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Information Content of Projections and Reconstruction of Objects in Discrete Tomography

Summary of the PhD Dissertation

by

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1 Introduction

Tomography is a technique for reconstructing and examining the inner structure of objects from their projections, without the destruction of the objects themselves. It is a widely used tool in various applications, like medical diagnostics, crystallography, non-destructing testing of materials, geology, etc.

In *transmission tomography* [7, 11] a *projection* is taken by exposing the object of study to some penetrating radiation on one side, and measuring the energy of the transmitted beams at different points on the other side, as illustrated in Figure 1. In this way, one can calculate the attenuation of the energy, and deduce the absorption properties of the object on the paths of the beams. If the projections are gathered from enough directions (which might mean hundreds of projections), one can reconstruct the material properties at arbitrary positions of the object.

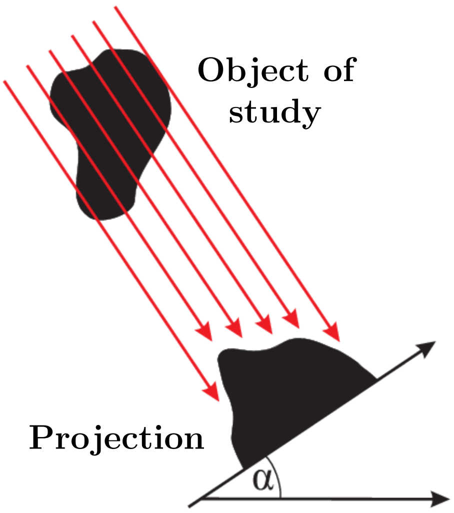

Figure 1: Illustration of a projection of an object.

In *discrete tomography* [8, 9], we assume that the examined object consists of only a few known materials. This extra information can be used to drastically reduce the number of projections required for the reconstructions, and by this to minimize the cost, or unwanted damaging effects of the projection acquisition process. Moreover, in *binary tomography* we assume that the reconstructed image contains only two intensity levels, one for the homogeneous material and another one for the empty space between.

The thesis is a summary of the Author's research in the field of discrete tomography. The central concept of this work was to examine the information content of projections, and try to understand what kind of information the projection data holds, and how this information determines the reconstruction of objects. This analysis of the information content of projections is essential for developing more reliable and robust methods for discrete tomography, and can lead to entirely new approaches to the reconstruction problem.

2 Formulation of the discrete reconstruction problem

We use the algebraic formulation of binary tomography (see, e.g., chapter 7 of [11]), and assume that the object to be reconstructed is represented on a two dimensional discrete image of size $n \times n$. Also, we assume a parallel beam projection geometry where each projection value is

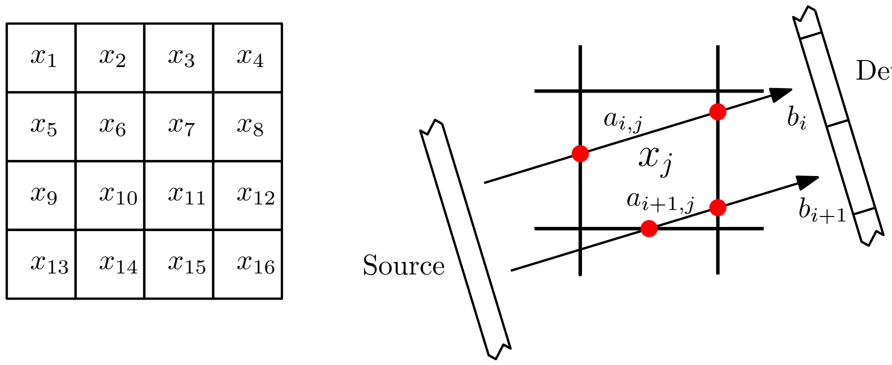

Figure 2: Representation of the ordering of the pixels and the parallel beam geometry.

calculated by the weighted sum of the pixel values of the image intersecting with the projection line. With these assumptions the *discrete reconstruction* problem can be written in a form of a linear equation

$$\mathbf{Ax} = \mathbf{b} \ , \qquad \mathbf{x} \in \Phi^{n^2} \ , \tag{1}$$

where

- $\mathbf{x}$ is the vector of all n^2 unknown image pixels,
- $\mathbf{b}$ is the vector of all m measured projection values,
- $\mathbf{A}$ is a projection coefficient matrix of size $m \times n^2$, that describes the projection geometry by all a_{ij} elements representing the length of the intersection of the i-th projection line through the j-th pixel,
- and $\Phi = \{\phi_0, \phi_1, \ldots, \phi_c\}$ is the set of the possible intensities (assuming that $\phi_0 < \phi_1 < \ldots < \phi_c$),

as illustrated in Figure 2. With the above formulation, one can acquire a solution to the reconstruction problem by solving (1).

3 Direction-dependency in discrete tomography

With discrete tomography, one can reconstruct the inner structure of an image even from only few (say, up to 2-10) projections. In this case, the low number of projections provides a big variety for choosing their directions (see, Figure 3 for an illustration).

In a previous work [15], the authors briefly showed, that this freedom of the choice of projections can influence the accuracy of the reconstruction. We gave an extension of the former experiments for the further investigation of the *direction-dependency* of reconstructions. The aim was to determine if a better choice of projections can significantly improve the quality of the reconstructed results, and if there are regularities which make this phenomenon predictable.

