MOBILE ATOMS IN A CAVITY FIELD: STATISTICAL AND QUANTUM ASPECTS

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न वा अरे पत्युः कामाय पतिः प्रियो भवत्यात्मनस्तु कामाय पतिः प्रियो भवति ।
न वा अरे जायायै कामाय जाया प्रिया भवत्यात्मनस्तु कामाय जाया प्रिया भवति ।
न वा अरे पुत्राणां कामाय पुत्राः प्रिया भवन्त्यात्मनस्तु कामाय पुत्राः प्रिया भवन्ति ।
न वा अरे वित्तस्य कामाय वित्तं प्रियं भवत्यात्मनस्तु कामाय वित्तं प्रियं भवति ।
न वा अरे ब्रह्मणः कामाय ब्रह्म प्रियं भवत्यात्मनस्तु कामाय ब्रह्म प्रियं भवति ।
न वा अरे क्षत्रस्य कामाय क्षत्रं प्रियं भवत्यात्मनस्तु कामाय क्षत्रं प्रियं भवति ।
न वा अरे लोकानां कामाय लोकाः प्रिया भवन्त्यात्मनस्तु कामाय लोकाः प्रिया भवन्ति ।
न वा अरे देवानां कामाय देवाः प्रिया भवन्त्यात्मनस्तु कामाय देवाः प्रिया भवन्ति ।
न वा अरे भूतानां कामाय भूतानि प्रियाणि भवन्त्यात्मनस्तु कामाय भूतानि प्रियाणि भवन्ति ।
न वा अरे सर्वस्य कामाय सर्वं प्रियं भवत्यात्मनस्तु कामाय सर्वं प्रियं भवत्य्-
आत्मा वा अरे द्रष्टव्यः श्रोतव्यो मन्तव्यो निदिध्यासितव्यो ।
मैत्रेय्यात्मनो वा अरे दर्शनेन श्रवणेन मत्या विज्ञानेनेदꣳ सर्वं विदितम् ॥

ब्रह्म तं परादाद्योऽन्यत्रात्मनो ब्रह्म वेद ।
क्षत्रं तं परादाद्योऽन्यत्रात्मनः क्षत्रं वेद ।
लोकास्तं परादुर्योऽन्यत्रात्मनो लोकान्वेद ।
देवास्तं परादुर्योऽन्यत्रात्मनो देवान्वेद ।
भूतानि तं परादुर्योऽन्यत्रात्मनो भूतानि वेद ।
सर्वं तं परादाद्योऽन्यत्रात्मनः सर्वं वेदेदं ब्रह्मेदं क्षत्रम् ।
इमे लोका इमे देवा इमानि भूतानीदꣳ सर्वं यदयमात्मा ॥

1 Preliminaries and Motivations of the Work

1.1 Optical Cavity QED

The past decade saw an amazing development in the field of optical resonators and that of cold atoms. Recently, by the fusion of these two fields, a new microscopic system has been created: a tiny optical resonator (cavity) embedding an atom, which is trapped for a long time and is strongly coupled to the electromagnetic field of the resonator. Here the light-matter coupling allows us to study the fundamental features of interacting systems in general. The atom-cavity interaction can be tuned very precisely, and the system eventually realizes a controllable coupled dissipative quantum dynamics on a quite generic level.

Previously, when dealing with *light-matter* interaction, one of the components could be utterly simplified to render it a parameter in the dynamics of the other component. On one hand there is optics, where *matter* is used to manipulate *light*: *matter* is put in parametric form (refractive index) into the Maxwell equations to describe the propagation of *light*. On the other hand, there is also a plethora of examples for the complementer case when *light* is used to manipulate atoms. When treating these phenomena, the amplitude of the *light* field is given as a prescribed function in the Newton or Schrödinger equation governing the behaviour of *matter*.

In a cavity, however, light-matter interaction is realized on a more general level since both the light and the matter components are dynamic. Cavity QED (CQED), though at first sight only a very special case in quantum optics, can therefore also be looked on as a generalisation of both optics, and matter manipulation by light, because of the introduction of a new dynamical element. These latter can in principle be obtained as certain limiting cases of CQED, cases when either the dynamics of the atom or that of the field can be omitted.

The origins of CQED can be traced back to the 1940s, when it was discovered that the radiation properties of an atom are determined not only by its electronic structure, but also by the mode density of the surrounding electromagnetic field. This latter can be modified by boundary conditions, which allows us to manipulate the radiation properties of atoms. Indeed, in the 1980s it has been demonstrated experimentally that for excited atoms traversing a cavity the excited state's lifetime differs from the one measured in free-space.

