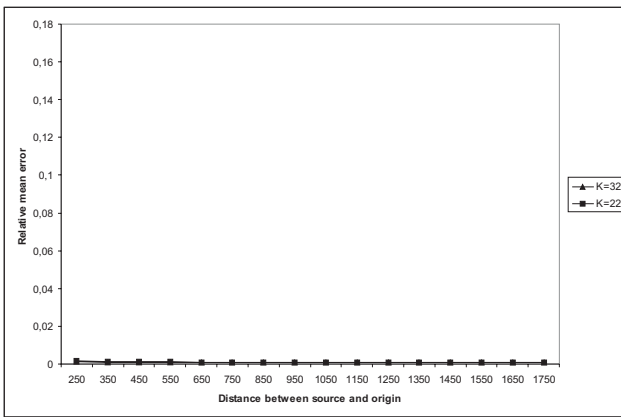


(a) Noiseless

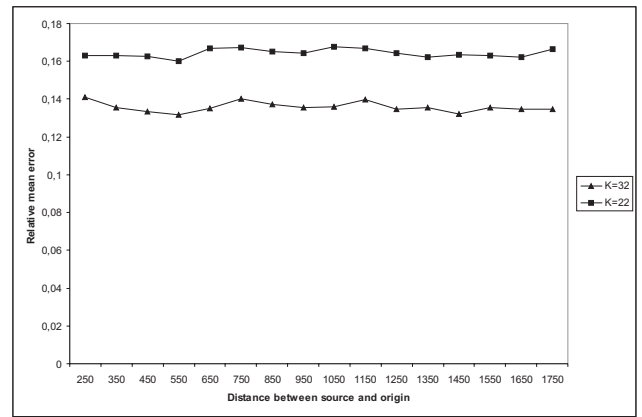


(b) 5% Gaussian noise

**Figure 2:** Relative mean error as a function of the distance between source and origin in case of using half-lines integrals.



(a) Noiseless



(b) 5% Gaussian noise

**Figure 3:** Relative mean error as a function of the distance between source and origin in case of using strip integrals.

## 2.4 Applications

We show two possible applications of the Discrete Tomography method in chapter 2 of the Thesis. We give solutions for a problem of the non-destructive testing in these applications.

We have applied successfully the Discrete Tomography method for these problems. The regularization terms used during the method affected the quality of the reconstruction results. We have concluded that we can get nice results with this DT method using more than one regularization term.

### 2.4.1 Non-destructive testing with X-ray

The acquired projection values can be below the noise level in case of the large absorption in oblong objects (e.g. metal). Accordingly these projections are useless. The conventional reconstruction algorithms (e.g. filtered back projection) do not give results with necessary accuracy in these circumstances.



Before reconstructing from real projection data we simulated projection data and analyzed the sensitivity of DT to distortions. We have used in all experiments 2D reconstruction method and special phantom (Fig. 4).



**Figure 4:** The phantom used in the experiments.

In the chapter 2.5 of the Thesis we have compensated the missing information with *a priori* knowledge using discrete tomography method (Fig. 5).



(a) without  $\Psi_{sm}(x)$  regularization term



(b) Using  $\Psi_{sm}(x)$  regularization term

**Figure 5:** Reconstructions for reduced dynamic range in noiseless case.

Polychromatic X-rays cause systematic distortions in the projection data. For polychromatic X-rays the Beer-law (1) describes the symptom approximately, since exponential is additionally integrated over the spectrum (1). The result of the beam hardening correction was also used in the evaluation of efficiency of the reconstruction system.

The simulations, experiments, and the beam hardening correction were performed by “Corporate Technology PS 9, Siemens AG”. The reconstructions with discrete tomography were done at the University of Szeged.

