Non-equilibrium dynamics of one-dimensional isolated quantum systems

Ph.D. Thesis statements

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Chapter 1

Introduction

The dynamics of the isolated quantum systems after the change of an external parameter is a field of active research [1–4]. One limiting case according to the speed of the variation of the external parameter is the *quench dynamics*, when the parameter is changed suddenly. In experiments the sudden change of the Hamiltonian can be realized using the phenomenon of the Feshbach resonance [1–4]. According to quench dynamics, one might be interested in the functional form of the expectation value of the physical quantities short times after the quench, or one might be interested in the asymptotic values very long times after the quench. [5, 6, 18–39]. The other limiting case (according to the speed of the variation of the external parameter) is the nearly adiabatic dynamics, also called to Kibble-Zurek process, when the external parameter is varied very slowly. The external parameter is usually varied linearly with time (as $\sim 1/\tau \times t$), and the system crosses a phase transition during the dynamics. At the beginning of the process the system is in the ground state of the instantaneous Hamiltonian. If the speed of the process $(1/\tau)$ would be much smaller than the smallest gap during the whole dynamics, the system would stay exponentially close to the instantaneous ground state, according to the adiabatic theorem. However, the smallest gap goes to zero as the system approach to the critical point, and the dynamics can not be slow enough.
The question, how far the system is from the instantaneous ground state at the end of the process (after crossing the critical point) was target of intensive studies [32, 38, 40–42, 44]. Kibble [40] and Zurek [41] developed a heuristic picture which connects the distance from the instantaneous ground state (also called defect density) with the speed of the process $1/\tau$ using the critical exponents of the phase transition: $P(\tau) \sim$ (density of deffects) $\sim \frac{1}{\xi^d} \sim \tau^{-\frac{d\nu}{\nu z+1}}$, where d is the dimension of the system, ν is the correlation-length exponent, z is the dynamical exponent.
Among the quench dynamics and Kibble-Zurek process other processes connecting to different time dependence of the external parameter were investigated in the literature, for example, periodic driven systems, citebordia [46], ans systems in randomly fluctuating environment [47].
There is a lot of articles about the quench dynamics of the homogeneous systems [5, 7–18, 20, 32, 33, 37]. One of the most notable results is the quasi-classical description of the quench dynamics. At the time of the quench, quasi-particle pairs with opposite momentum starts from every site of the model. After the quench the quasi particle moves ballisticsally (with constant velocity), and reflects from the free ends of the system [23, 33, 38]. . With the quasi classical description asymptotically exact results can be obtained for homogeneous systems. The dynamics of the inhomogeneous systems was only investigated in a few special case: the entanglement entropy in disordered spin chains [48–50], and in the models of the many-body localizationn [51] [52]. A special type of inhomogeneity id the quasi-periodic order, which means a non-periodic but deterministic system [53–55]. The quasi-periodic systems have unusual dynamical properties, for example the spreading of the wave packet in not "ballistic" as in the homogeneous systems, but follows an anomalous diffusion [56, 57].
The dissertation is based on the following four articles:

1. Ferenc Iglói, Gergő Roósz, Yu-Cheng Lin *Nonequilibrium quench dynamics in quantum quasicrystals* New J. Phys. 15, 023036 (2013)

2. Ferenc Iglói, Gergő Roósz, Loïc Turban *Evolution of the magnetization after a local quench in the critical transverse-field Ising chain* J. Stat. Mech. (2014) P03023

3. Gergő Roósz., Uma Divakaran, Heiko Rieger, Ferenc Iglói *Non-equilibrium quantum relaxation across a localization-delocalization transition* Phys. Rev. B 90, 184202 (2014)

4. Gergő Roósz., Yu-Cheng Lin, Ferenc Iglói *Critical quench dynamics of random quantum spin chains: Ultra-slow relaxation from initial order and delayed ordering from initial disorder* New J. Phys. 19, 023055 (2017)

Chapter 2

Methods

2.1 Local quench

In this Section I summarize the results of [70]. In the literature the usually the global quenches are investigated, for example instantaneous turn on of a homogeneous magnetic field. On other interesting phenomenon is the local quench, when the Hamiltonian is changed instantaneously in the neighborhood of a given site. In experiments the adsorption of X-rays in metals realize local quenches [62].