In these experiments the cavity was present as a passive element merely to tailor the mode density of the electromagnetic field surrounding the atom. The dynamics of the cavity field was irrelevant. It must be accounted for only in the regime of strong atom-field coupling. The parameter describing the atom-cavity coupling is the single-photon Rabi frequency: This is the frequency of the oscillation between atomic and cavity excitation (Rabi oscillation) when there is only one excitation quantum (atomic excitation or photon — hence the name "single-photon" Rabi frequency) in the system. Let us overview the characteristic frequencies of the system:

- single-photon Rabi frequency g,
- spontaneous emission rate γ,
- cavity decay rate κ,
- inverse of the atom-cavity interaction time (atomic flight time through the cavity).

Strong coupling is achieved when the first one dominates all the rest. This clearly means that the atom and the cavity exchange the excitation several times before it is dissipated into the environment via either dissipation channels (cavity decay or spontaneous emission) or the atom leaves the cavity. In a sense, the identities of atom and field are lost and we are left with a single new object.

Strong coupling was first achieved in the micro-wave regime in the 1990s. With this system a series of very fundamental quantum mechanical experiments were performed such as the direct proof of field quantisation and decoherence of quantum superpositions. Reaching strong coupling in the optical domain is a much more challenging task, since due to the short wavelength very short cavity length (10μm – 100μm) is needed. Since the cavity length is short, photons are reflected more often, therefore to obtain small κ extremely good mirrors are needed (with transmission coefficient $\lesssim 10^{-5}$).

Hence, strong coupling can be interpreted as one single photon making several round trips in the cavity, each time impinging on the atom. An interesting consequence is that the optical resolution, which in free space equals the half of the wavelength is improved by a factor of the square root of the number of round trips. This has lead to the creation of the so called atom microscope: the trajectory of atoms in a cavity can be reconstructed from the time-resolved analysis of the light intensity escaping the cavity.

Entering the optical domain yields a substantial difference as compared to the micro-wave regime since the momentum of optical photons is big enough to significantly act on the atomic centre-of-mass, making it a part of the dynamics. In an optical resonator even a very weak field, in fact, a single photon agitates the atom considerably. Hence, in optical CQED we have three degrees of freedom: the atomic internal one (electronic configuration), the atomic CM motion, and the state of the cavity-field.

When an optical field impinges on a fixed atom, two types of forces emerge:

Dipole force originates from photon absorption from the field followed by *stimulated* emission into the populated laser mode.

Radiation pressure arises from photon absorption from the field followed by *spontaneous* emission into a vacuum mode.

Their duality on the microscopic level clearly corresponds to the duality of the real part (phase shift) and imaginary part (absorption) of a macroscopic dielectric's complex refraction index. The dipole force is conservative and can be rendered by an optical potential. The radiation pressure is dissipative: the stochasticity of the emitted photon's direction yields momentum diffusion via atomic recoil (recoil diffusion).

1.2 Laser Cooling

When the atom moves, light forces depend not only on its position, but also on its velocity. In the simplest case this dependence is via the Doppler effect. Suppose a counter-propagating pair of laser beams red-detuned from the atomic resonance impinge on the atom. In this case a moving atom is more likely to absorb a photon with momentum opposite to its direction of motion since the counter-propagating mode is Doppler shifted nearer to resonance. The spontaneous re-emission of the photon, on the other hand, is isotropic and yields no momentum transfer on average. The momentum of the atom is therefore damped on average, which means cooling for an ensemble of atoms. This simplest method of laser cooling is called Doppler cooling. When applied in three dimensions, it resembles the atom moving in a viscous medium, hence the field of three pairwise orthogonal counter-propagating pair of red-detuned laser beams is often called "optical molasses". Similarly to classical Brownian motion, the final temperature is determined by the contest of the above described friction and recoil-induced diffusion, the lowest achievable temperature scaling with the spontaneous emission rate.

More elaborate laser cooling schemes include polarisation-gradient cooling, where the limiting temperature is determined by the stochasticity of the last spontaneously emitted photon, so that it scales with the recoil frequency. For certain atomic transitions even this recoil limit can be penetrated eg by velocity-selective coherent population trapping. Here the temperature is already in the nano-Kelvin regime. Preparation of ultracold atoms and molecules has allowed for the study of many phenomena stemming from the quantum nature of matter (just think of eg atomic Bose-Einstein condensates), and has already yielded many applications such as atomic clocks, atomic interferometers, and lithography. Quantum objects absent from thermal noise and prepared in their ground state are vital for quantum information processing. Eg with laser-cooled ions in ion traps, quantum teleportation and multi-bit quantum operations have been performed.

In the above cooling schemes spontaneous emission is of paramount importance. At first sight it may seem that it merely heats the atoms via the recoil noise, and it may not be as transparent that this alone is responsible for cooling. Indeed: cooling is irreversible damping of the kinetic energy, and a dissipative channel must be inherent. In the laser-based cooling schemes the only irreversible process is light scattering from the laser into surrounding vacuum modes, that is, spontaneous emission.