Such studies are motivated by practical problems. In many applications of discrete and binary tomography, there are limitations on the number of available projections, because the projection acquisition process can have a high cost, or can damage the object of study. Therefore, we could benefit from reducing the number of required projections, or increasing the accuracy of the reconstructions by only improving the projection acquisition process by proper projection selection strategies.

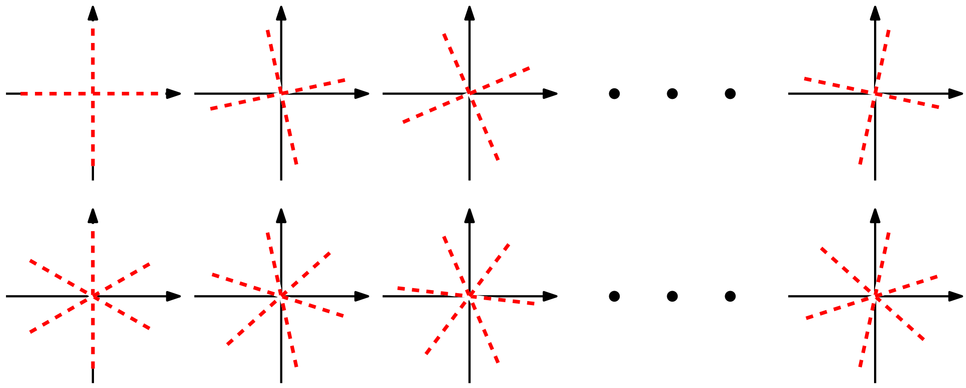

Figure 3: Some possible projection directions with low projection numbers. Dashed lines indicate the directions of the projection beams.

3.1 Experimental studies

We implemented a test frameset in which it was possible to examine the direction-dependency phenomenon in the binary case of discrete tomography. In the experiments we produced projection sets of 22 binary phantom images from several different direction sets, and reconstructed the phantoms from the simulated projection data. Then, we compared the results of the same phantom, produced from projection sets having the same projection counts, but different projection directions. In this way, we could determine if the choice of projection directions influenced the accuracy of the reconstructed results.

3.1.1 Direction-dependency with equiangular projection sets

First of all, reconstructions were performed from equiangular projection sets, in which case the angles of the projections were determined by a p projection count and an α_0 starting angle as

$$S(p, \alpha_0) = \left\{ \alpha_0 + i\frac{180^\circ}{p} \;\middle|\; i = 0, \ldots, p-1 \right\} . \tag{2}$$

We reconstructed the phantoms from different $S(p, \alpha_0)$ projection sets with p ranging between 2 and 18, and integer α_0 starting angles from 0° to $\left\lceil \frac{180^\circ}{p} - 1^\circ \right\rceil$. These experiments simulated the rotation of an examined object in a scanner.

We also designed a direction-dependency measurement for the equiangular case, with which we could find extreme cases of the direction-dependency, i.e., when an object could be perfectly reconstructed from a certain number of projections, if the projection angles were well chosen, but one could also get highly inaccurate results from the same amount of projections taken from other directions. This direction-dependency measurement has the form

$$D^{\sigma}_{Alg}(\mathbf{x}^*, p) = \left(E^{max}_{Alg}(\mathbf{x}^*, p) - E^{min}_{Alg}(\mathbf{x}^*, p)\right) \cdot \exp\left(-\frac{(E^{min}_{Alg}(\mathbf{x}^*, p))^2}{\sigma^2}\right) , \tag{3}$$

where $E^{min}_{Alg}(\mathbf{x}^*, p)$ and $E^{max}_{Alg}(\mathbf{x}^*, p)$ are, respectively, the measured error (expressed as Relative Mean Error [10]) of the best and worst reconstruction of the $\mathbf{x}^*$ image with a given p projection count, but varying α_0 starting angle, when the reconstruction is performed by a chosen Alg reconstruction algorithm.