The most of the theoretical studies consider one-dimensional critical systems, for which conformal description can be applied. The conformal field theory describes the continuous limit of the system, in this limit the system lives in a two dimensional space-time (x,t). In this Section we investigate a critical transverse field Ising chain. The parameters of a (generalized) defect site are changed instantaneously at time 0. Here "generalized" means that a coupling and the two neighboring magnetic fields differs from the critical value. The Hamiltonian of the investigated system is:

$$\mathcal{H}_i = -\frac{1}{2}\left[\sum_{n=1}^{L-1}\sigma_n^x\sigma_{n+1}^x + (J_i-1)\sigma_{L/2}^x\sigma_{L/2+1}^x + \sum_{n=1}^{L}\sigma_n^z + (h_{i1}-1)\sigma_{L/2}^z + (h_{i2}-1)\sigma_{L/2+1}^z\right], \tag{2.1}$$

where σ_n^x, σ_n^y, σ_n^z are the Pauli matrices.

The defect site is at the middle of the chain. Before the quench $t<0$ the coupling at the defect site is J_1, the left-neighbor magnetic field is h_{11}, the right-neighbor magnetic field is h_{12}. After the quench the parameters of the defect site are h_{11}, h_{12}, J_2. We investigate the time dependence of the local magnetization at the defect site. The local magnetization can be calculated as the matrix element of the $\sigma_n^x(t)$ operator, (which is in the Heisenberg picture), between the ground state and the first excited state of the initial Hamiltonian [64] $m_n(t) = \langle\Phi_0|\sigma_n^x(t)|\Phi_1\rangle$ where n is the site where the local magnetization is measured. The local magnetization on the defect site is: $m_d(t) = m_{n=L/2}(t)$.

The system can considered as the transfer matrix of a two dimensional classical spin system. With this identification the dynamics of the magnetization can be calculated for imaginary times in the thermodynamic ($L\to\infty$) limit. The results (for the imaginary time dynamic) can be summarized as follows: The dynamic is the function of the $\kappa_i = \frac{J_i}{h_{i1}\,h_{i2}}$ $\quad(i=1,2)$ effective interactions, and the magnetization follows a power law:

$$m_d(\tau) \sim \tau^{x_{12}-x_2}\,, \quad 0<\tau\ll L\,, \tag{2.2}$$

where the exponents are: $x_i = \frac{\sqrt{2}}{\pi}\arctan(\frac{1}{\kappa_i})$ and $x_{12} = \sqrt{x_1 x_2}$.

For the real time dynamics there are results in the literature for special cases: $h_{11}=h_{12}=h_{21}=h_{22}=1$,and $J_1=\infty$ (the boundary spins are fixed initially) or $J_1=1$ and $J_2=0$ (cutting a homogeneous chain to two halves).

For the fixed spin initial condition [63] the conform result is:

$$m_d^{(+)}(t) \sim t^{-2x_m} \quad 0<t\ll L\,. \tag{2.3}$$

which was tested with numerical simulations in the transverse field Ising model [58]. In a finite chain with open boundary conditions [58] the time dependence of the magnetization is:

$$m_d^{(+)}(t,L) \sim \left[L\sin\left(\pi\frac{t}{L}\right)\right]^{-2x_m}, \quad 0<t<L\,. \tag{2.4}$$

For the case when a homogeneous chain is cut to two parts the magnetization behaves as [58]:

$$m_d^{(fb)}(t,L) \sim L^{-1/2}\left[L\sin\left(\pi\frac{t}{L}\right)\right]^{1/4}, \quad 0<t<L\,. \tag{2.5}$$

which takes the following form for the $t \ll L$ limit:

$$m_d^{(fb)}(t) \sim m_0(L)\, t^{1/4}, \quad 0 \leq t \ll L\,, \tag{2.6}$$

where $m_0(L) \sim L^{-x_{ms}}$ is the equilibrium value of the magnetization in the initial state.