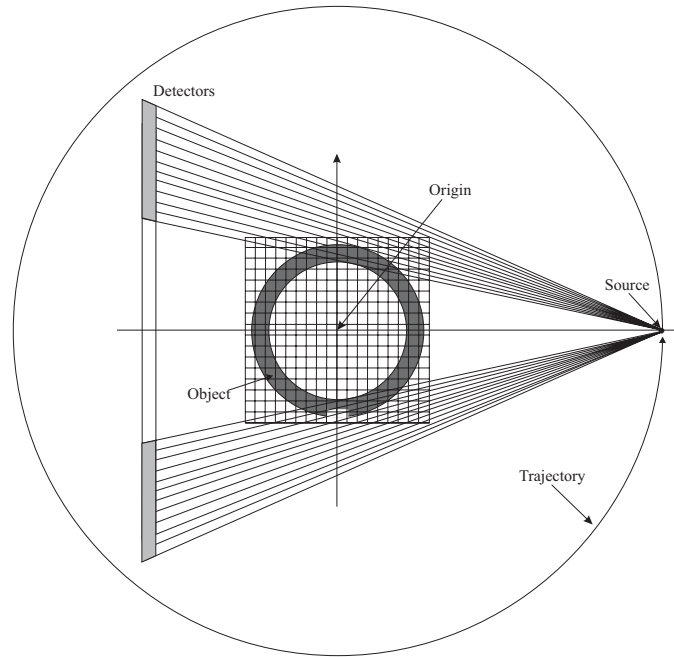
### 2.4.2 Pipe corrosion study using neutron radiation sources

We describe another application of the Discrete Tomography, wherein this system is used for non-destructive testing of pipelines transporting liquids and gases. Pipeline safety and reliability are of key importance for the nuclear- and petrochemical industries. One of the most important parameters in a pipeline to be monitored and measured is the wall thickness (7 mm, 9 mm, 11 mm és 13 mm). Only tomographic methods may provide inspections without the costly removal of insulation material during the operation of the plant. An additional advantage is that this technique can even be applied in high temperature environments.

First, we created geometric ring phantoms for simulating the cross-sections of the pipes with different wall thicknesses [3]. These ring phantoms also have corresponding holes.

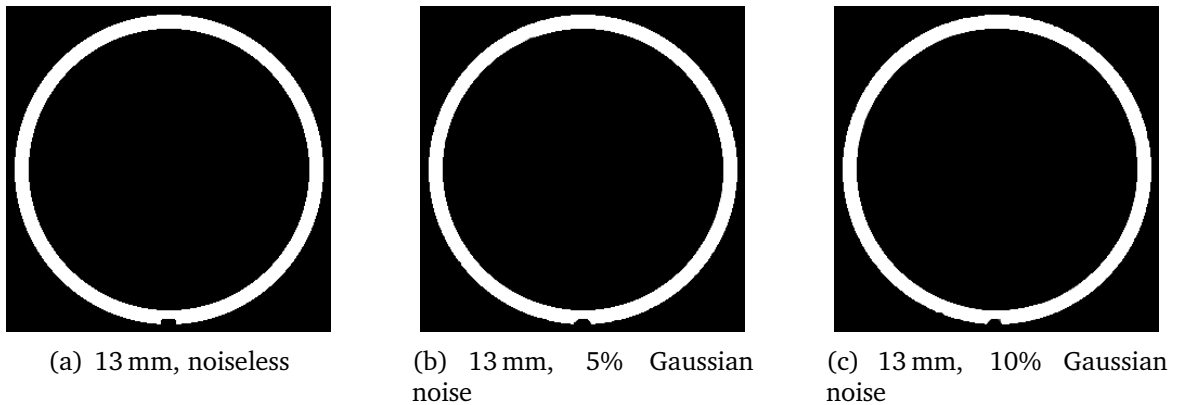
Fan-beam projections of the phantoms were then computed. We supposed that the attenuation in the inner volume of the pipe was so high that the rays were completely absorbed. That is, the projections along the straight lines intersecting the inner volume are 0. In this way only the rays passing through the wall of the pipe can be used for the reconstruction part. It also means as well that we have no information about the inner volume (Fig. 6), hence we

did not try to reconstruct it. The absorption coefficient of the insulation material is negligible when using neutron radiation source.



**Figure 6:** The geometry model of the experiment. Only the grey detector positions were used for the reconstruction.

We have reconstructed the wall of the pipe with corrosion defects using discrete tomography method (Fig. 7) during our experiments.



**Figure 7:** The reconstructed average images in case of 32 projections.

### 3 Emission Discrete Tomography System

Recently, a new kind of discrete tomography problem has been introduced [13]. This type of problems can be considered as the topic of the *emission discrete tomography*, shortly *EDT*,

connected to a kind of emission model. In this model the whole space is filled with some homogeneous absorbing material and the function to be reconstructed represents an object emitting radioactive rays into the surrounding space. Accordingly, the measurements in EDT are so-called *absorbed projections*. They depend on both the emitting object and the absorption.