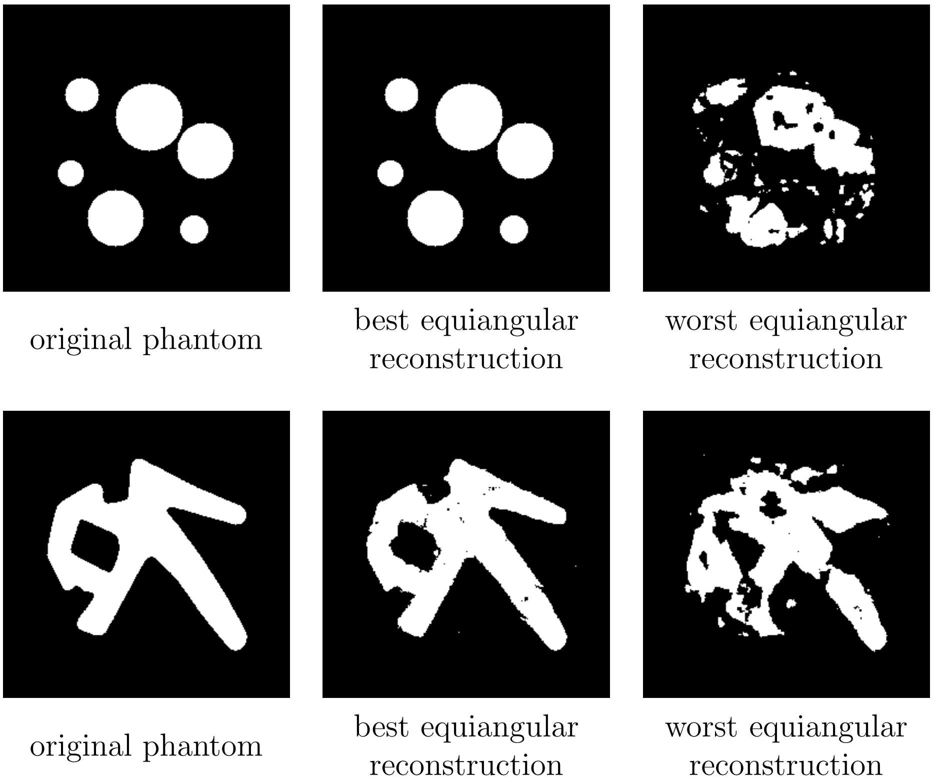

Figure 4: Example reconstructions of phantoms with the same number of four projections but different projection directions.

We found that the reconstructed results can highly depend on the choice of projections, and one can get entirely different results only by changing the orientation of the object in the scanner. Two examples for the direction-dependency can be seen in Figure 4.

3.1.2 Direction-dependency with non-equiangular projection sets

After the equiangular studies, we examined if further improvement of the accuracy could be reached by allowing arbitrary projection angles. We defined two non-equiangular projection selection strategies for improving the angles.

One such strategy was a greedy projection insertion. This method starts from an empty set of projections. Then, it iteratively tests a set of candidate projections, and adds the one to the current projection set, that yields the biggest improvement in the reconstruction.

Another non-equiangular strategy was based on simulated annealing [14], that is a randomized optimization meta-heuristic. In this approach, we started from the predefined $S(p, 0°)$ equiangular projection set for each image and projection number, and minimized the error of the reconstructed image by altering the projection angles with simulated annealing.

Based on the experiments, we found that further improvement of the results is achievable by not only optimizing the orientation of the object, but choosing each projection direction individually. An example is shown in Figure 5.

3.1.3 Direction-dependency with changing conditions of the reconstructions

We also studied the direction-dependency of objects under different conditions, by reconstructing the image with different algorithms, and by investigating the effects of projection noise, and

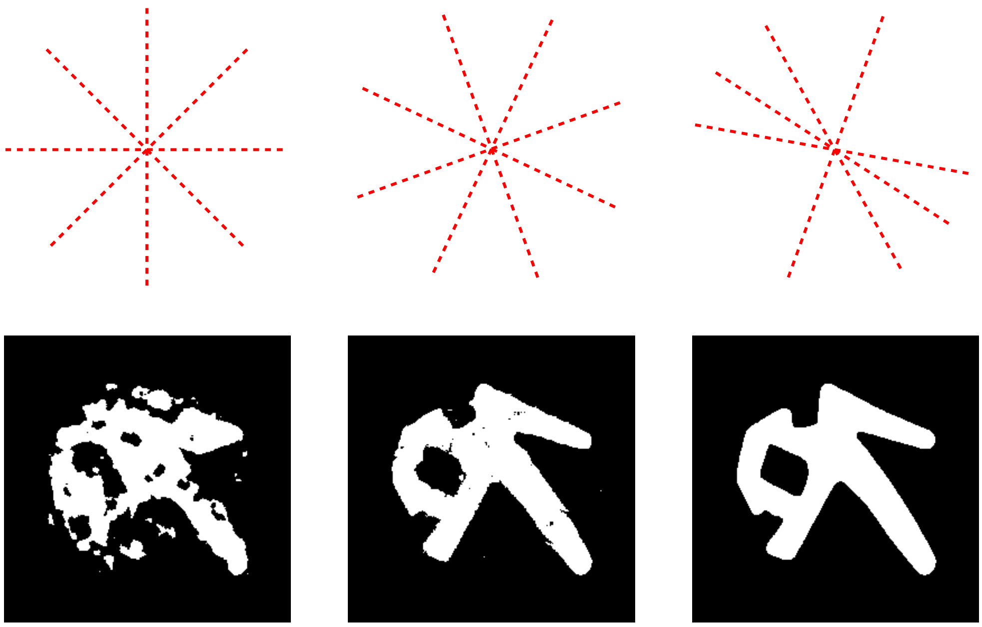

Figure 5: Reconstructions of a phantom with $S(4, 0°)$, $S(4, 19°)$, and $S = \{29°, 57°, 80°, 160°\}$ projection sets, from left to right, respectively. Dashed lines indicate the directions of the projections, images below are the corresponding reconstructions.

the small modifications of the object. The aim was to find out if the direction-dependency characteristics of an object is consistent under different conditions, i.e., if the optimal projection angles are not affected by the parameters of the reconstruction, and the distortions of the data. If it is so, then the direction-dependency phenomenon can be explained by the information content of the projections themselves, and it might be exploited in practical applications.

First, we examined the usage of different reconstruction algorithms. This, again, was done by reconstructing the phantom images from various sets of projections with three reconstruction algorithms, a thresholded Simultaneous Iterative Reconstruction Technique (SIRT – see Chapter 7 of [11]), the Discrete Algebraic Reconstruction Technique (DART [2]) and a reconstruction technique based on energy minimization with D.C. programming (that will be referred to as DC [24]). We compared the accuracy of the outputs of the different methods to see if the projection sets producing better results with one algorithm also lead to better results with another reconstruction technique. We found that the basic tendencies were the same with all three reconstruction algorithms, and the same projection directions lead to optimal results, regardless of the applied reconstruction technique. This indicates that the direction-dependency is a property of the objects themselves, and originates from the different information content of the different projections, which is independent from the reconstruction algorithm used.