The exponent in equation (2.6) equals with $1/4 = 2(x_{ms}/2 - x_m)$, where $x_2 = x_m$ and $x_{12} = x_{ms}/2$. For the fixed spin initial condition (equation (2.4)) the same holds with $x_{12} = 0$. If t^2 is substituted for the imaginary time in equation (2.2), one got the real time behavior (equations (2.4) and (2.6)). One arrives to the following conjecture:

$$m_d(t) \sim m_0(L)\, t^{2(x_{12}-x_2)}, \quad 0 < t \ll L\,, \tag{2.7}$$

where $m_0(L) \sim L^{-x_1}$. In a finite system with open boundary condition the magnetization varies periodically:

$$m_d(t, L) \sim L^{-x_1} \left[L \sin\left(\pi \frac{t}{L} \right) \right]^{2(x_{12}-x_2)}, \; 0 < t < L\,. \tag{2.8}$$

I validated the conjectures with precise numerical simulations.

2.2 Dynamics of the Finonacci Ising quasi-crystal

In this section, I summarize the results about the dynamics of the Fibonacci Ising quasi crystal. The Hamiltonian defining the model is the following:

$$\mathcal{H} = -\frac{1}{2} \left[\sum_i J_i \sigma_i^x \sigma_{i+1}^x + h \sum_i \sigma_i^z \right] , \tag{2.9}$$

(σ_i^x, σ_i^z are the Pauli matrices at site i.) The J_i couplings are site dependent, and parameterized as follows:

$$J_i = J r^{f_i} , \tag{2.10}$$

here $r > 0$ characterize the strength of the inhomogeneity, $r = 1$ corresponds to the homogeneous system and the smaller r corresponds to the stronger inhomogeneity. The f_i numbers are 0 or 1, and forms the quasi-periodic Fibonacci sequence.

The J coupling strength in the (2.10) equation is $J = r^{-\rho}$, where

$$\rho = \lim_{L\to\infty} \frac{\sum_{i=1}^{L} f_i}{L} = 1 - \frac{1}{\omega} , \tag{2.11}$$

is the proportion of the 1s in the Fibonacci sequence (in the $L \to \infty$ limit).

The Fibonacci sequence is defined by the following expression:

$$f_i = 1 + \left[\frac{i}{\omega} \right] - \left[\frac{i+1}{\omega} \right] , \tag{2.12}$$

where $[x]$ is the (lower) integer part of x and $\omega = (\sqrt{5}+1)/2$. With the above definition the critical point is at $h = 1$, $h < 1$ is the ferromagnetic phase, and $h > 1$ is the paramagnetic phase. The Fibonacci sequence is an irrelevant perturbation according to the Harris-Luck criteria [71].
I investigated a global quench, I changed the value of the transverse field instantaneously. Before the quench the magnetic field is h_0, after the quench the magnetic field is h. We investigated the time dependence of the local magnetization, the entanglement entropy, and the so-called wave packet.

The definition of the entanglement entropy is the following: The system is in a pure state $|\Psi\rangle$, which can be described with the density matrix $\rho = |\Psi\rangle\langle\Psi|$. The system is split spatially to two parts A and B. The reduced density matrices are defined as: $\rho_A = \mathrm{Tr}_B \rho$ and $\rho_B = \mathrm{Tr}_A \rho$. The entanglement entropy is the von Neumann entropy of the reduced density matrices: $S = -\mathrm{Tr}_B \rho_B \ln \rho_B = -\mathrm{Tr}_A \rho_A \ln \rho_A$.

With numerical simulations we obtained the following results: The magnetization follows a stretched exponential after the quench:

$$m_{\mathrm{b}}(t) \sim A(t) \exp\left(-C t^{\mu}\right) . \tag{2.13}$$

where the $A(t)$ prefactor is $\mathcal{O}(1)$. Two different regions can be distinguished in the behavior of the $A(t)$ prefactor: if $h < h^*(r)$ then $A(t) > 0$ and if $h < h^*(r)$ then $A(t)$ oscillates. The $h^*(r)$ dynamical phase transition point is power law of the parameter r: $h^*(r) \sim r^{\omega}$, where $\omega \approx 0.24$. The magnetization oscillates if the neighborhood of the investigated spin is locally paramagnetic, and remains positive if the neighborhood is locally ferromagnetic. The entanglement entropy grows with a power function of the time:

$$\mathcal{S}(t) \sim t^{\sigma} , \tag{2.14}$$

The σ exponent approximately equals with the μ exponent, and with the spreading exponent of the wave packet, I interpreted this behavior with a quasi-classical reasoning. If one suppose that a quasi-classical description holds in a similar form as in the homogeneous Ising chain, but the quasi-particles follow an anomalous diffusion (and not a ballistic motion), then the diffusion exponent naturally appear in the expressions of the entanglement entropy and the magnetization.

2.3 Non-equilibrium dynamics of the Harper-modell

The Harper modell is defined by the following Hamiltonian:

$$\mathcal{H} = -\frac{1}{4}\sum_{n=1}^{L}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y) - \sum_{n=1}^{L} h_n\sigma_n^z \ . \tag{2.15}$$

where h_n is a quasi-periodic potential:

$$h_n = h\cos(2\pi\beta n) \ , \tag{2.16}$$

where $\beta = \frac{\sqrt{5}-1}{2}$ is the inverse of the golden mean. In the Harper model there is a localization-delocalization phase transition [59], for $|h| < 1$ the eigenstates are extended, the spectra is absolutely continuous, for $|h| > 1$ the eigenstate are localized, and the spectra is point-spectra. At hte critical point ($|h| = 1$) the spektra is fractal-like, singular-continuous [61]. The quench here means the sudden change of the h amplitude from the pre-quench value h_0 to the after quench value h. I organize the results according the after quench parameter h.

If the quench ends in the extended phase the dynamic is similar to the dynamic of a homogeneous system: The magnetization decrease exponentially, the entanglement entropy increase linearly, and the width of the wave-packet also increase linearly. If the quench ends in the localized phase ($h > 1$) the entanglement entropy and the magnetization both remain finite. If the quench ends at the critical point ($h = 1$), the local magnetization decrease with a stretched exponential $m_{\mathrm{b}}(t) \sim A(t)\exp(-Ct^{\mu})$ where $\mu \approx 0.43(5)$, the entanglement entropy grows with power function of the time $S(t) \sim t\sigma$ with $\sigma \approx 0.47(5)$. The wave packet spreads as $d(t) \sim t^{0.477}$ [59]. The exponents of the entropy, magnetization and wave-packet width are close to each other, which can be understood with the quasi classical interpretation.

2.4 Nearly adiabatic dynamics in the Harper model

In this section I summarize the methods about the nearly adiabatic dynamic of the Harper model. I used two protocol types: during the first the parameter h starts from $-\infty$, varies as $h = t/\tau$ and goes to ∞ as $t \to \infty$. In the second protocol is similar, but it ends at $t = 0$ when the parameter h is zero. In the first protocol the system cross the phase transition twice, in the second protocol the system cross the phase transition only once. When the system cross the phase transition excitations (also called defects) are created. The standard Kibble-Zurek scaling [40] predict, that the density of the defects behaves as $P \sim 1/\tau^{1/2}$. My numerical findings are compatible with a sightly different value $P \sim 1/\tau^{0.45}$. I modified the standard Kibble-Zurek scaling, and created a scaling theory specially for the Harper model which, is in correspondence with the numerical results.

2.5 Non-equilibrium dynamics of the disordered Ising model

I this section I investigate the dynamics of the disordered transverse Ising model after a quench, when the transverse field is changed instantaneously from h_0 to h. There are results about the dynamics of the entanglement entropy in disordered systems after a quench [48–51, 66]. The entanglement entropy of the disordered Ising chain shows an ultra slow dynamics [49]:

$$\mathcal{S}(t) \sim a\ln\ln t \, , \tag{2.17}$$

The asymptotic value is found to be:

$$\mathcal{S}(\ell) \sim b\ln\ell \, , \tag{2.18}$$

where ℓ is the length of the investigated subsystem. These results were explained with a modified version of the strong disorder renormalization-group (SDRG). The SDRG method describe the equilibrium properties of disordered systems [60]. This method has been generalized to excited states [67–69], and the generalized version is often referred as RSRG-X [67].