The results of the chapter 3 has been published in [11, 12, 17] and article [4] will appear in a book chapter (accepted for publication).

### 3.1 EDT for a special absorption value

The reconstruction of binary matrices from their row and column sums is a basic problem in discrete tomography (DT). There are several theories, algorithms, and applications connected with this problem. As a collection of related papers see [6]. We investigate the same problem in EDT.

We measure absorption projections in EDT. The measured values are dependent not only on the radioactive material but depend on the absorption of the surrounding material also. Quantitatively, the detected activity emitted from a point of the object can be described as

$$I = I_0 \cdot e^{-\mu x} , \quad (12)$$

where  $I_0$  denotes the initial activity in the point of the object,  $I$  is the detected activity,  $\mu \geq 0$  denotes the absorption coefficient of the homogeneous material and  $x$  is the length of the path between the point and the detector.

Let  $\beta = e^{-\mu}$ . Let  $A = (a_{ij})_{m \times n}$  be a  $(0,1)$ -matrix (in other words: binary matrix) with size  $m \times n$ , i.e.,  $a_{ij} \in \{0, 1\}$  for  $i = 1, \dots, m$ ,  $j = 1, \dots, n$ . Its *absorbed row* and *column sum vectors*,  $R_\beta(A) = (r_1, \dots, r_m)$  and  $S_\beta(A) = (s_1, \dots, s_n)$ , respectively, are defined as

$$\begin{aligned} r_i &= \sum_{j=1}^n a_{ij} \beta^{-j}, & i &= 1, \dots, m , \\ s_j &= \sum_{i=1}^m a_{ij} \beta^{-i}, & j &= 1, \dots, n . \end{aligned} \quad (13)$$

**Definition 1.** We say that a binary vector has the *convexity property* if there is no 0 between two 1s in it.

**Definition 2.** We say that a binary matrix has the *h- and v-convexity* if the rows or columns vectors of the matrix have the convexity property.

**Definition 3.** We say that a binary matrix has the *hv-convexity* if the rows and columns vectors of the matrix have the convexity property.

Then consider the following reconstruction problem for *hv-convex* binary matrices from their absorbed row and column sums.

## RECONSTRUCTION *hvMA*

Given:  $m, n \in \mathbb{N}$  and  $R \in \mathbb{R}_0^m, S \in \mathbb{R}_0^n$  ( $\mathbb{R}_0$  denotes the set of non-negative real numbers).

Task: Construct an *hv-convex* binary matrix  $\mathbf{A}$  with size  $m \times n$  such that

$$R_\beta(\mathbf{A}) = R \quad \text{és} \quad S_\beta(\mathbf{A}) = S .$$

Let  $R$  and  $S$  be the absorbed row and column sums of the binary matrix  $A = (a_{ij})_{m \times n}$ . Then, using the terminology of numeration system [14], we can say on the base of (13) that the word  $a_{i1} \cdots a_{in}$  is a (finite) representation in base  $\beta$  of  $r_i$  or it is a (finite)  $\beta$ -representation of  $r_i$  for  $i = 1, \dots, m$ . Similarly,  $a_{1j} \cdots a_{mj}$  is a  $\beta$ -representation of  $s_j$  for  $j = 1, \dots, n$ .

Let henceforward

$$\beta = \frac{1 + \sqrt{5}}{2} ,$$

which is the golden ratio. It is easy to see that constant  $\beta$  has the following property.

$$\beta^{-1} = \beta^{-2} + \beta^{-3} . \quad (14)$$

The  $\beta$ -representation is not unique:

$$100 = 011 , \quad (15)$$

because  $1 \cdot \beta^{-1} + 0 \cdot \beta^{-2} + 0 \cdot \beta^{-3} = 0 \cdot \beta^{-1} + 1 \cdot \beta^{-2} + 1 \cdot \beta^{-3}$  on the base of (14).

It is proved in the Thesis that such a reconstruction problem can be solved in  $\mathcal{O}(m \times n)$  time from absorbed projections when the absorption is represented by  $\beta = (1 + \sqrt{5})/2$ . Also a reconstruction algorithm is given to determine the whole structure of *hv-convex* binary matrices from such projections.