We also examined the effects of noise on the projection data. We distorted the projection values with additive Gaussian noise of different magnitude. According to the results, the noise did not change the relation between the accuracies of the reconstructions of an object gained from different projection sets. The same projections gave optimal results regardless of the strength of the noise on the data.

Finally, we performed experiments to see if slight modifications of the objects of study change the direction-dependency of the objects. We constructed slightly altered versions of the phantoms simulating fractures and bubbles in the material. We found that such small changes of the object did not change the accuracy of the reconstructions and the same directions proved to be optimal as for the original, distortion free phantoms.

As a summary, we can say that the reconstructions of objects might significantly be improved only by finding the optimal directions for the projections. Moreover, this phenomenon originates from the information content of the projections themselves, and it is independent from the applied reconstruction algorithm. The angles of the optimal projections remain consistent if the projection data is corrupted by measurement error, or in case the object slightly changes. These regularities of the direction-dependency makes it exploitable in real applications.

The results of this thesis group were published in two conference proceedings [16, 17] and two journals [18, 19]. Up to the date of the release of the dissertation, there were three independent citations to the results [4, 6, 13].

4 Energy minimization reconstruction algorithm for multivalued discrete tomography

Discrete reconstruction algorithms have to cope with various difficulties of tomography. The reconstruction problem commonly requires the restoration of the structure of an object from incomplete projection data, possibly affected by errors coming from the discrete formulations of the reconstruction problem and stochastic noise affecting the projection acquisition process. Also, the general case of discrete tomography is proved to be NP-hard, and efficient algorithms providing perfect results can only be defined for some special cases [3, 5]. There is a variety of different approaches to overcome these problems. Some methods provide heuristic strategies for discretizing the results of continuous reconstructions (see, e.g., [2]), some other techniques reformulate the task as an optimization problem that can be solved by different meta-heuristics [1, 12].

We describe a reconstruction algorithm, that we developed for the general case of DT, and that can perform the reconstruction by minimizing a suitably constructed energy function. The basic idea behind this method is based on the DC algorithm [24]. Unfortunately, the DC method is only capable of reconstructing binary images, and the aim was to provide a more general algorithm for multivalued discrete tomography, that is not restricted to the binary case, and can compete with the current cutting-edge reconstruction algorithms in the literature.

4.1 The energy function of the algorithm

The energy function is designed to take its minima at solutions of the discrete reconstruction problem. It can be given in the form

$$\mathcal{E}_\mu(\mathbf{x}) := f(\mathbf{x}) + \mu \cdot g(\mathbf{x}) \ , \quad \mathbf{x} \in [\phi_0, \phi_c]^{n^2} \quad , \tag{4}$$

where $f(\mathbf{x})$ is a function formalising a continuous reconstruction problem, and $\mu \cdot g(\mathbf{x})$ is a term formulating the discreteness prior. In more detail, the first term is

$$f(\mathbf{x}) = \frac{1}{2} \cdot \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + \frac{\alpha}{2}\mathbf{x}^{\mathrm{T}}\mathbf{L}\mathbf{x} \tag{5}$$

where the $\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2$ is a projection correctness term, that measures the difference between the expected projections and the projections of the $\mathbf{x}$ solution. It also contains the $\frac{\alpha}{2}\mathbf{x}^{\mathrm{T}}\mathbf{L}\mathbf{x}$ smoothness prior, where $\mathbf{L}$ is a matrix such that

$$\mathbf{x}^{\mathrm{T}}\mathbf{L}\mathbf{x} = \sum_{i=1}^{n^2} \sum_{j \in N_4(i)} (x_i - x_j)^2 \tag{6}$$

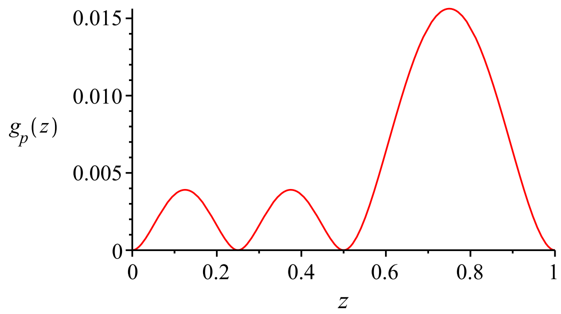

Figure 6: Example of the $g_p(z)$ one-variable discretization function with intensity values $\Phi = \{0, 0.25, 0.5, 1\}$.

and $N_4(i)$ denotes the set of pixel indexes 4-adjacent to the i-th pixel. By this term, one can use an additional smoothness prior with an α weight that will prefer final results with large homogeneous areas on them.

Furthermore, the $g(\mathbf{x})$ discretizing term of the energy function of (4) is written as the sum of one-variable discreteness terms on the pixels in the reconstructed image

$$g(\mathbf{x}) = \sum_{i=1}^{n^2} g_p(x_i) \ , \quad i \in \{1, 2, \ldots, n^2\} \ , \tag{7}$$

where each g_p function is given as the composition of fourth grade polynomials over the intervals of the Φ set of expected intensities as

$$g_p(z) = \begin{cases} \frac{[(z-\phi_{j-1})\cdot(z-\phi_j)]^2}{2\cdot(\phi_j-\phi_{j-1})^2}, & \text{if } z \in [\phi_{j-1}, \phi_j] \text{ for each } j \in \{1, \ldots, c\}, \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

The g_p function is illustrated in Figure 6. This discretization function assigns a small value to each pixel if its intensity in the reconstruction is close to an element of Φ and higher values (increasing with the distance) otherwise.