The RSRG-X prediction of the fraction of the prefactors in equation (2.17) and equation (2.18) is $b/a = \psi_{\mathrm{ne}}$, where $\psi_{\mathrm{ne}} = 1/2$. The relation between the non-equilibrium time and length scale is:

$$\ln t \sim L^{\psi_{\mathrm{ne}}} \ . \tag{2.19}$$

The investigated model (disordered transverse Ising chain) is defined by the following Hamiltonian:

$$\mathcal{H} = -\sum_{i=1}^{L-1} J_i \sigma_i^x \sigma_{i+1}^x - \sum_{i=1}^{L} h_i \sigma_i^z \ , \tag{2.20}$$

where J_i chosen from the interval $[0, 1]$ and; h_i is chosen from $[0, h]$ according to uniform distribution.
I used two types of initial states: One is totally ferromagnetic initial state with $h \to 0$, the other is totally paramagnetic initial state which corresponds to $J_i = 0$. If the quench starts from the totally ferromagnetic phase ($h_0 = 0$), and ends in the ferromagnetic phase ($0 < h < 1$), the magnetization remains finite.

If the system starts from the totally paramagnetic phase ($J_i = 0$). And ends at the critical point the magnetization *increase* and reach an asymptotic value. The asymptotic average value of the magnetization follows a power law of the system length: $[m_p]_{av}(L) \sim L^{-b'}$, $\quad b' = 1.4$, quasi-long-range order are present. If the quench starts from the totally ferromagnetic phase and ends at the critical point, the magnetization decrease very slowly:

$$m(t) \sim (\log(t))^{-a} \ , \tag{2.21}$$

where $a \approx 0.14$. After the decreasing period the magnetization reach an asymptotic value which has the following finite-size dependence:

$$m_p(L) \sim L^{-b} \ , \tag{2.22}$$

where $b \approx 0.068(5)$. The fraction of the two exponents is $b/a = 0.48(5) \approx 1/2$ in agreement with the RSRG-X results.

Chapter 3

Thesis statements

In this dissertation I investigated the dynamics of inhomogeneous one dimensional systems with free-fermion techniques. Different types of inhomogeneities were investigated (local defect, two types of quasi periodic modulation, and a disordered system). The main results are the following:

1. For the generalized local defect I checked with large scale numerical simulations the conjuncture about the dynamics of the local magnetization at the defect site. Here the term "generalized" means, that a coupling and the two neighboring magnetic field differs from the critical value. The dynamics is function of the quench parameter κ_i which is a combination of the coupling, and the magnetic fields: $\kappa_i = \frac{J_i}{h_{1i}h_{2i}}$. Here κ_1 describes the pre-quench system, and κ_2 describes the after quench system. The time evolution of the magnetization in a finite system of length L is:

$$m_d(t,L) \sim L^{-x_1} \left[L \sin\left(\pi \frac{t}{L}\right) \right]^{2(x_{12}-x_2)}, \; 0 < t < L \, .$$

   where the exponents are $x_i = \frac{\sqrt{2}}{\pi} \arctan(\frac{1}{\kappa_i})$ and $x_{12} = \sqrt{x_1 x_2}$.
   I found a precise agreement between the analytic formula and the numerical results.

   The results stated here were published in:
   Ferenc Iglói, Gergő Roósz, Loïc Turban *Evolution of the magnetization after a local quench in the critical transverse-field Ising chain* J. Stat. Mech. (2014) P03023

2. For the Fibonacci Ising chain I investigated the dynamics of the local magnetization after a quench numerically with free fermion methods. I found that the magnetization follows a stretched exponential decrease:

$$m_{\mathrm{b}}(t) \sim A(t) \exp\left(-Ct^{\mu}\right) \, .$$

   I also found a dynamical phase transition in the behavior of the magnetization: There is a critical magnetic field h^* if the after quench magnetic field is bigger than h^*, the $A(t)$ prefactor of the magnetization oscillates, if the after quench magnetic field is below h^* the prefactor $A(t)$ remains positive. In a homogeneous system $h^* = 1$, in the Finonacci Ising chain $h^* < 1$, and h^* is power law of the inhomogeneity strength.
   For the entanglement entropy and for the wave-packet with power-law increase were found. The exponents of the magnetization, entropy, and wave packet are close to each other in the non-oscillatory phase. This phenomenon was understood using a quasi-classical reasoning.