1.3 Cavity Cooling

Recognition of the central role of spontaneous emission calls the Purcell effect into our mind, that is, the possibility of modifying spontaneous emission by a cavity. This must have some impact on laser cooling properties. Based on the prohibition or increase of spontaneous emission on certain frequencies many cooling methods have indeed been developed. In a general form it has been pointed out that in inelastic scattering processes — that is, when the scattered photon's frequency differs from the incoming one due to atomic recoil — the resonator as a spectral filter prefers converting the photon frequency upwards. The necessary energy must be taken from the atomic kinetic energy. Let us emphasise again that in this approach the resonator is a passive element to tailor the mode density of the surrounding vacuum field.

Laser cooling methods based on spontaneous emission have a very serious common barrier: Since a single spontaneously emitted photon carries away only very little energy, a closed optical cycle is necessary to be able to repeat the scattering process. Now in general an excited atom can end up in many different final states after spontaneous emission. For a special class of atoms by applying repumping lasers for the incidental dark states it can be achieved that the atoms remain in a closed subspace. However, there is no mean to develop a general method to cool arbitrary atoms or molecules. For the latter, the situation is even more complicated because the rotational and vibrational states are so abundant that they form an almost continuous band in which the population is spread after a few scattering cycles. In fact, cooling of molecules by optical means is an unsolved problem.

Partly there lies the significance of dynamical cavity cooling. The effect is an important motivation for the present work as well. The name "dynamical" refers to that here the role of the cavity is more than the passive role it plays in the Purcell effect. It introduces a new dynamical element to the system and with it a new dissipation channel — the cavity field and the cavity loss channel. In the strong coupling regime, the subsystems share all the available dissipation channels: The atomic kinetic energy can transform into the cavity field's energy and leak into the environment via cavity loss. As a result it can be achieved that the limiting temperature scale with the cavity decay rate, instead of the spontaneous emission rate as in Doppler cooling. The former can be much lower than the latter for a good cavity.

Ultimately, spontaneous emission is not even necessary, all the cooling relies on the cavity loss channel, which makes that the mechanism is free from the limitations mentioned above for the spontaneous-emission based methods. Indeed, the method is in principle applicable to general polarisable particles, ie even to molecules. Note that dynamical cavity cooling outlines a very general cooling concept: to the object to be cooled we couple another object and a new dissipation channel via this one.

1.4 Our Work

The work during our PhD years starting in 2003 was a logical continuation of what we did before for our Master's Thesis. This means that we have been working in the field for about four years.

The starting point was to study some new aspects of the dynamical cavity cooling phenomenon, and more generally maybe to apply statistical physics to optical CQED systems in a stricter sense than it had been done before.

In the first place these new aspects consisted of entering the transition regimes between classical and quantum descriptions for either the atomic motion or the cavity field or both. These turned out to be equivalent with transitions between dynamical and non-dynamical fields, so that we could really study the advantages of making the atom interact with a cavity rather than a fixed free field.

An important motivation was also to identify regimes where there are possibilities for *trapping* by the cavity field in all three dimensions and study whether this, together with the cooling allows for the Bose-Einstein condensation of a gas of independent atoms. Hence the need has naturally arisen for a fully quantum description of optical CQED systems, which we have fulfilled during the work. Note that Bose-Einstein condensation fully by optical means has as yet never been achieved because the reabsorption of spontaneously scattered photons inside the sample makes it impossible to achieve the phase-space density necessary for condensation. Cavity cooling is a good candidate for all-optical condensation because here spontaneous scattering is not necessary for cooling.

Another fairly new aspect is the study of such cases when many atoms interact with the same cavity field. In this case the problem is inherently a many-body one, since even if not interacting directly, the atoms still interact, and strongly, via the cavity field. We have realized that this interaction is actually so strong that it cannot be accounted for by any kind of perturbative treatment. A convincing proof of this is that this alone yields a pattern-formation phenomenon, the so called self-organisation, which, when applying strict formalism of statistical physics has been proved to be a dynamical phase transition. The question has naturally arisen whether in the case when the cavity field and atomic motion are both quantum, and the field can yield a quantum feedback on the atomic motion, the self-organisation has a quantum phase transition counterpart. On this problem we gave an outlook in the work, but as yet it has not been sufficiently worked out to appear in the Theses. This will be a major direction of our future work.

The methods invoked are mostly standard quantum optical ones such as the Lindblad formalism and the Monte Carlo wave-function method adapted for the atom-cavity system. In the latter method heavy numerical optimisation and calculations are inherent. In the many-atom case we have applied field theoretical many-body description.