4.2 The optimization process

We also defined a new optimization process for minimizing the $\mathcal{E}_\mu(\mathbf{x})$ energy function. The basic idea is to make a distinction between the parts of the energy function and to assume that the projection correctness has a higher priority.

On this base, the algorithm optimizes the energy function with a simple projected subgradient method, while applying an automatic weighting between the two terms of the energy function. In each iteration step of the optimization process, one can calculate the gradient of the $\|\mathbf{Ax} - \mathbf{b}\|_2^2$ projection correctness term in the energy function, by computing the $\mathbf{A}^{\mathrm{T}}(\mathbf{Ax} - \mathbf{b})$ vector. For each pixel, this vector explicitly contains an estimation of its correctness in the current solution, according to the projections. If the gradient on a pixel is big in absolute value, then that pixel lies on incorrect projection beams, and its value should be modified. The magnitude of the gradient will correspond to the correctness of the related projections. If we apply a Gaussian function on these values we can get a weight, that is lower when the corresponding pixel needs further adjustments, and higher if the projection rays hitting that specific pixel are more or less satisfied. By weighting the discretization with this value calculated from the gradient of the projection correctness, one can apply an automatic adjustment of the discretizing term for each pixel, omitting it when the projections are not satisfied, and slowly increasing its effect as the pixel values are getting closer and closer to an acceptable reconstruction.

In practice, this means that the method starts with an arbitrary initial solution, and first approximates a continuous reconstruction based on the given set of projections. Later, when the projections of the intermediate reconstruction are approaching the required ones, the weight of the discretizing term begins to increase for each pixel, and the pixels will be slowly steered towards discrete values of Φ. The formal description of the method is given in Algorithm 1. The algorithm uses the $\mathcal{T}_\Phi(\mathbf{x})$ notation for the thresholding of the $\mathbf{x}$ vector into the Φ set of intensities, defined as

$$(\mathcal{T}_\Phi(\mathbf{x}))_i = \begin{cases} \phi_0, & \text{if } x_i < (\phi_0 + \phi_1)/2 \ , \\ \phi_j, & \text{if } (\phi_{j-1} + \phi_j)/2 \leq x_i < (\phi_j + \phi_{j+1})/2 \ , \quad j \in \{2, \dots, c-1\} \\ \phi_c, & \text{if } (\phi_{c-1} + \phi_c)/2 \leq x_i \ . \end{cases} \tag{8}$$

Algorithm 1 Energy-Minimization Algorithm for Multivalued DT

Input: $\mathbf{A}$ projection matrix; $\mathbf{b}$ expected projection values; $\mathbf{x}^{(0)}$ initial state; $\gamma, \mu, \sigma \geq 0$ predefined constants; Φ list of expected intensities; ϵ step size bound; k_{max} maximal iteration count.

$\lambda \leftarrow$ an upper bound for the largest eigenvalue of the $(\mathbf{A}^\mathrm{T}\mathbf{A} + \alpha \cdot \mathbf{L})$ matrix.
$k \leftarrow 0$
repeat
  $\mathbf{v} \leftarrow \mathbf{A}^\mathrm{T}(\mathbf{A}\mathbf{x}^{(k)} - \mathbf{b})$
  $\mathbf{w} \leftarrow \mathbf{L}\mathbf{x}^{(k)}$
  for each $i \in \{1, 2, \dots, n^2\}$ **do**
    $y_i^{(k+1)} \leftarrow x_i^{(k)} - \frac{v_i + \gamma \cdot w_i + \mu \cdot G_{0,\sigma}(v_i) \cdot \frac{\partial g}{\partial x_i}\big|_{\mathbf{x}=\mathbf{x}^{(k)}}}{\lambda + \mu}$
    $x_i^{(k+1)} \leftarrow \begin{cases} \theta_0, & \text{if } y_i^{(k+1)} < \phi_0, \\ y_i^{(k+1)}, & \text{if } \phi_0 \leq y_i^{(k+1)} \leq \phi_c, \\ \theta_c, & \text{if } \phi_c < y_i^{(k+1)}. \end{cases}$
  end for
  $k \leftarrow k + 1$
until $\|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\|_2^2 < \epsilon$ or $k > k_{max}$
Apply a thresholding and output $\mathcal{T}_\Phi(\mathbf{x}^{(k)})$ to gain fully discrete results.

4.3 Results

We also validated the performance of the proposed reconstruction algorithm by comparing it to other techniques from the literature. We performed experimental tests on a set of software phantoms by reconstructing them with the proposed method, and also, two other techniques, the DART [2] and DC [24] algorithms, mentioned above.

We found that the proposed method consistently gave the best results when it reconstructed multivalued images (i.e., phantoms containing at least 3 intensity levels) from a low number of projections. In other cases, however, the new method did not always have the best performance, but still, the accuracy of its results were only slightly behind the ones of the other two methods. When using projection data affected by random noise, the proposed method, again, performed better than the other two algorithms, and proved to be highly robust against projection noise. Nevertheless, the time requirement of our method was on the same scale as the other two techniques. With all the above findings, we can say that the proposed method is a vital alternative for the discrete reconstruction of objects.