   The results stated here were published in:
   Ferenc Iglói, Gergő Roósz, Yu-Cheng Lin *Nonequilibrium quench dynamics in quantum quasicrystals* New J. Phys. 15, 023036 (2013)

3. I investigated the quench dynamics of the Harper model, which model shows a localization-delocalization phase transition. I calculated the after-quench dynamics of the entanglement entropy and the local magnetization. If the quench ends in the localized phase, the dynamics is similar to the dynamics of an homogeneous system: The entanglement entropy grows linearly in time, the magnetization decreases exponentially, the wave packet shows a ballistic spread. If the quench ends in the localized phase, the entropy and the magnetization both remains finite in the long-time limit, and the width of the wave packet remains small, comparable with the static localization length.
   If the quench ends at the critical point, the magnetization decrease with a stretched exponential function $m_{\mathrm{b}}(t) \sim A(t) \exp\left(-Ct^{\mu}\right)$, and the entropy grows with a power law $S \sim t^{\sigma}$ of the time. The spreading of the wave packet follows a power law $d(t) \sim t^{0.477}$ [59]. The exponents μ and σ are close

to the exponent of the wave-packet spread. This fact has been understood using a quasi-classical reasoning.

The results stated here were published in:
G. Roósz., U. Divakaran, H. Rieger, F. Iglói *Non-equilibrium quantum relaxation across a localization-delocalization transition* Phys. Rev. B 90, 184202

4. I investigated a Kibble-Zurek process in the Harper model. The defect density scales with a power of the speed of the process $P \sim 1/\tau^{\kappa}$. The prediction of the standard Kibble-Zurek scaling for the exponent is $\kappa = 1/2$. My numerical data suggest smaller exponent, $\kappa \approx 0.45$. I developed a modified version of the Kibble-Zurek scaling for the Harper model. This modified scaling is in good agreement with the numerical data.

   The results stated here were published in:
   Gergő Roósz., Uma Divakaran, Heiko Rieger, Ferenc Iglói *Non-equilibrium quantum relaxation across a localization-delocalization transition* Phys. Rev. B 90, 184202

5. I investigated the after quench dynamics of the magnetization in the disordered transverse Ising model. I investigated three types of quenches: from a totally ferromagnetic ($h_0 = 0$) state to the ferromagnetic phase ($0 < h < 1$), from the totally ferromagnetic phase ($h_0 = 0$) to the critical point ($h = 1$), and from the totally paramagnetic state ($h_0 \to \infty$) to the critical point ($h = 1$).
   If the quench starts from the totally ferromagnetic state, and ends at the critical point, the magnetization remains constant in the long time limit.
   If the quench starts from the infinitely paramagnetic phase and ends at the critical point, the magnetization of the finite system increases, which is a unique fact of this disordered system. The asymptotic average value of the magnetization follows a power law of the system length: $[m_p]_{av}(L) \sim L^{-b'}$, $b' = 1.4$, quasi-long-range order are present.
   If the quench starts from the totally ferromagnetic phase ($h_0 = 0$) and ends in the critical point, the magnetization shows an extremely slow decrease of the form:

$$m(t) \sim (\log(t))^{-a}$$

   where $a \approx 0.14$. After the decreasing period the magnetization reach an asymptotic value which has the following finite-size dependence:

$$m_p(L) \sim L^{-b} \,, \tag{3.1}$$

   where $b \approx 0.068(5)$. The fraction of the two exponents is $b/a = 0.48(5) \approx 1/2$ in agreement withthe RSRG-X results.

   The results stated here were published in:
   Gergő Roósz, Yu-Cheng Lin, Ferenc Iglói *Critical quench dynamics of random quantum spin chains: Ultra-slow relaxation from initial order and delayed ordering from initial disorder* New J. Phys. 19, 023055 (2017)

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