2 Results in the form of Theses

Thesis I. [i] Based on the Monte Carlo Wave Function method we have created a simulation to study one atom moving in a single-mode cavity field with the atomic internal degree of freedom eliminated. It takes into account the whole coupled atom-field dynamics on a fully quantum mechanical basis. Exploiting the special form of the potentials, the whole evolution of the wave function could be implemented in momentum space with adaptive time step which is a huge gain both in terms of CPU time and accuracy as compared to conventional methods for solving Schrödinger equations. We are able to calculate and numerically treat the full joint atom-field steady-state density operator to decipher quantum properties of the system pertaining to coherence, entanglement, and nonclassicality.

Thesis II. [i] We have shown that the regime where the cavity decay rate is smaller than the spontaneous decay rate by about an order of magnitude is the limit where quantum effects such as atom-field entanglement and finite coherence length of the atomic wave packet commence to play a role in the dynamics. This quantum-classical transition regime is optimal for cavity cooling and trapping since here besides the fairly low temperature, localisation and trapping time is maximal while cooling time is minimal. In the quantum field regime we have shown that even when there is on average only one photon in the cavity, it still fully exhibits the cavity cooling effect. We have found good agreement between the results of former semiclassical simulations and MCWF ones in the regime of bigger cavity decay rate.

Thesis III. [i] We have shown that if an atom is trapped by a quantum field, the trapping time differs significantly from the one measured for a classical field of the same intensity. The atom can escape via the zero photon component of the field which yields no potential. The effect can serve as a proof for testing the graininess of the field.

Thesis IV. [ii] We proved that in the standard dynamical cavity cooling scheme the cooling force at a fixed rate of spontaneous photon scattering does not vanish in the regime of very far detuning associated with large optical potential depths. This is at sharp variance with standard Doppler cooling. From another point of view, the very popular far-off-resonance dipole trap scheme was generalised and its efficiency was shown to be greatly enhanced, embedding a cooling mechanism, if the field is enclosed in a cavity. The embedded cooling mechanism provides for trapping in steady state, which means that in an experiment trapping would be limited only by technical noise.

Thesis V. [i, ii] We have demonstrated cavity cooling and trapping with a model in which there is no reference to the internal structure of the particle to be cooled. Hence, cavity cooling is shown to be applicable for general linearly polarisable particles, in particular, for molecules.

Thesis VI. [iii] We have shown that if an atom moves in a monochromatic laser field and is strongly coupled to a mode with weak damping — such as a high-Q cavity mode — nearly resonant with the field then the velocity-dependence of its polarisation deviates substantially from the standard Doppler shift. This "anomalous" Doppler shift is proportional to the square of the ratio of coupling constant and decay rate. Depending on the parameters the Doppler-shift can even change sign: in this case a field counter-propagating to the motion is felt as red-shifted by the atom.

Thesis VII. [iii, iv] We have shown that if in the standard Doppler-cooling scheme a resonant object is coupled to the atom then the efficiency of cooling both in terms of limiting temperature and cooling time can be greatly enhanced. In addition, long-time trapping is achieved up to the range of seconds. The effect is based on the anomalous Doppler effect, which at proper but general enough parameters is an enhancement to the standard Doppler effect. An alternative explanation on a higher level of understanding is in terms of a peculiar resonance (polariton) of the coupled atom-cavity system, which in the strongly coupled case is very different from the simple Lorentzian resonance used for Doppler cooling.

Thesis VIII. [v] We have shown that the atom-cavity system in the case of atom pumping realizes a quantum seesaw with the cavity field standing for the seesaw, that is, a system capable of leaving its unstable equilibrium point via entanglement and without any noise. The effect has implications on phenomena in which spontaneous symmetry breaking is inherent, such as the self-organisation of ultra-cold atoms in a cavity field.

3 Publications related to the Theses

[i] A. Vukics, J. Janszky, and P. Domokos. Cavity cooling of atoms: a quantum statistical treatment. *J. Phys. B: At. Mol. Opt. Phys.*, 38:1453, 2005.

[ii] A. Vukics and P. Domokos. Simultaneous cooling and trapping of atoms by a single cavity-field mode. *Phys. Rev. A*, 72:031401, 2005.

[iii] P. Domokos, A. Vukics, and H. Ritsch. Anomalous Doppler effect and polariton-mediated cooling of two-level atoms. *Phys. Rev. Lett.*, 92:103601, 2004.

[iv] A. Vukics, P. Domokos, and H. Ritsch. Multidimensional and interference effects in atom trapping by a cavity field. *J. Opt. B: Quant. Semiclass. Opt.*, 6:143, 2004.

[v] C. Maschler, H. Ritsch, A. Vukics, and P. Domokos. Entanglement driven self-organization via a quantum seesaw mechanism. quant-ph/0512101, 2005.