## 3.2 EDT application for factor structures

First, consider the following problem. Let us suppose that there is a 3D dynamic object, which can be represented by a non-negative function  $f(r, t)$ , where  $r$  and  $t$  denote the position in space and time, respectively. Suppose that  $f$  can be expressed as a weighted composite of a set of (so far unknown) binary valued functions  $f_k(r)$ ,  $k = 1, 2, \dots, K$  ( $K \geq 1$ ) being constant in time, such that

$$f(r, t) = c_1(t) \cdot f_1(r) + c_2(t) \cdot f_2(r) + \cdots + c_K(t) \cdot f_K(r) + \eta(r, t), \quad (16)$$

where  $c_k(t)$  denote the  $k$ -th weighting coefficient, which depends on time, and  $\eta(r, t)$  represents the noise or residual in  $(r, t)$ . Given the assumption that  $\eta$  and  $f$  are uncorrelated,  $c_k(t)$  and  $f(r)$  are to be determined such that  $f_i$  are independent from  $f_j$  for all  $i \neq j$ . If the values of  $f(r, t)$  are available then the problem can be solved by factor analysis.

However, it can happen that we cannot measure the function  $f$  in the points of the space, but we can measure certain projections only. This is frequently the case, for example, in nuclear medicine, where the dynamic object is the radioactivity distribution in some human organ and the projections are gamma camera images from different directions. In this case SPECT imaging is applied to reconstruct the cross-sections of the object.

Let  $f(r, t)$  denote the intensity function of the object to be reconstructed. Suppose that the absorption in the space is constant, that is the absorption coefficient is  $\mu \geq 0$  everywhere. All half-lines in the space can be described as  $\ell(S, v) = \{S + u \cdot v \mid u \geq 0\}$ , where  $S$  and  $v$  are

the point and direction of the half-line, respectively. Then the projections of  $f$  in time  $t$  can be measured along  $\ell(S, v)$  half-lines by point detectors as follows

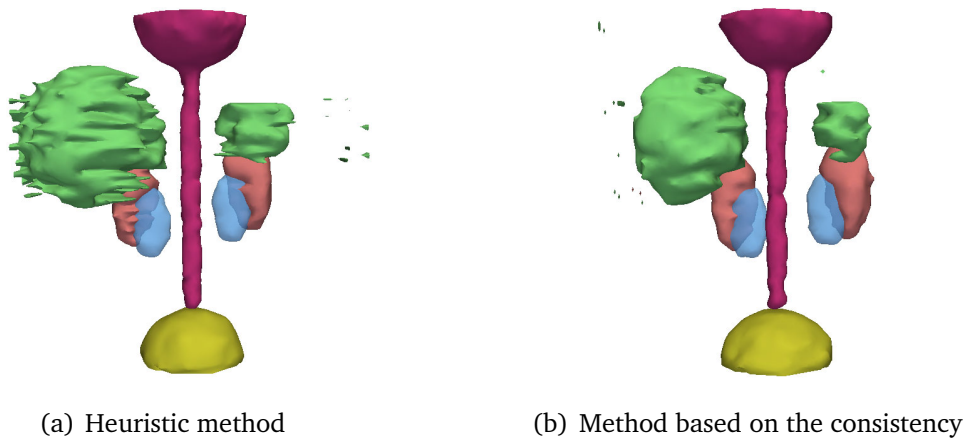
$$[\mathcal{P}^{(\mu)} f](S, v, t) = \int_0^{\infty} f(S + u \cdot v, t) \cdot e^{-\mu u} du . \quad (17)$$

Usually, the absorbed projection values are measured along many parallel half-lines simultaneously (e.g., by using line or plane detectors).

The method was tested on 3D phantom experiment. Our phantom (i.e., the function  $f$  in (16)) was a simplified 3D mathematical model of the human renal system (it was provided by Dr. Werner Backfrieder, AKH Vienna, Austria). Each simulated factor structure of the whole 3D object had specific dynamics (radioactivity changes with time) according to (16), so, their projections seemed to be separable from the projections of other structures by factor analysis. The factor analysis was performed on each sequence of projections by the method published in [21, 22] using spatial constraints (Dr. Martin Samal, Charles University Prague, Czech Republic).

The projection images cannot be considered as the absorbed projections of the factor. However, the absorbed projections of the factor structures can be computed from these images by suitable multiplications. Therefore, before using any kind of reconstruction method, we need to determine the multiplicative constants. We have given two methods to determine these intensity values. The first is a heuristic method and the second method based on the consistency condition derived for absorbed projections [24].