The results of this thesis point were published in two conference proceedings [20, 21].

5 Local and global uncertainty in binary reconstructions

Although one can perform binary tomographic reconstructions of objects from a low number of projections, limitations on the projection count might be too strict even for binary tomography. We developed a method, to measure the information content of the projections, and discover the local uncertainties of binary reconstructions, i.e., to what degree the projection data determines each pixel of the reconstruction.

5.1 Measuring local uncertainties in reconstructions

The uncertainty in the incomplete and insufficient projection data puts the binary reconstruction task into a probabilistic context. In this case, the projection data usually do not determine the results entirely, and there are many possible reconstructions satisfying the projections. On the other hand, we can assume, that only the – yet unknown – original image of the object of study is accepted as a correct solution, that can be regarded as a random element of the set of reconstructions with the correct projections. In this way, each binary point of $\mathbf{x} \in \{0,1\}^{n^2}$ (i.e., each binary reconstruction) can be assigned a

$$P(\mathbf{x} \,|\, \mathbf{A}, \mathbf{b}) \tag{9}$$

probability of being correct. Naturally, the more a reconstruction satisfies the $\mathbf{b}$ projections the higher its probability should be. From the probabilities of the reconstructions, we can also calculate for each pixel the probability that the pixel takes a value of 1 by

$$P(x_i = 1 \,|\, \mathbf{A}, \mathbf{b}) = \sum_{\substack{\mathbf{y} \in \{0,1\}^{n2} \\ y_i = 1}} P(\mathbf{y} \,|\, \mathbf{A}, \mathbf{b}) \,. \tag{10}$$

Similarly, $P(x_i = 0 \,|\, \mathbf{A}, \mathbf{b}) = 1 - P(x_i = 1 \,|\, \mathbf{A}, \mathbf{b})$ can also be calculated. As a further step, one can calculate the entropy

$$\begin{aligned} H(x_i) = &- P(x_i = 0 \,|\, \mathbf{A}, \mathbf{b}) \cdot \log_2(P(x_i = 0 \,|\, \mathbf{A}, \mathbf{b})) - \\ &- P(x_i = 1 \,|\, \mathbf{A}, \mathbf{b}) \cdot \log_2(P(x_i = 1 \,|\, \mathbf{A}, \mathbf{b})) \,, \end{aligned} \tag{11}$$

and describe the uncertainty of each pixel in the reconstruction. This value is based only on the parameters of the projections and the information content of the measured projection data. Pixels with high entropy are ambiguous, and one cannot hope to get a reliable reconstruction of them without additional information. The projections simply do not hold enough data, to determine these pixels. Although the model is clear, the direct measurement of the probabilities is unfortunately impossible to carry out, due to the exponential number of possible reconstructions. On the other hand, these values can be approximated by using heuristic methods.

In binary tomography, we are looking for results satisfying the given projections which are taken from the binary domain. If we try to find a reconstruction that has the correct projections,

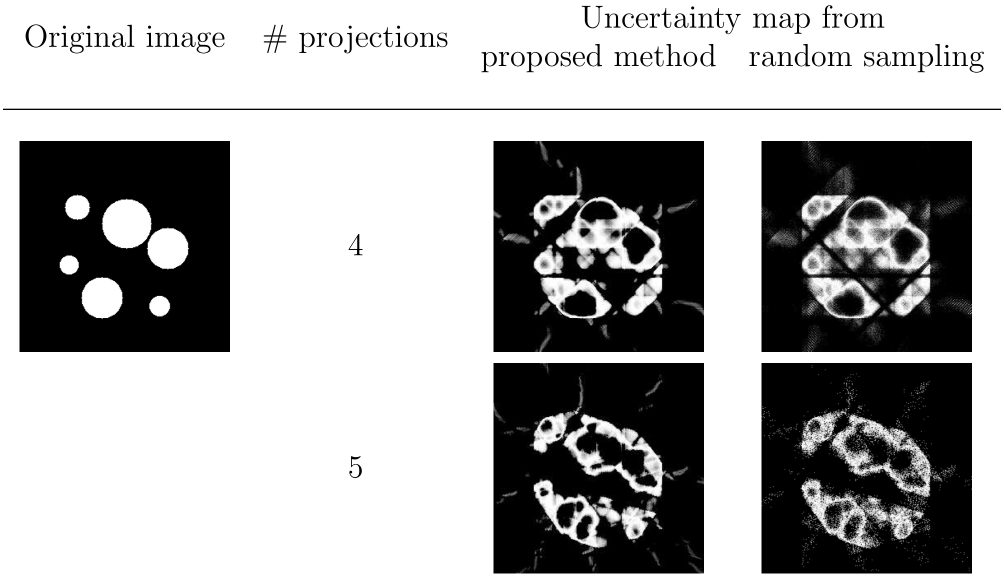

Figure 7: Uncertainty maps of projection sets. Dark areas are determined by the projections, while brighter areas are not, and hold uncertainty.

but in which the pixel values are the farthest away from the $\{0, 1\}$ set, we can measure how easy it is to change the pixel values. If the value of a pixel is easy to be pushed towards 0.5, then that pixel is evenly likely to take 0 or 1 in the final result. With this concept, we can measure the variability of pixels from only the projections, even if we have no prior knowledge of the original object.

By the framework of Algorithm 1 this can be done by setting a discreteness prior that discourages binary results. One such prior can be given by the function

$$g(\mathbf{x}) = \frac{1}{2} \cdot \left\| \mathbf{x} - \frac{1}{2} \cdot \mathbf{e} \right\|_2^2 , \tag{12}$$

where $\mathbf{e}$ stands for a vector with all n^2 positions having a value of 1. When the resulting $\mathbf{x}$ is computed, calculating the entropy

$$H(x_i) = -(x_i \cdot \log_2(x_i) + (1 - x_i) \cdot \log_2(1 - x_i)) \tag{13}$$

for each pixel value would approximate the values of the uncertainty map given in (11).

For validating this method, we performed an experimental test on a set of software phantoms. We took a set of 22 binary phantom images, produced their projection sets, and performed a random sampling on the space of possible reconstructions. This provided a statistics on the probability on the pixel intensities in the reconstructions, and also the uncertainty of the pixels. This statistical data was then compared to our proposed local uncertainty measure. An example of the outputs of the two methods can be seen in Figure 7. The results indicated that the local uncertainty measure, indeed, describes the uncertainty of the pixels.

5.2 Global uncertainty in binary tomography

The local uncertainties can also be summarized into a global measurement that describes the information content of the whole projection set. This can be done by the formula

$$\mathcal{U}(\mathbf{x}) = \frac{\sum_{i=1}^{n} H(x_i)}{\frac{1}{p}\sum_{j=1}^{m} b_j} \ . \tag{14}$$

This measurement sums up the $H(x_i)$ pixel uncertainties in the local uncertainty measurement, and normalizes this sum with the approximated number of non-zero values on the original image. In the ideal case, the sum of the projection values for each projection would contain the number of ones of the original image to reconstruct. In a practical application, however, the measured data can hold rounding errors, therefore we normalized with the average value of the cumulated projection values of the projections.

We validated the global uncertainty measure empirically by reconstructing a set of software phantoms from their different projection sets, and compared the results with the proposed measurement. If the global uncertainty measurement is correct, then projection sets with smaller uncertainty should lead to less error when an actual reconstruction is performed, and a connection would be visible between the uncertainty of a projection set and the actual reconstruction. The results indicated that, indeed, this is the case, and the global uncertainty measurement describes the overall information content of the projections.

5.3 Applications and results

We also proposed different possible applications of the local and global uncertainty measurements. First of all, the local measurement can be used to aid reconstruction algorithms like the DART, to highlight the problematic parts of the objects. The process of the DART is based on deforming and fine-tuning the boundary of an initial reconstruction. Coming from the nature of the process, this method has difficulties in finding small formations (i.e., bubbles in the material) if they are far from an edge of the initial solution. We designed a modified version of the DART that uses the uncertainty maps for finding problematic areas of the reconstructed image, and rules out the weakness of the original DART.

Furthermore, the global uncertainty measure can be used to grade the overall quality of the reconstruction, and the local uncertainty can highlight the possible falsely reconstructed parts of the results. This can be used to detect false reconstructions and, by this, to aid a decision process based on the binary reconstructions. On the other hand, the global uncertainty describes the information content and the reliability of the projection set, therefore it can be used for finding ideal projection angles in the non-destructive testing of objects, when a blueprint of the object of study is available (i.e., finding projections which lead to less uncertainty).

The results of this thesis group are published in one conference proceedings [22]. A further submitted journal paper [23] is currently under review.

Summary of the author's contributions

The findings of the research can be divided into three thesis groups. Table 1 gives the connection between the results and the publications of the Author.

In the first thesis group, I examined the direction-dependency in binary tomographic reconstructions. The results were published in two conference proceedings [16, 17], and two journal papers [18, 19].

I/1. I designed an experimental test environment for examining the correspondence between the quality of a binary reconstruction and the choice of projection directions. I showed, that the quality of binary reconstructions strongly rely on the choice of directions used in the projection acquisition. I found that some projection sets have higher information content leading to better reconstructions.

I/2. I gave various projection selection strategies for improving the projection angles, and thus to enhance the accuracy of the reconstruction, when a blueprint of the object is available.

I/3. I showed that the direction-dependency phenomenon is caused by the different information content of the different projections and it is only slightly influenced by the choice of the reconstruction algorithm, or the distortions of the data. Based on this observation, I concluded that the direction-dependency is a predictable phenomenon that can be exploited in the non-destructive testing of objects.

In the second thesis group, I gave a new reconstruction algorithm for the general case of discrete tomography. The results were published in two conference proceedings [20, 21].

II/1. I developed a new reconstruction algorithm for the general case of discrete tomography. This algorithm reformulates the reconstruction problem into the minimization of an energy function. I defined a function that has its minima corresponding to accurate discrete reconstructions, and designed a novel optimization scheme for minimizing this energy function. I validated the performance of the proposed method in experimental tests and compared it to other state-of-the-art reconstruction algorithms. I found that the proposed method could compete with other algorithms in both the speed of the computation, and the accuracy of the results. Moreover, I found that it is highly robust when the projection data is affected by random noise.

In the third thesis group, I gave measures for the local and global uncertainty in binary reconstructions. The results were in part published in a conference proceedings [22], and are submitted as a journal paper [23].