We have successfully reconstructed the binary matrices from the absorption projections after determination of the intensity values (Fig. 8).



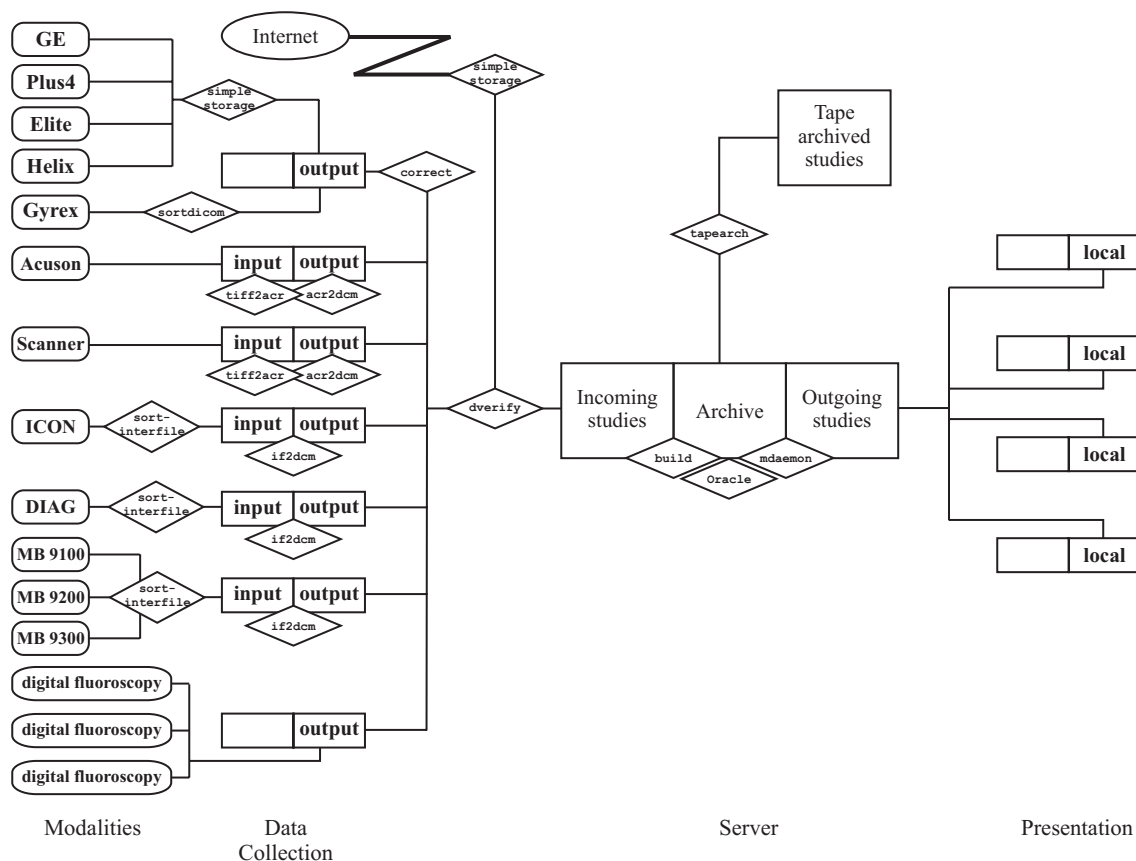
**Figure 8:** The reconstructed 3D structures in front view.

## 4 Picture Archiving and Communication System

SZOTE-PACS is a DICOM [23] based Picture Archiving and Communication System (PACS) developed at the Universities of Szeged.

The SZOTE-PACS development was started at the two Universities of Szeged in 1995. The development was supervised by Dr. László Csernay professor. The research and the development was sponsored by FEFA III. and FEFA IV. grants. Due to these came to fruition the first Hungarian Picture Archiving and Communication System at the Medical University in Szeged.

It is able to collect studies from different modalities and convert them into DICOM format. The DICOM studies can be edited, modified by RIS data, then verified and transferred to the archiving server. (The RIS is such a database which contains patient data as well as information for actuation of the radiology department). The archived studies can be presented and/or processed on the viewing workstation. There is a graphical application base on Oracle for searching and other database management functions of the Archive. The basic structure of SZOTE-PACS is shown on Fig. 9.