III/1. I provided a probability based formulation of the uncertainty of pixels in binary reconstructions, i.e., which ascertains how the projection data determines each pixel of a reconstructed image. I gave a method that is capable of approximating the local uncertainty maps of the reconstructions on an acceptable level, and validated the results in experimental studies.

III/2. I gave a formula that can summarize the local uncertainties into a global measure, that describes the overall uncertainty in the projections and predicts the expected error of a reconstruction.

III/3. I also described how the uncertainty measure can be used to improve the performance of the DART reconstruction algorithm, and to aid the examination processes using binary tomography.

	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
I/1.	•							
I/2.		•						
I/3.			•	•				
II/1.					•	•		
III/1.							•	•
III/2.								•
III/3.								•

Table 1: The connection between the thesis points and the Author's publications

References

[1] P. Balázs, M. Gara, *An evolutionary approach for object-based image reconstruction using learnt priors*, Lecture Notes in Computer Science, 5575, 520–529 (2009).

[2] K.J. Batenburg, J. Sijbers, *DART: a fast heuristic algebraic reconstruction algorithm for discrete tomography*, IEEE Conference on Image Processing IV, 133–136 (2007).

[3] S. Brunetti, A. Del Lungo, F. Del Ristoro, A Kuba, M. Nivat, *Reconstruction of 4- and 8-connected convex discrete sets from row and column projections*, Lin. Alg. Appl. 339, 37–57 (2001).

[4] J. Chen, A. Gao, C. Zhang, *An efficient algorithm for reconstruction of discrete sets from horizontal projections with absorption*, Communications in Computer and Information Science CCIS 307:(PART 1), 762–772 (2012).

[5] A. Daurat, *Determination of q-convex sets by X-rays*, Theoretical Computer Science, 332(1-3), 19–45 (2005).

[6] A. Frost, E. Renners, M. Hotter, J. Ostermann, *Probabilistic evaluation of three-dimensional reconstructions from X-ray images spanning a limited angle*, SENSORS 13:(1), 137–151 (2013).

[7] G.T. Herman, *Fundamentals of Computerized Tomography, Image Reconstruction from Projections*, 2nd edition, Springer-Verlag, London, 2009.

[8] G.T. Herman, A. Kuba (Eds.), *Discrete Tomography: Foundations, Algorithms and Applications*, Birkhäuser, Boston, 1999.

[9] G.T. Herman, A. Kuba (Eds.), *Advances in Discrete Tomography and Its Applications*, Birkhäuser, Boston, 2007.

[10] A. Kuba, G.T. Herman, S. Matej, A. Todd-Pokropek: *Medical applications of discrete tomography*, Discrete Mathematical Problems with Medical Applications, DIMACS Series in Discrete Mathematics and Theoretical Computer Science, AMS, 55, 195–208 (2000).

[11] A.C. Kak, M. Slaney, *Principles of Computerized Tomographic Imaging*, IEEE Press, New York, 1999.

[12] T. Lukić, *Discrete tomography reconstruction based on the multi-well potential*, Lecture Notes in Computer Science 6636, 335–345 (2011).

[13] T. Lukić, B. Nagy, *Energy-minimization based discrete tomography reconstruction method for images on triangular grid*, Lecture Notes in Computer Science vol. 7655, 274–284 (2012).

[14] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller, E. Teller, *Equation of state calculation by fast computing machines*, J. Chem. Phys. 21, 1087–1092 (1953).

[15] A. Nagy, A. Kuba, *Reconstruction of binary matrices from fan-beam projections*, Acta Cybernetica, 17(2), 359–385 (2005).

[16] L. Varga, P. Balázs, A. Nagy, *Direction-dependency of a binary tomographic reconstruction algorithm*, Lecture Notes in Computer Science 6026, 242–253 (2010).

[17] L. Varga, P. Balázs, A. Nagy, *Projection selection algorithms for discrete tomography*, Lecture Notes in Computer Science 6474, 390–401 (2010).

[18] L. Varga, P. Balázs, A. Nagy, *Projection selection dependency in binary tomography*, Acta Cybernetica, 20, 167–187 (2011).

[19] L. Varga, P. Balázs, A. Nagy, *Direction-dependency of binary tomographic reconstruction algorithms*, Graphical Models 73(6), 365–375 (2011).

[20] L. Varga, P. Balázs, A. Nagy, *An energy minimization reconstruction algorithm for multivalued discrete tomography*, 3rd International Symposium on Computational Modelling of Objects Represented in Images, Rome, Italy, Proceedings (Taylor & Francis): 179–185 (2012).

[21] L. Varga, P. Balázs, A. Nagy, *Gradiens módszerek automatikus súlyozásán alapuló diszkrét tomográfiai eljárás*, A Képfeldolgozók s Alakfelismerők Társaságának 9. országos konferenciája - KÉPAF 2013, 210–223, (2013).

[22] L. Varga, L.G. Nyúl, A. Nagy, P. Balázs, *Local uncertainty in binary tomographic reconstruction*, Proceedings of the 10th IASTED International Conference on Signal Processing, Pattern Recognition and Applications, IASTED - Acta Press, 490–496 (2013).

[23] L. Varga, L.G. Nyúl, A. Nagy, P. Balázs, *Local and global uncertainty in binary tomographic reconstruction*, Submitted to IEEE Transactions on Image Processing.

[24] S. Weber, *Discrete tomography by convex concave regularization using linear and quadratic optimization*, PhD thesis, Heidelberg University, 2009.