**Figure 9:** The structure of the SZOTE-PACS. The storage units of the workstations were nominated by squares. The main processes were signed with diamonds.

The modalities are connected to the university network mostly via converter stations. On the converter stations the studies are automatically converted into DICOM format and the verified studies transmitted to the server automatically or manually. The main part of the system is the central server that receives the incoming DICOM studies. The studies can be processed and presented on the viewing workstations. All stations are connected to the system via Network File System (NFS) or File Transfer Protocol (FTP) and, in certain cases, via DICOM protocol.

The result of the SZOTE-PACS have been published in [1, 2, 10, 18–20] proceedings articles.

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## 6 Contributions of the Thesis

Contributions in the first group described in Chapter 2 were published in journal articles [3, 9, 16]. Article [8] will appear in a book chapter (accepted for publication).

I/1. We have devised and implemented a Discrete Tomography System. This system is suitable to study the effects of the changing of the parameters of the cone-beam data acquisition geometry and the applied reconstruction method for noisy and noiseless case.

I/2. We have carried out experiments by varying the parameters of the cone-beam data acquisition geometry in case of using line and strip integrals. According to the results of the experiments there is no substantive difference between the cone-beam and the parallel beam data acquisition geometry using the given DT reconstruction method on the studied phantoms. We found that there is no major difference when line and strip integrals were used during the reconstruction in the case of the same phantoms.

We have examined the effect of changing of the applied reconstruction algorithm for different regularization terms in case of cone-beam geometry. It was an important result that further quality improvement can be achieved by using prototype ( $\Psi_{\text{poz}}$ ) and regularization terms preferring large coherent areas ( $\Psi_{\text{sm}}$ ) in the case of noisy projections of the given phantoms.

I/3. We have studied the behavior of the implemented Discrete Tomography System against the physical distortions in the measured projection data in the collaboration with “Corporate Technology PS 9, Siemens AG” in Munich. We have successfully applied the DT method on simulated and real data.

We have given a possible discrete tomography reconstruction method for testing corrosion in pipe lines using cone-beam data acquisition geometry (International Atomic Energy Agency research HUN-12109 at KFKI Atomic Energy Research Institute). The given software phantoms were successfully reconstructed from a limited amount of available information using the implemented DT method.

Contributions in the second group described in Chapter 3 and were published in journal articles [11, 12, 17]. Article [4] will appear in book chapter (accepted for publication).

- II/1. It is proved that the reconstruction problem in the class of the  $h\nu$  convex binary matrices of size  $m \times n$  can be solved in  $\mathcal{O}(m \times n)$  time from their absorbed projections when the absorption is represented by the coefficient  $\mu = \log((1 + \sqrt{5})/2)$ . Also a reconstruction algorithm was given.
- II/2. We have given two methods for the determination of the intensity value of a non-binary two valued object from the absorbed projections. These methods were applied on separated projections of a 3D mathematical model (Dr. Werner Backfrieder, AKH Vienna, Austria), (Dr. Martin Samal, Charles University Prague, Czech Republic).
- II/3. We have successfully reconstructed 3D structures by emission discrete tomography method from 4 corrected absorbed projections separated by factor analysis. The volumes of the reconstructed structures are close to the original ones when the projections were corrected with the intensity values determined in two different ways.

Contributions in the third group described in Chapter 4 were published in proceedings articles [1, 2, 10, 18–20].

- III. We have developed a picture archiving and communication system (PACS) which is able to store digital medical images in a central database in DICOM format. The PACS users are able to search and download the results of the search from the database onto their workstations. During the implementation we had to solve the following problems:
- (a) converting studies given in different formats into a common format (DICOM),
  - (b) making connection to the existing Radiology Information System (RIS),
  - (c) providing controlled automated processes for the PACS users,
  - (d) correcting the wrong DICOM studies which do not comply with the DICOM standard,
  - (e) providing long term tape archiving.

This system was the first such system in Hungary, and it had been used successfully in clinical environment for 10 years.

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A. Nagy and A. Kuba. Reconstruction of binary matrices from fan-beam projections. *Acta Cybernetica*, 17(2):359–385, 2005.

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## Full papers in conference proceedings

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<sup>1</sup>Science Citation Index, Impact factor: 1.166

<sup>2</sup>Science Citation Index, Impact factor: 0.503

<sup>3</sup>1996, XX. Neumann kollokvium, Competition of the Young researchers, Veszprém (II. prize)

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